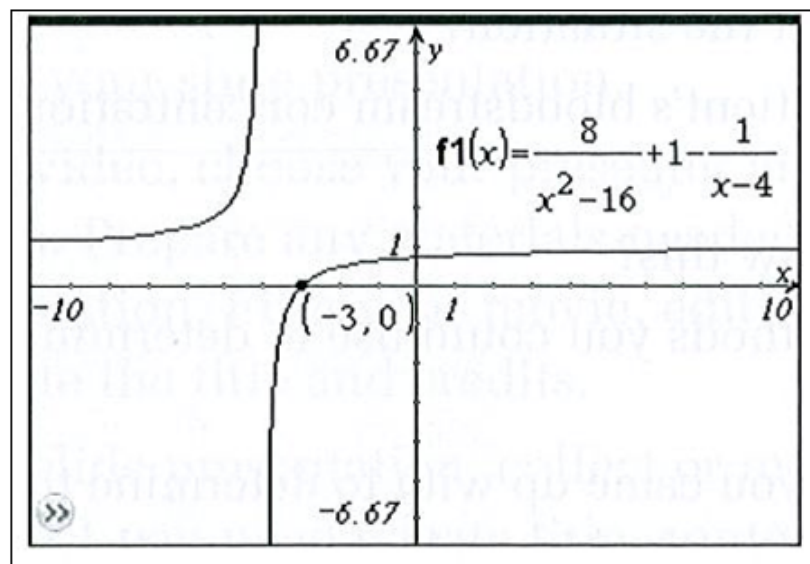


## 9.3 Connecting Graphs and Rational Functions

Just as with many other types of equations, rational equations can be solved algebraically or graphically. Solving rational equations using an algebraic approach will sometimes result in extraneous roots. For example, an algebraic solution to the equation  $\frac{8}{x^2 - 16} + 1 = \frac{1}{x - 4}$  results in  $x$ -values of  $-3$  and  $4$ . Why is  $x = 4$  not a valid solution?



Solving a rational equation graphically involves using technology to graph the corresponding rational function and identify the  $x$ -intercepts of the graph. The  $x$ -intercepts of the graph of the corresponding function give the roots of the equation.

Ex.1 a) Determine the roots of the rational equation  $x + \frac{6}{x+2} - 5 = 0$  algebraically.

$$x \neq -2$$

$$(x+2)x + \frac{6 \cancel{(x+2)}}{\cancel{x+2}} - 5 \stackrel{(x+2)}{=} 0 \quad (x+2)$$

$$x^2 + 2x + 6 - 5x - 10 = 0$$

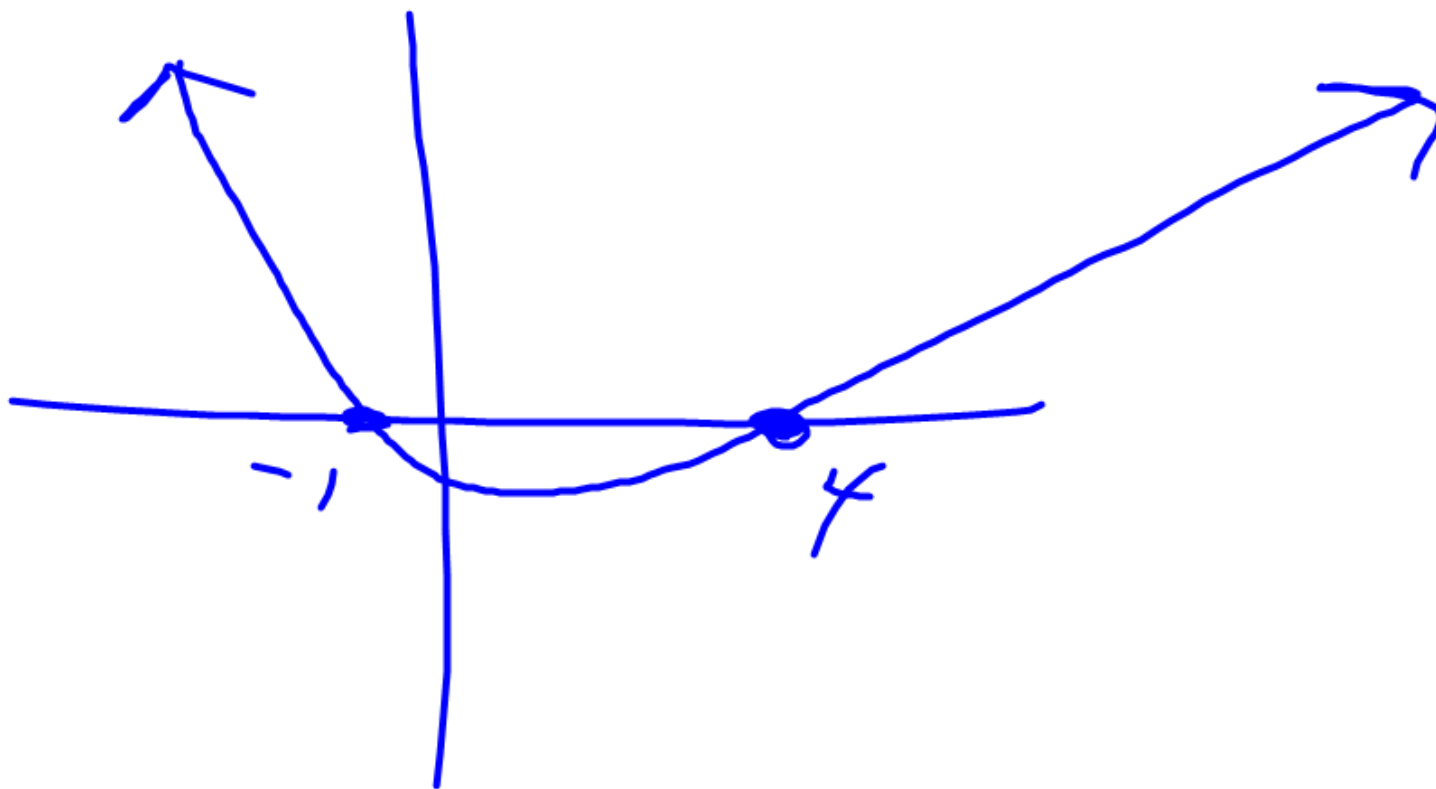
$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4 \text{ OR } x = -1$$

b) Using a graphing calculator, graph the rational function  $y = x + \frac{6}{x+2} - 5$  and determine the x-intercepts.

$$x = -1 \quad \text{or} \quad x = 4$$



c) What is the connection between the roots of the equation and the x-intercepts of the graph of the function?

Roots = x intercepts

## Your Turn

a) Determine the roots of the equation  ~~$\frac{14}{x} - x + 5 = 0$~~  algebraically.

$$x \neq 0$$

$$14 - x^2 + 5x = 0$$

$$0 = x^2 - 5x - 14$$

$$0 = (x - 7)(x + 2)$$

$$x = 7 \text{ OR } x = -2$$

**b)** Determine the  $x$ -intercepts of the graph of the corresponding function

$$y = \frac{14}{x} - x + 5.$$

$$x \neq -5/2$$

### Your Turn

a) Solve the equation  $2 - \frac{3x}{2} = \frac{1 + 4x - x^2}{4x + 10}$  graphically. Express your answer to the nearest hundredth.

LCM

$$2(2x+5) \quad 2(2x+5) \quad 2(2x+5)$$
$$2 - \frac{3x}{2} = \frac{1 + 4x - x^2}{2(2x+5)} \quad \cancel{(2)(2x+5)}$$

$$4(2x+5) - 3x(2x+5) = 1 + 4x - x^2$$

$$8x + 20 - 6x^2 - 15x = 1 + 4x - x^2$$

$$0 = 5x^2 + 11x - 19$$



$$x \neq -5/2$$

### Your Turn

a) Solve the equation  $2 - \frac{3x}{2} = \frac{1 + 4x - x^2}{4x + 10}$  graphically. Express your answer to the nearest hundredth.

LCM

$$2(2x+5) \quad 2(2x+5) \quad 2(2x+5)$$
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$$4(2x+5) - 3x(2x+5) = 1 + 4x - x^2$$

$$8x + 20 - 6x^2 - 15x = 1 + 4x - x^2$$

$$0 = 5x^2 + 11x - 19$$

$$a = 5$$

$$b = 11$$

$$c = -19$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(5)(-19)}}{2(5)}$$

$$x = \frac{-11 \pm \sqrt{501}}{10}$$

$$x = \frac{-11 \pm 22.38}{10}$$

$$x = \frac{-11 + 22.38}{10}$$

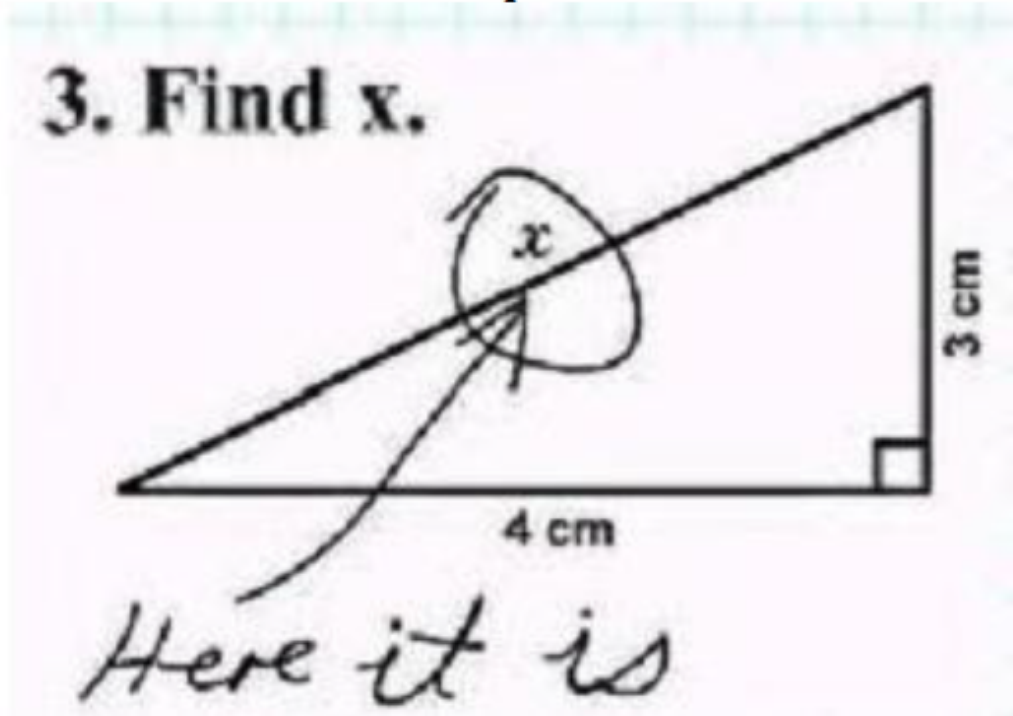
$$x = 1.238$$

$$\text{OR } x = \frac{-11 - 22.38}{10}$$

$$x = -3.338$$

**b)** Verify your solution algebraically.

Do not be the person who does this:



## Key Ideas

- You can solve rational equations algebraically or graphically.
- The solutions or roots of a rational equation are equivalent to the  $x$ -intercepts of the graph of the corresponding rational function. You can use either of the following methods to solve rational equations graphically:
  - Manipulate the equation so that one side is equal to zero; then, graph the corresponding function and identify the value(s) of the  $x$ -intercept(s).
  - Graph a system of functions that corresponds to the expressions on both sides of the equal sign; then, identify the value(s) of  $x$  at the point(s) of intersection.
- When solving rational equations algebraically, remember to check for extraneous roots and to verify that the solution does not include any non-permissible values.

Assignment

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