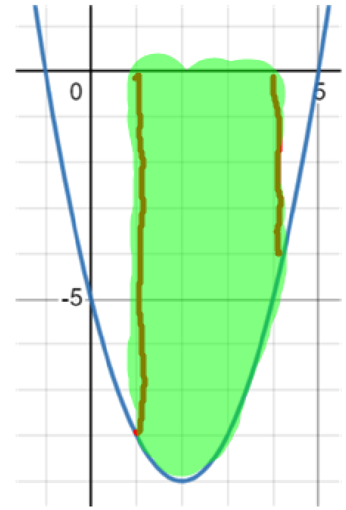
9.2 Area Above a Curve

Suppose $f(x)$ is a **continuous** function defined on a closed interval [a,b] and $f(x) \leq 0$, for all points in the interval. The area bounded by the x-axis, the vertical lines $x = a$ and $x = b$, and the function $y = f(x)$ is given by

$$\int_a^b -f(x)dx$$

Example #1

Find the area bounded by the x-axis, the lines $x = 1$ and $x = 4$, and the curve $f(x) = x^2 - 4x - 5$.



$$\text{Area} = \int_1^4 -(x^2 - 4x - 5)\,dx$$

$$= \int_1^4 (-x^2 + 4x + 5)\,dx$$

$$= -\frac{x^3}{3} + 2x^2 + 5x \Big|_1^4$$

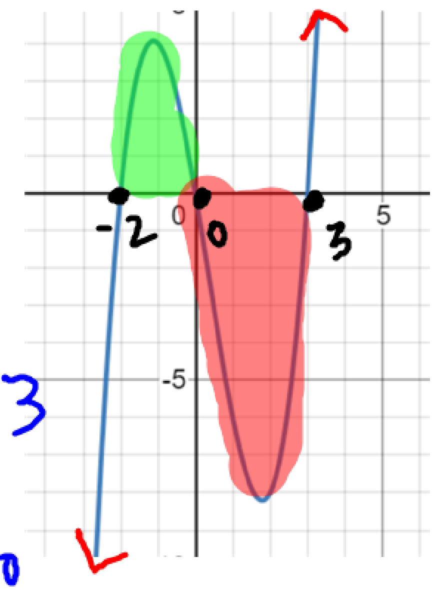
$$= \left(-\frac{(4)^3}{3} + 2(4)^2 + 5(4)\right) - \left(-\frac{1}{3} + 2 + 5\right)$$

$$= -\frac{64}{3} + 32 + 20 + \frac{1}{3} - 2 - 5 = -21 + 45$$

$$= 24$$

Example #2

Find the total area trapped by the x-axis, and the curve $f(x) = x^3 - x^2 - 6x$.



$$\text{Area} = \int_{-2}^{0} (x^3 - x^2 - 6x)\,dx + \int_{0}^{3} -(x^3 - x^2 - 6x)\,dx$$

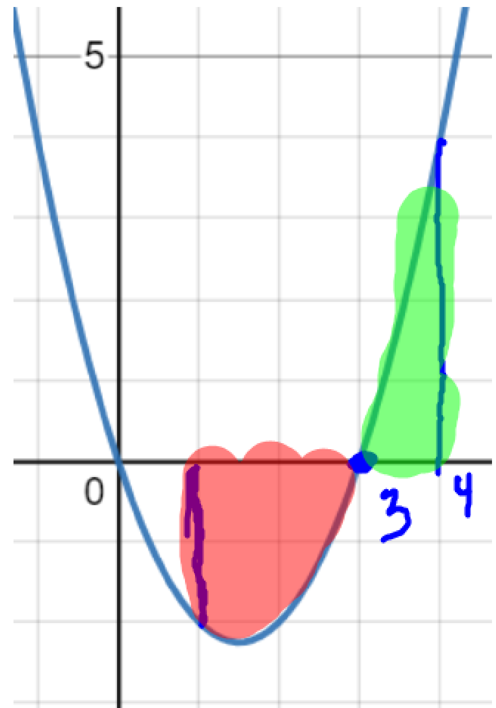
$$= \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 \Big|_{-2}^{0} + \left(-\frac{x^4}{4} + \frac{x^3}{3} + 3x^2\right)\Big|_{0}^{3}$$

$$= 0 - \left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3(-2)^2\right) + -\frac{(3)^4}{4} + \frac{(3)^3}{3} + 3(3)^2$$

$$= -4 - \frac{8}{3} + 12 - \frac{81}{4} + 9 + 27 = {}^{253}/_{12}$$

Your turn

Find the total area enclosed by the x-axis, and the curve $f(x) = x^2 - 3x$, and the lines $x = 1$ and $x = 4$.



$$\text{Area} = \int_2^3 -(x^2 - 3x)\,dx + \int_3^4 (x^2 - 3x)\,dx$$

$$= \left. -\frac{x^3}{3} + \frac{3x^2}{2} \right|_2^3 + \left. \frac{x^3}{3} - \frac{3x^2}{2} \right|_3^4$$

$$= \left(-\frac{(3)^3}{3} + \frac{3(3)^2}{2}\right) - \left(-\frac{(2)^3}{3} + \frac{3(2)^2}{2}\right) + \left(\frac{4^3}{3} - \frac{3(4)^2}{2}\right) - \left(\frac{3^3}{3} - \frac{3(3)^2}{2}\right)$$

$$= -9 + \frac{27}{2} + \frac{8}{3} - 6 + \frac{64}{3} - 24 - 9 + \frac{27}{2} = 3$$

Assignment

Page 381

#'s 2,3,4,5,6,7,8,13,15