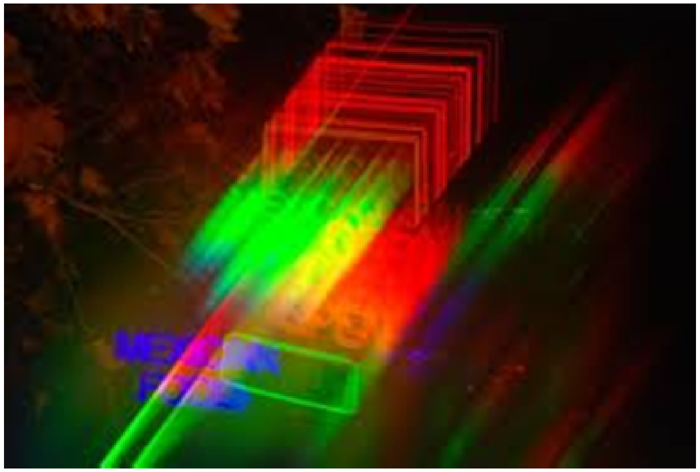
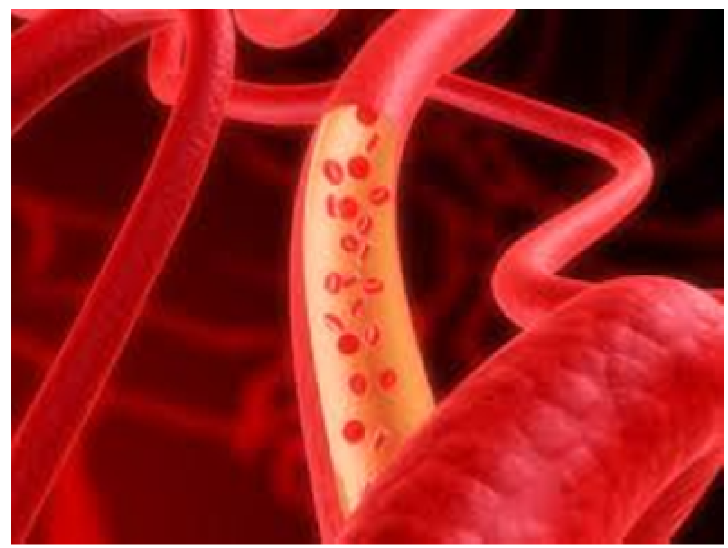
Chapter 9 Rational Functions

9.1 Exploring Rational Functions Using Transformations

Why does the lens on a camera need to move to focus on objects that are nearer or farther away? What is the relationship between the travel time for a plane and the velocity of the wind in which it is flying? How can you relate the amount of light from a source to the distance from the source? The mathematics behind all of these situations involves rational functions.



A simple rational function is used to relate distance, time, and speed. More complicated rational functions may be used in a business to model average costs of production or by a doctor to predict the amount of medication remaining in a patient's bloodstream.



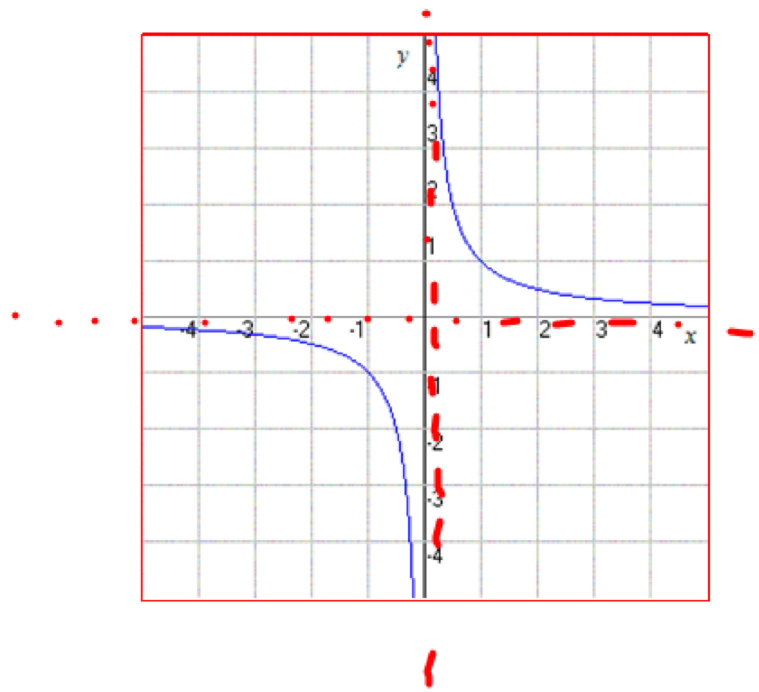
In this chapter, you will explore a variety of rational functions. You have used the term *rational* before, with rational numbers and rational expressions, so what is a rational *function*?

Definition

rational function

- a function that can be written in the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
- some examples are $y = \frac{20}{x}$, $C(n) = \frac{100 + 2n}{n}$, and $f(x) = \frac{3x^2 + 4}{x - 5}$

Let us start with the basic rational function $y = \frac{1}{x}$



Characteristics of the graph:

- x=0 is a vertical asymptote
- y=0 is a horizontal asymptote
- domain is all reals except x=0
- range is all reals except y=0
- no x-intercept
- no y-intercept
- as x approaches 0, $|y|$ becomes large
- as $|x|$ becomes large, y approaches zero

$$y = \frac{1}{\left(\frac{1}{1000}\right)} = 1000$$

$$y = \frac{1}{1000}$$

$y = \frac{1}{x}$ $a = 6$

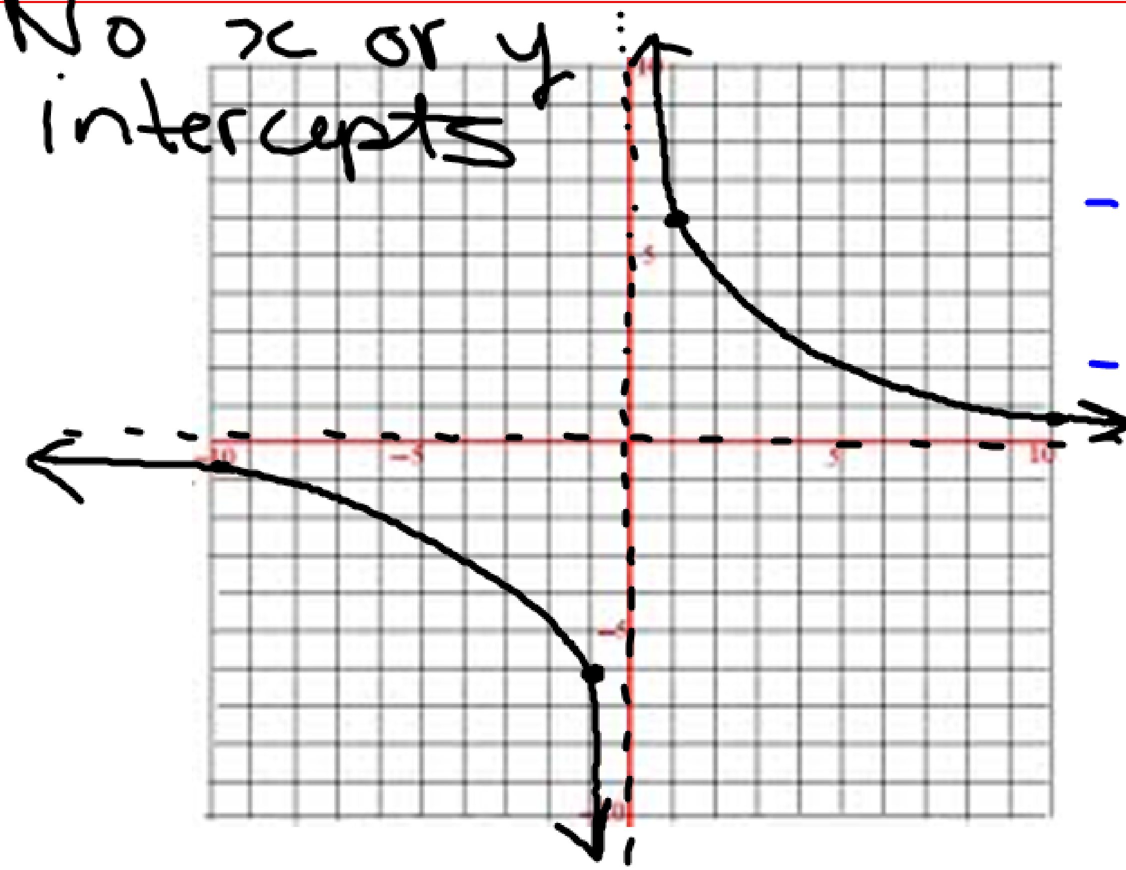
$(x, y) \rightarrow (x, 6y)$

Your Turn

Analyse the function $y = \frac{6}{x}$ using a table of values and a graph. Identify characteristics of the graph, including the behaviour of the function for its non-permissible value.

x	y
-10	-1/10
-1	-1
0	∞
1	1
10	1/10

No x or y intercepts



x	y
-10	$-\frac{6}{10}$
-1	-6
0	∞
1	6
10	$\frac{6}{10}$

To obtain the graph of a rational function of the form $y = \frac{a}{x - h} + k$ from the graph of $y = \frac{1}{x}$, apply a vertical stretch by a factor of a, followed by translations of h units horizontally and k units vertically.

- The graph has a vertical asymptote at $x = h$.
- The graph has a horizontal asymptote at $y = k$.
- Knowing the location of the asymptotes and drawing them first can help you graph and analyse the function.

Let's learn how to graph a rational function using transformations.

$a = 3$ $k = 5$
$h = -4$

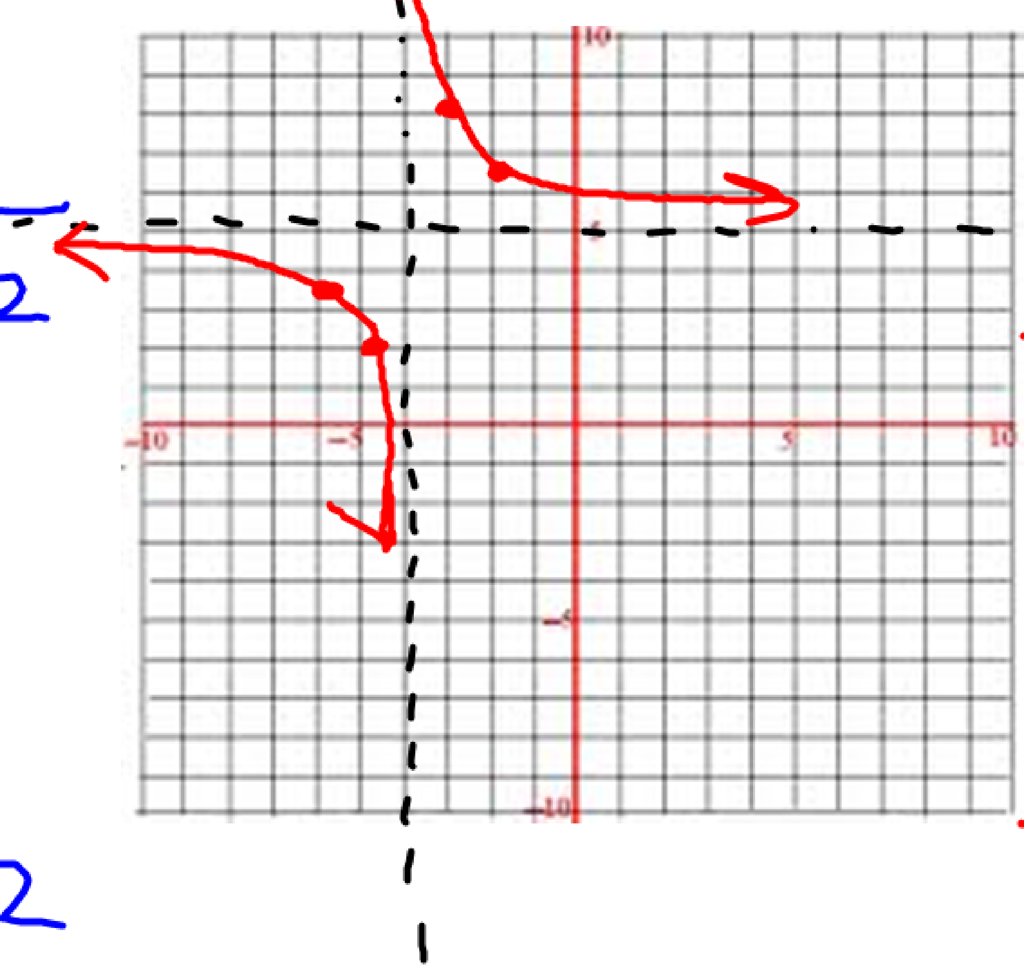
Example 1: Sketch the graph of the function $y = \frac{3}{x+4} + 5$ using transformations, and identify any important characteristics of the graph.

What graph could we start with?

$(x-4,\ 3y+5)$

$y = \frac{1}{x}$

x	y
-2	$-\frac{1}{2}$
-1	-1
0	∞
1	1
2	$\frac{1}{2}$



x	y
-6	3.5
-5	2
-4	∞
-3	8
-2	6.5

$$y = \frac{3}{x+4} + 5$$

<u>y int</u>

let $x = 0$

$$y = \frac{3}{0+4} + 5$$

$$y = 5\,{}^{3}/_{4}$$

<u>x int</u>

let $y = 0$

$$0 = \frac{3}{x+4} + 5$$

$$(x+4)(-5) = \frac{3}{\cancel{x+4}}\cancel{(x+4)}$$

$$-5x - 20 = 3$$

$$-5x = 23$$

$$x = -\frac{23}{5} = -4\,{}^{3}/_{5}$$

Domain

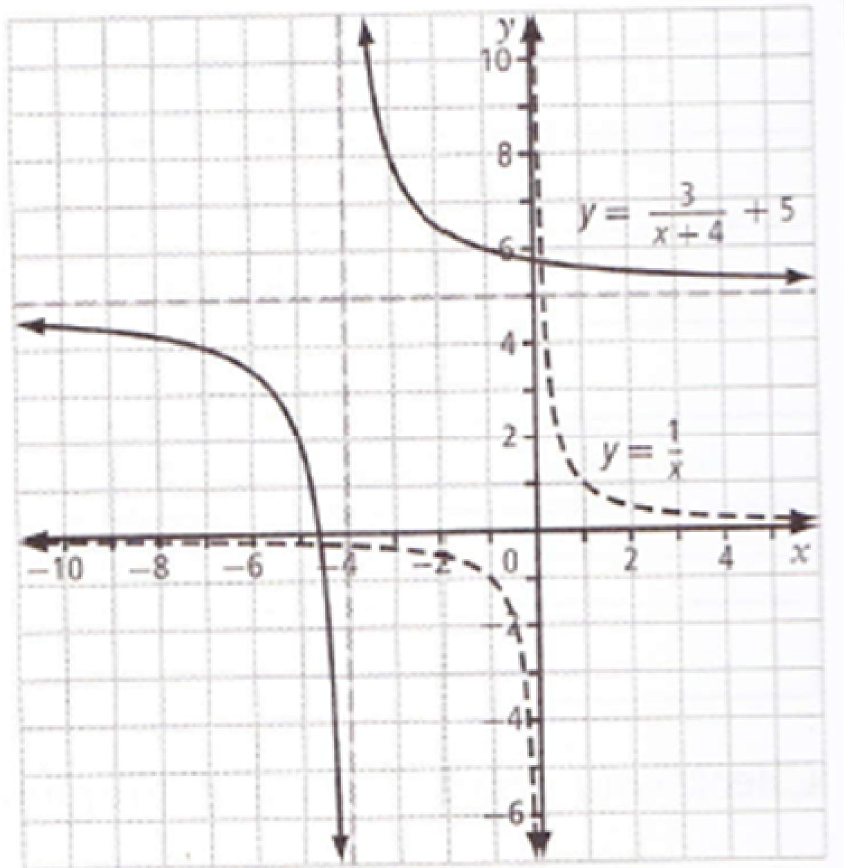
$x \in R$

$x \neq -4$

Range

$y \in R$

$y \neq 5$


y
10
8
6
4
2
-2
-4
-6
-10
-8
-6
-4
-2
0
2
4
x
$y = \frac{3}{x+4} + 5$
$y = \frac{1}{x}$

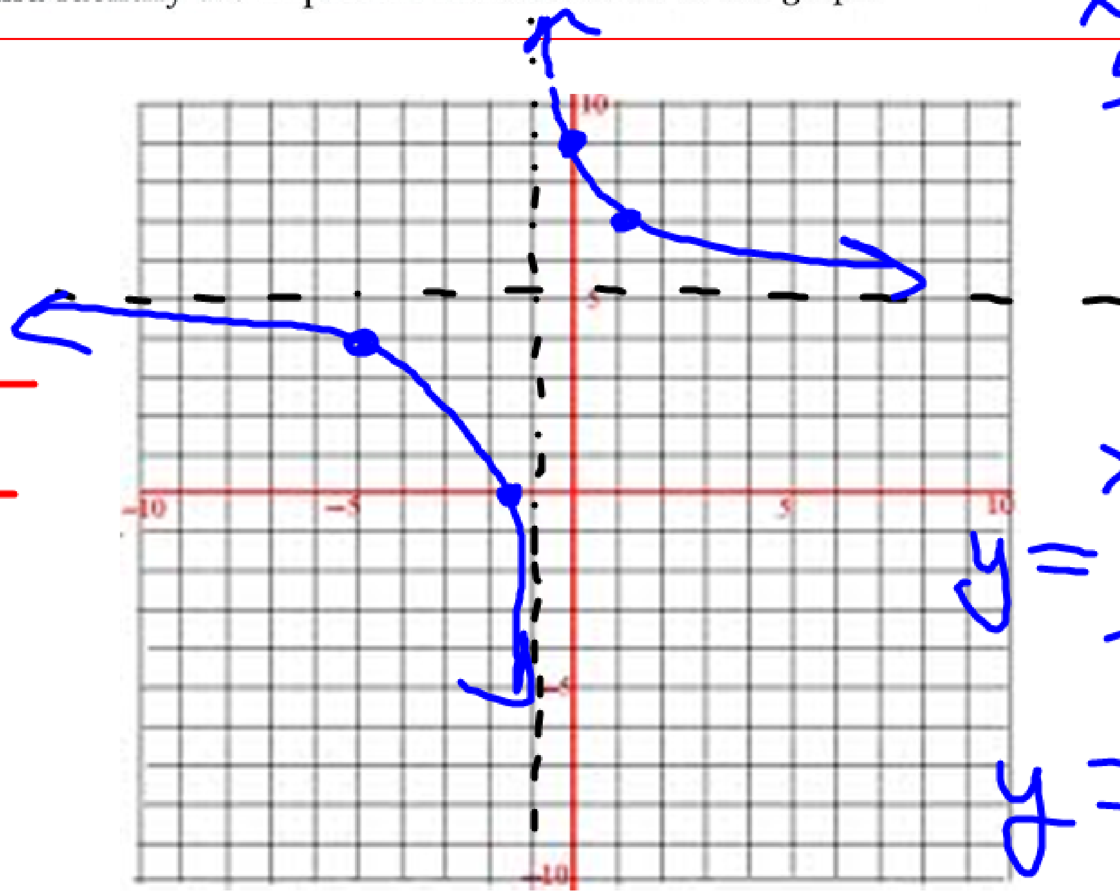
$a = 4 \quad h = -1 \quad k = 5$

Your Turn

Sketch the graph of the function $y = \frac{4}{x+1} + 5$ using transformations, and identify the important characteristics of the graph.

$y = \frac{1}{x}$

x	y
-2	-1/2
-1	-1
0	∞
1	1
2	1/2



$x = 1$

$\frac{4}{2} + 5$

$= 7$

$x = -5$

$y = \frac{4}{-4} + 5$

$y = 4$

$$y = \frac{4}{x+1} + 5$$

y int

let $x = 0$

$$y = \frac{4}{1} + 5$$

$$y = 9$$

$$(0, 9)$$

x int

let $y = 0$

$$0 = \frac{4}{x+1} + 5$$

$$-5 = \frac{4}{x+1}$$

$$-5(x+1) = \frac{4}{\cancel{x+1}}\cancel{(x+1)}$$

$$-5x - 5 = 4$$

$$-5x = 9$$

$$x = -9/5 = -1\,4/5$$

$$\left(-\frac{9}{5}, 0\right)$$

$y = \frac{a}{x-h} + k$

Graph a Rational Function With Linear Expressions in the Numerator and the Denominator

Graph the function $y = \frac{4x-5}{x-2}$. Identify any asymptotes and intercepts.

a) Find the y-intercept

let $x = 0$

$y = \frac{-5}{-2}$

$y = \frac{5}{2}$

$(0, 5/2)$

b) Find the x-intercept

let $y = 0$

$0 = \frac{4x-5}{x-2}$

$0 = 4x - 5$

$5 = 4x$

$\frac{5}{4} = x$

$(5/4, 0)$

VA

$x = 2$

restriction

HA

$y = 4$

c) Manipulate the rational equation to obtain the form $y = \frac{a}{x-h} + k$

This form will reveal both the vertical and horizontal asymptote.

$$\boxed{y = \frac{4x-5}{x-2}}$$

$$y = 4(3)-5 \over 3-2$$

$$y = 7$$

$$(3, 7)$$

$$y = \frac{a}{x-h} + K$$

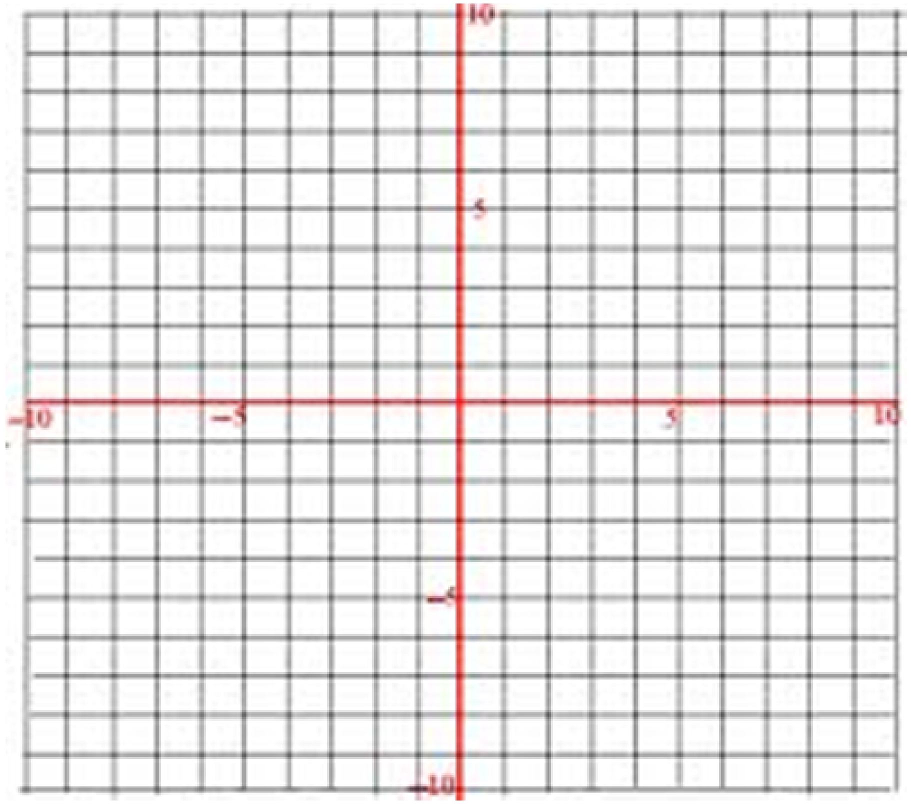
$$y = \frac{3}{x-2} + 4$$

$$7 = \frac{a}{3-2} + 4$$

$7 = a + 4$

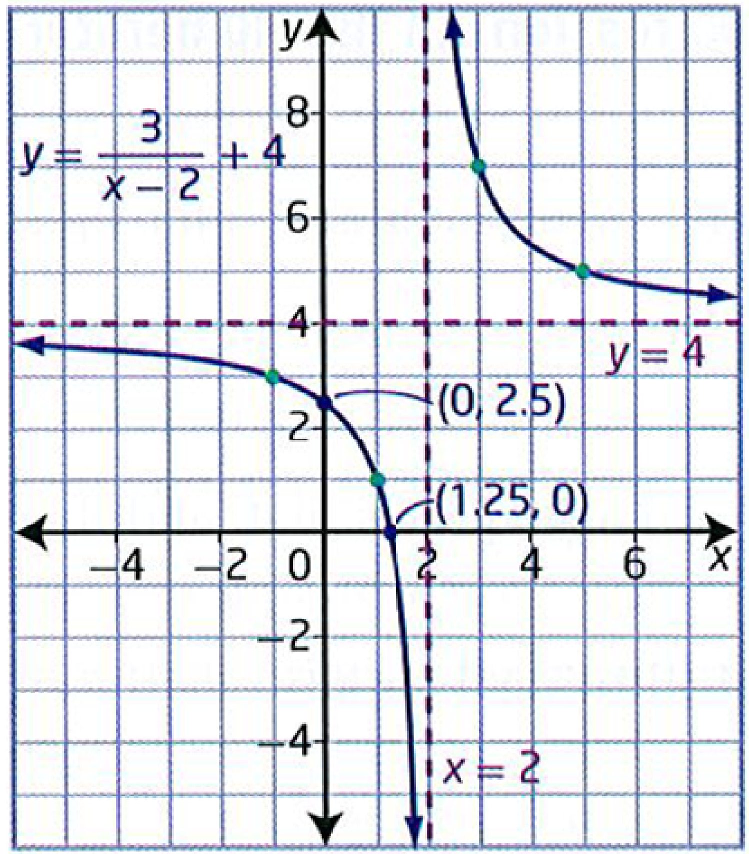
$3 = a$

How could making a table of values for $y = \frac{1}{x}$, help us?



x	y
1	
2	
-1	
-2	

This is what your graph should look like!



Manipulate the rational equation to obtain the form $y = \frac{a}{x-h} + k$

$$y = \frac{3x+7}{x-1}$$

$h = 1 \quad k = 3$

$(2, 13)$

$$y = \frac{10}{x-1} + 3$$

$$13 = \frac{a}{2-1} + 3$$

$$13 = a + 3$$

$$10 = a$$

Manipulate the rational equation to obtain the form $y = \frac{a}{x-h} + k$

~~Using h,k, and a coordinate~~

$$y = \frac{5x+1}{x-2}$$

$$y = \frac{5(x-2)+10 \quad +1}{x-2}$$

$$y = \frac{5(x-2) + 11}{x-2}$$

$$y = \frac{11}{x-2} + 5$$

Assignment

Page 442, #'s 1,3,4,5a,b,7,8,11

desmos