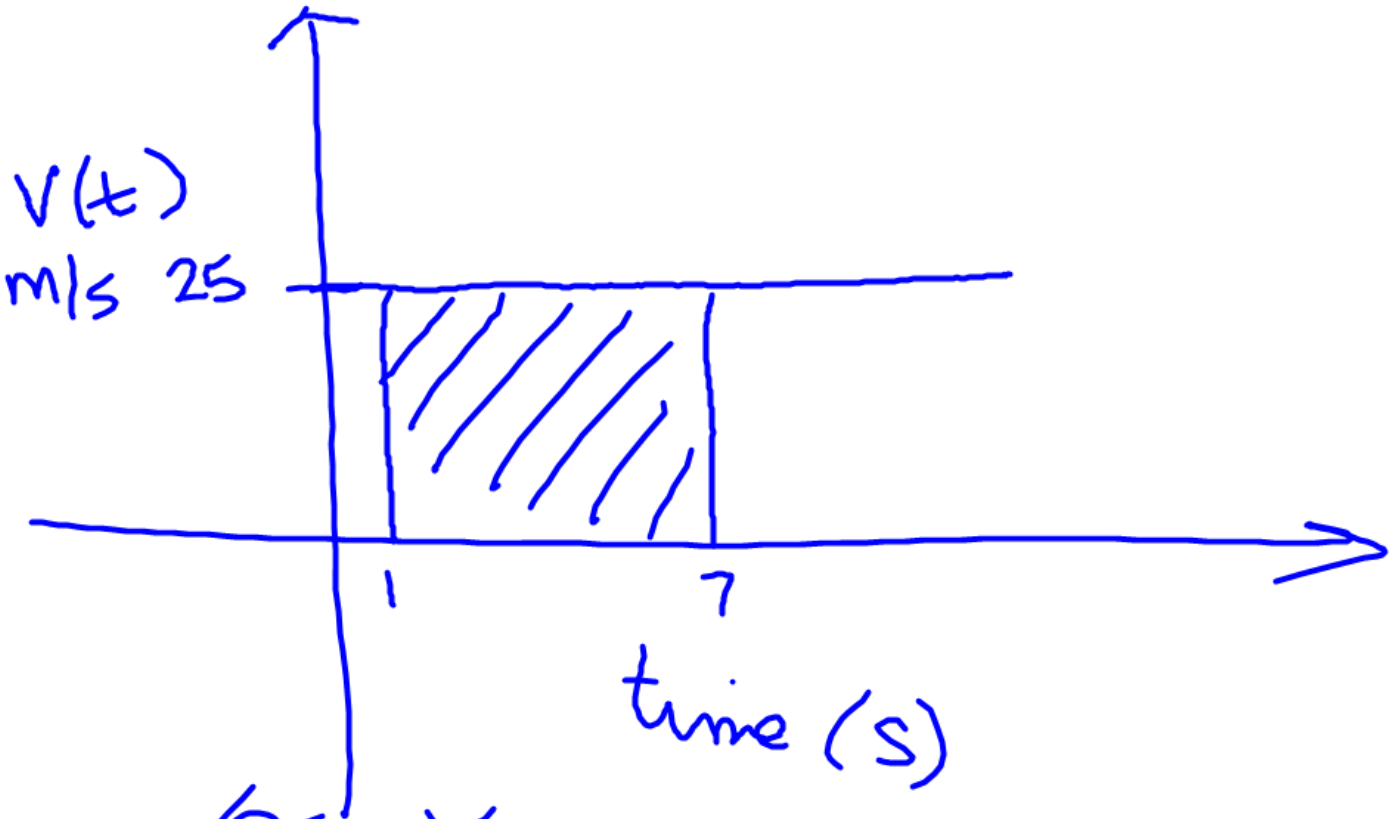
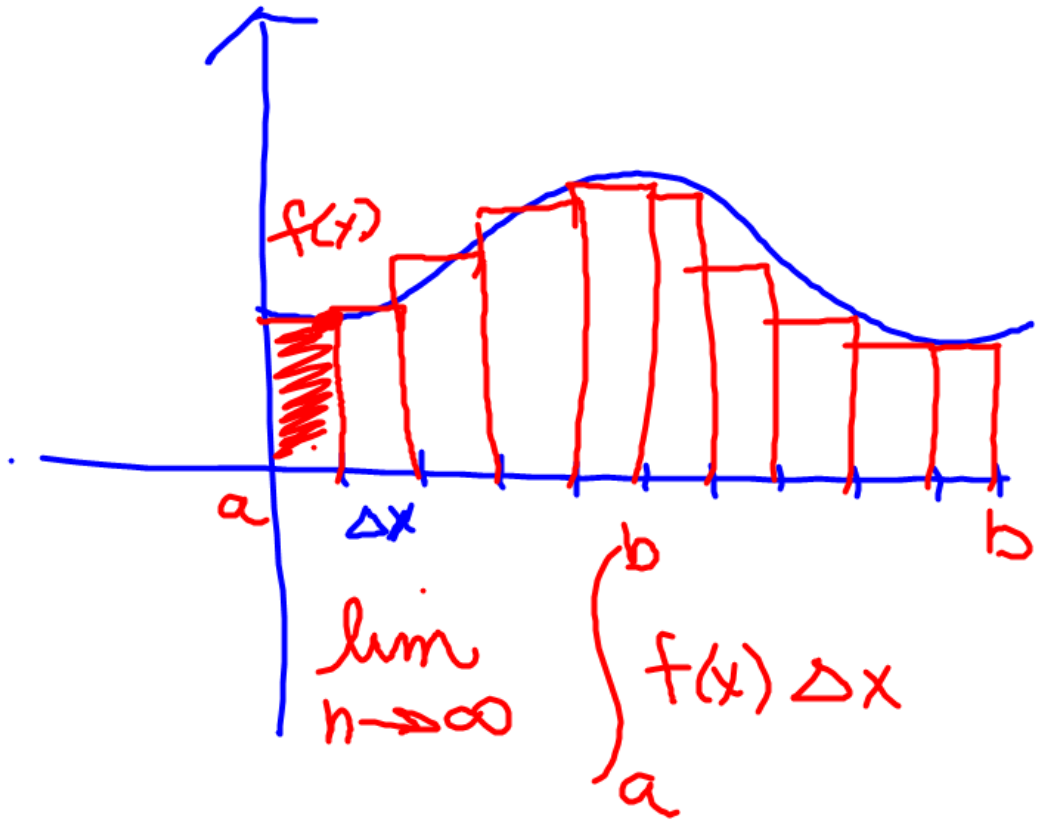
Chapter 9 Area and The Definite Integral

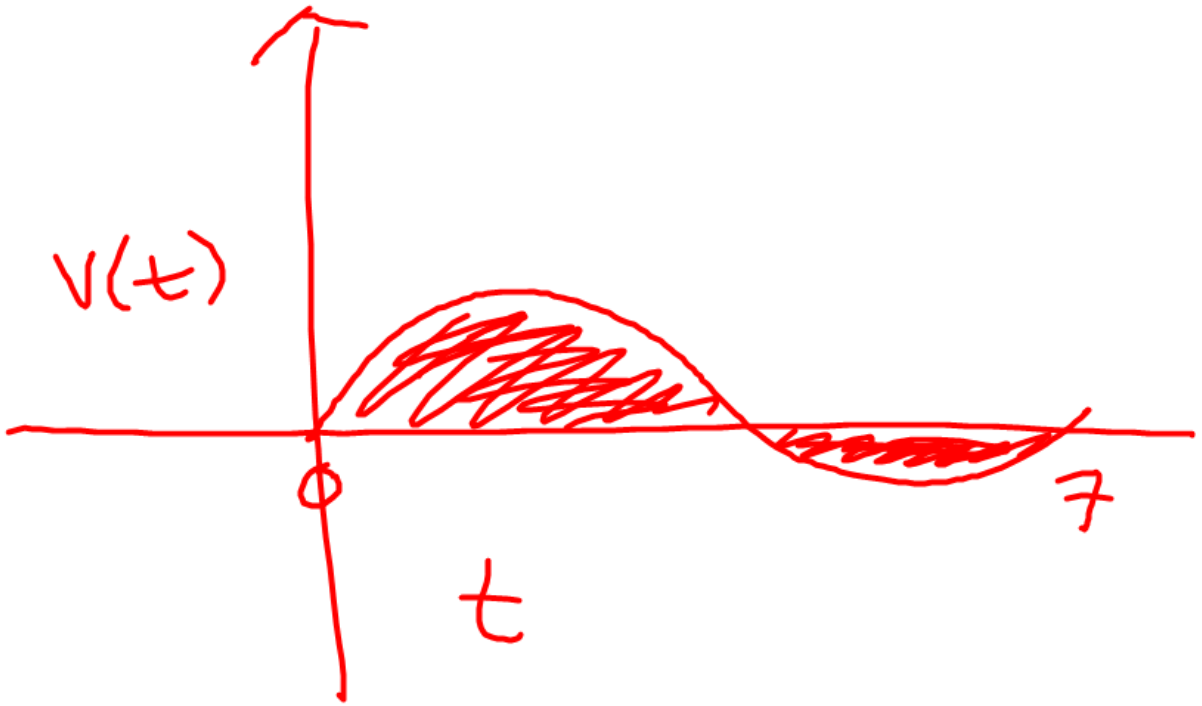
9.1 Area Under a Curve



$(25\,m/s)(6\,s) = 150\,m$



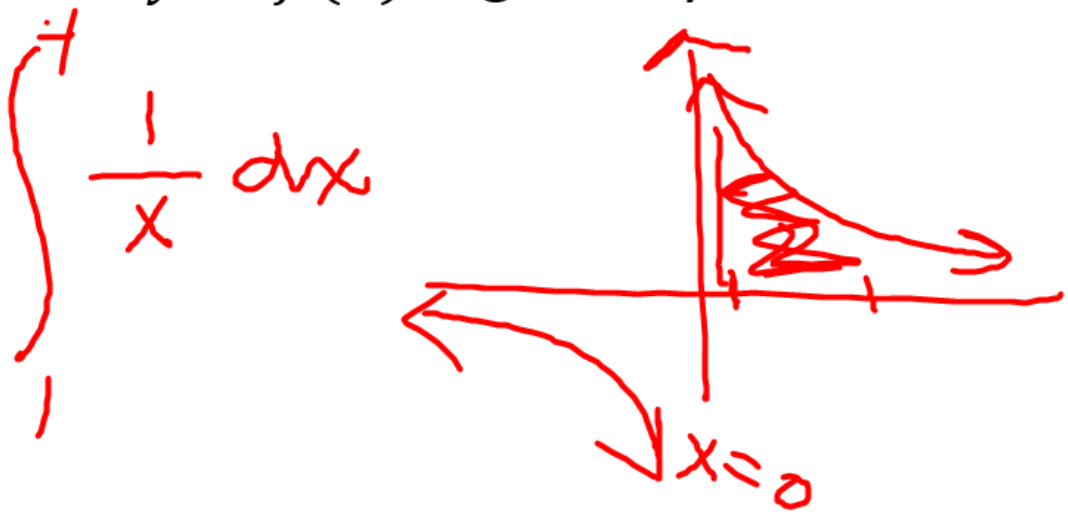
$$\lim_{n \to \infty} \int_a^b f(x)\,\Delta x$$


V(t)
0
T
t

The Fundamental Theorem of Calculus

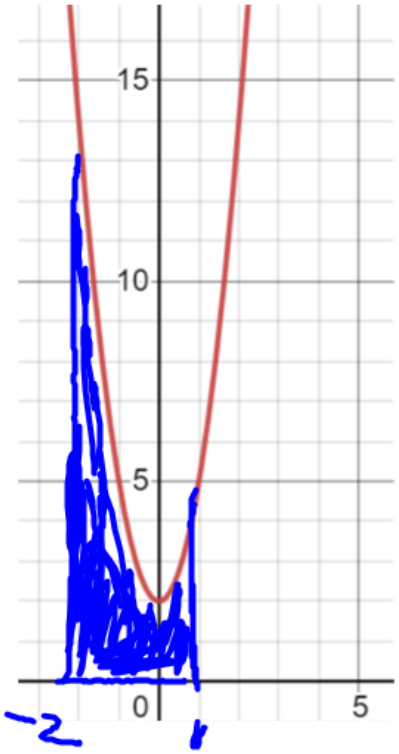
Suppose $f(x)$ is a **continuous** function defined on a closed interval $[a,b]$ and $f(x) \geq 0$, for all points in the interval. The area bounded by the x-axis, the vertical lines $x = a$ and $x = b$, and the function $y = f(x)$ is given by

$$\int_a^b f(x)dx$$



Example #1

Find the area bounded by the x-axis, the lines $x = -2$ and $x = 1$, and the curve $f(x) = 3x^2 + 2$



$$\text{Area} = \int_{-2}^{1} (3x^2 + 2)\,dx$$

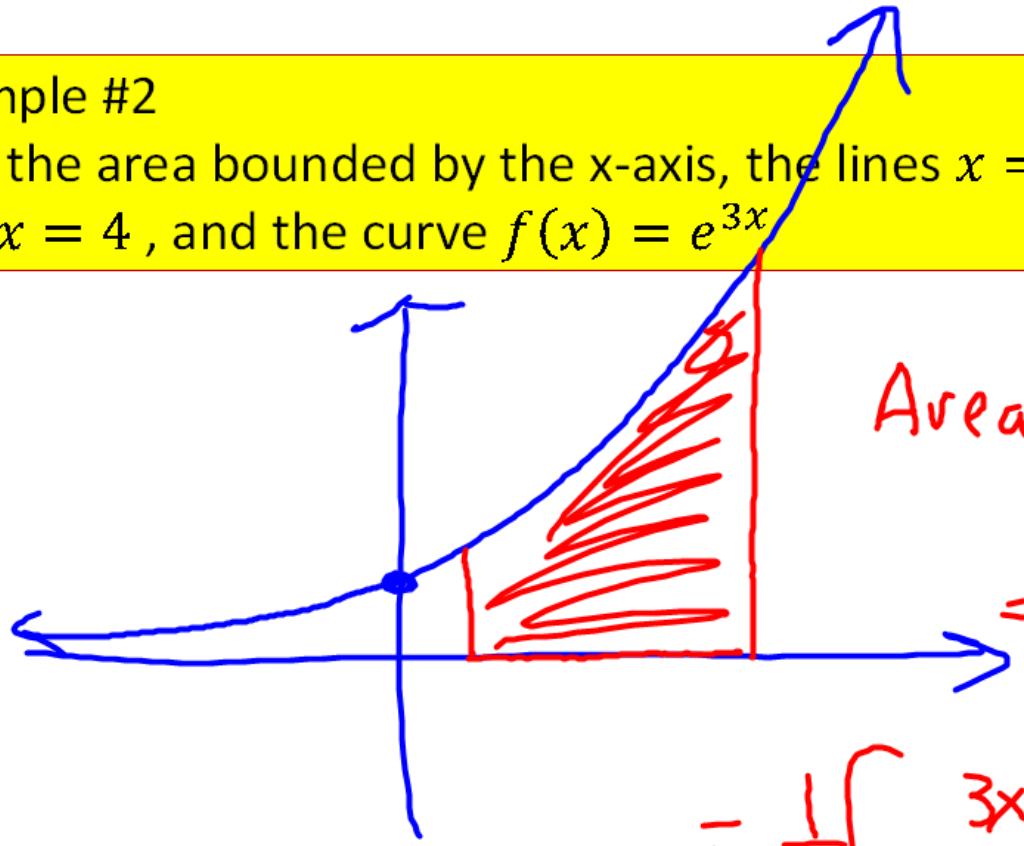
$$\text{Area} = x^3 + 2x \Big|_{-2}^{1}$$

$$= [(1)^3 + 2(1)] - [(-2)^3 + 2(-2)]$$

$$= 3 - (-12) = 15$$

Example #2

Find the area bounded by the x-axis, the lines $x = 1$ and $x = 4$, and the curve $f(x) = e^{3x}$



$$\text{Area} = \int_1^4 e^{3x}\,dx$$

$$= \frac{1}{3} e^{3x} \Big|_1^4$$

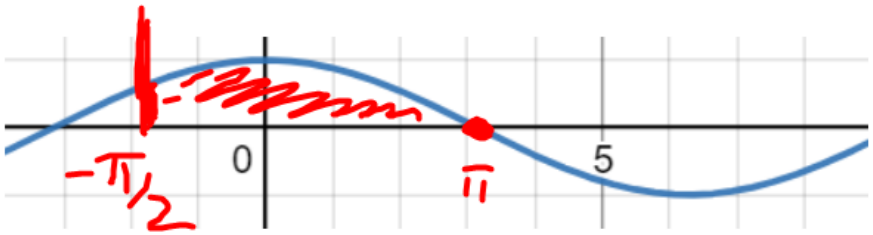
$$= \frac{1}{3}\left[e^{3x} \Big|_1^4 \right]$$

$$= \frac{1}{3}\left[e^{12} - e^{3} \right]$$

Example #3

Find the area bounded by the x-axis, the lines $x = \frac{-\pi}{2}$ and $x = \pi$, and the curve $f(x) = \cos\frac{1}{2}x$

$$\text{Area} = \int_{-\frac{\pi}{2}}^{\pi} \left(\cos\tfrac{1}{2}x\right) dx$$



$$= 2\sin\tfrac{1}{2}x \Big|_{-\pi/2}^{\pi}$$

$$= 2\left[\sin\tfrac{1}{2}\pi - \sin\left(\tfrac{1}{2}\left(-\tfrac{\pi}{2}\right)\right)\right]$$

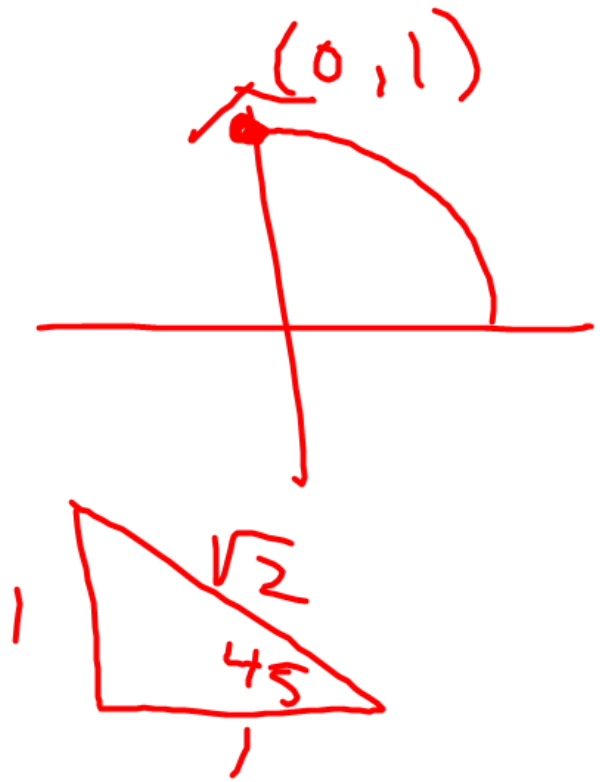
$$= 2\left[\sin\tfrac{\pi}{2} - \sin\left(-\tfrac{\pi}{4}\right)\right]$$

$$= 2\left[\sin\tfrac{\pi}{2} + \sin\tfrac{\pi}{4}\right]$$

$$= 2\left[1 + \tfrac{1}{\sqrt{2}}\right]$$

$$= 2\left[1 + \tfrac{\sqrt{2}}{2}\right]$$

$$= 2 + \sqrt{2}$$



Assignment
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