8.4 Definite Integral P. 370 1-36

1. $\int_2^5 10\,dx$

$= 10x \Big|_2^5$

$= 10(5) - 10(2)$

$= 50 - 20$

$= 30$

2. $\int_{-2}^1 (4-x)\,dx$

$= 4x - \frac{1}{2}x^2 \Big|_{-2}^1$

$= \left(4(1) - \frac{1}{2}(1)^2\right) - \left(4(-2) - \frac{1}{2}(-2)^2\right)$

$= \left(4 - \frac{1}{2}\right) - (-8 - 2)$

$= \left(\frac{7}{2}\right) - (-10) = \frac{27}{2}$

3. $\int_2^6 x^2\,dx$

$= \frac{1}{3}x^3 \Big|_2^6$

$= \left(\frac{1}{3}(6)^3\right) - \left(\frac{1}{3}(2)^3\right)$

$= 72 - \left(\frac{8}{3}\right)$

$= \frac{208}{3}$

4. $\int_{-3}^3 (x-1)^2\,dx$ let $u = x-1$ $du = dx$

$= \frac{1}{3}(x-1)^3 \Big|_{-3}^3$

$= \left(\frac{1}{3}(3-1)^3\right) - \left(\frac{1}{3}(-3-1)^3\right)$

$= \left(\frac{1}{3}(2)^3\right) - \left(\frac{1}{3}(-4)^3\right)$

$= \left(\frac{8}{3}\right) - \left(\frac{-64}{3}\right)$

$= 24$

5. $\int_{-5}^{-1} (t^2 + 4t - 5)\,dt$

$= \frac{1}{3}t^3 + 2t^2 - 5t \Big|_{-5}^{-1}$

$= \left(\frac{1}{3}(-1)^3 + 2(-1)^2 - 5(-1)\right) - \left(\frac{1}{3}(-5)^3 + 2(-5)^2 - 5(-5)\right)$

$= \left(\frac{-1}{3} + 2 + 5\right) - \left(\frac{-125}{3} + 50 + 25\right)$

$= \left(\frac{20}{3}\right) - \left(\frac{100}{3}\right) = \frac{-80}{3}$

6. $\int_0^1 \sqrt[3]{x^2}\,dx$

$= \frac{3}{5}x^{5/3} \Big|_0^1$

$= \left(\frac{3}{5}(1)^{5/3}\right) - (0) = \frac{3}{5}$

7. $\int_{-4}^{-2} \frac{1}{x^2}\,dx$

$= -1x^{-1} \Big|_{-4}^{-2}$

$= \left(-1(-2)^{-1}\right) - \left(-1(-4)^{-1}\right)$

$= \left(\frac{-1}{-2}\right) - \left(\frac{-1}{-4}\right)$

$= \frac{1}{4}$

8.4- Continued

8. $\int_1^3 \frac{w^2+1}{w}\,dw$

$= \int_1^3 w + \frac{1}{w}\,dw$

$= \frac{1}{2}w^2 + \ln|w| \Big|_1^3$

$= \left(\frac{1}{2}(3)^2 + \ln|3|\right) - \left(\frac{1}{2}(1)^2 + \underbrace{\ln(1)}_{=0}\right)$

$= 4 + \ln 3$

$b\sqrt{b}$

$b(b^{1/2})$

9. $\int_1^4 \sqrt{b}\,(b-2)\,db$

$\int_1^4 b^{3/2} - 2\sqrt{b}\,db$

$\frac{2}{5}b^{5/2} - 2\left(\frac{2}{3}\right)b^{3/2} \Big|_1^4$

$= \left(\frac{2}{5}(4)^{5/2} - \frac{4}{3}(4)^{3/2}\right) - \left(\frac{2}{5}(1)^{5/2} - \frac{4}{3}(1)^{3/2}\right)$

$= \left(\frac{64}{5} - \frac{32}{3}\right) - \left(\frac{2}{5} - \frac{4}{3}\right)$

$= \frac{32}{15} - \left(\frac{-14}{15}\right)$

$= \frac{46}{15}$

10. $\int_0^{\frac{\pi}{2}} \cos x\,dx$

$= \sin x \Big|_0^{\frac{\pi}{2}}$

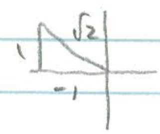
$= \sin\frac{\pi}{2} - \sin 0$

$= (1) - 0$

$= 1$

11. $\int_0^{\frac{3\pi}{4}} \sin x\,dx$

$= -\cos x \Big|_0^{3\pi/4}$



$= -\cos\frac{3\pi}{4} - (-\cos 0)$

$= -\left(-\frac{\sqrt{2}}{2}\right) + 1 = \frac{\sqrt{2}}{2} + 1$

8.4 - Continued

12. $\int_2^6 e^x dx$

$= e^x \Big|_2^6$

$= e^6 - e^2$

13. $\int_{\ln 2}^{\ln 6} e^x dx$

$= e^x \Big|_{\ln 2}^{\ln 6}$

$= e^{\ln 6} - e^{\ln 2}$

$= 6 - 2 = 4$

14. $\int_0^{\frac{3\pi}{2}} \cos \frac{x}{3} dx$ Let $u = \frac{x}{3}$ $du = \frac{1}{3} dx$

$= 3 \int_0^{\frac{3\pi}{2}} \cos \frac{x}{3} \frac{1}{3} dx$

$= 3 (\sin \frac{x}{3}) \Big|_0^{\frac{3\pi}{2}}$

$= (3 \sin \frac{\frac{3\pi}{2}}{3}) - (3 \sin \frac{0}{3})$

$= (3) - (0) = 3$

15. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x dx$ Let $u = 4x$ $du = 4dx$

$= \frac{1}{4} \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x \, 4dx$

$= \frac{1}{4} (-\cos 4x) \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}}$

$= (-\frac{1}{4} \cos 4(\frac{\pi}{4})) - (-\frac{1}{4} \cos 4(\frac{\pi}{8}))$

$= (\frac{1}{4}) - (0) = \frac{1}{4}$

16. $\int_3^6 \frac{1}{x} dx$

$= \ln |x| \Big|_3^6$

$= \ln 6 - \ln 3$

$= \ln (\frac{6}{3})$

$= \ln 2$

8.4 - Continued

17. $\int_e^{e^3} \frac{1}{x}\,dx$

$= \ln x \Big|_e^{e^3}$

$= \ln e^3 - \ln e^1$

$= 3 - 1 = 2$

18. $\int_1^4 \sqrt{t}\,dt$

$= \frac{2}{3} t^{3/2} \Big|_1^4$

$= \left(\frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{3}(1)^{3/2}\right)$

$= \left(\frac{16}{3}\right) - \left(\frac{2}{3}\right) = \frac{14}{3}$

19. $\int_4^9 \frac{x^2 - x}{\sqrt{x}}\,dx$

$\int_4^9 x^{3/2} - x^{\frac{1}{2}}\,dx$

$\left(\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2}\right)\Big|_4^9$

$= \left(\frac{2}{5}(9)^{5/2} - \frac{2}{3}(9)^{3/2}\right) - \left(\frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2}\right)$

$= \left(\frac{486}{5} - 18\right) - \left(\frac{64}{5} - \frac{16}{3}\right) = \left(\frac{396}{5}\right) - \left(\frac{112}{15}\right) = \frac{1076}{15}$

20. $\int_{-3}^{-2} e^{3x}\,dx$ let $u = 3x$ $du = 3dx$

$= \frac{1}{3}\int_{-3}^{-2} e^{3x}\,3dx$

$= \frac{1}{3}e^{3x}\Big|_{-3}^{-2}$

$= \left(\frac{1}{3}\left(e^{3(-2)}\right)\right) - \left(\frac{1}{3}\left(e^{3(-3)}\right)\right)$

$= \frac{e^{-6} - e^{-9}}{3}$

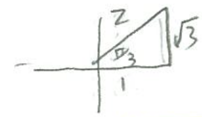
21. $\int_0^1 \frac{1}{1+x^2}\,dx$

$= \tan^{-1} x \Big|_0^1$

$= (\tan^{-1} 1) - (\tan^{-1} 0)$

$= 45 - 0$

$= 45^\circ$ or $\frac{\pi}{4}$



8.4 - Continued

22. $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\,dx$

$\sin^{-1} x \Big|_{\frac{1}{2}}^{\sqrt{3}/2}$

$= \left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) - \left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

$= 60 - 30 = 30$ or $\frac{\pi}{6}$

23. $\displaystyle\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x\,dx$ let u

du=

$= \frac{1}{2}\displaystyle\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x\, 2dx$

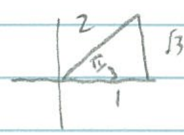
$= \frac{1}{2}(\sec 2x)\Big|_0^{\frac{\pi}{6}}$

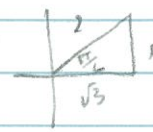
$= \left(\frac{1}{2}\sec 2\left(\frac{\pi}{6}\right)\right) - \left(\frac{1}{2}\sec 2(0)\right)$

$= \frac{1}{2}\left(\frac{2}{1}\right) - \left(\frac{1}{2}(1)\right)$

$= \frac{1}{2}$

24. $\displaystyle\int_{\frac{\pi}{18}}^{\frac{\pi}{9}} \sec^2 3x\,dx$ let $u = 3x$ $du = 3dx$



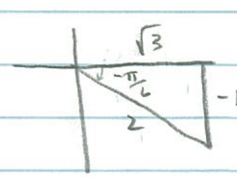


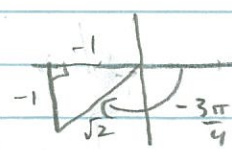
$= \frac{1}{3}\tan 3x \Big|_{\frac{\pi}{18}}^{\frac{\pi}{9}}$

$= \frac{1}{3}\tan 3\left(\frac{\pi}{9}\right) - \frac{1}{3}\tan 3\left(\frac{\pi}{18}\right)$

$= \frac{1}{3}\left(\frac{\sqrt{3}}{1}\right) - \frac{1}{3}\left(\frac{\sqrt{3}}{3}\right) = \frac{3\sqrt{3}-\sqrt{3}}{9} = \frac{2\sqrt{3}}{9}$

25. $\displaystyle\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{6}} \csc x \cot x\,dx$





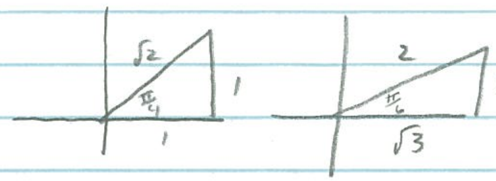
$= -\csc x \Big|_{-\frac{3\pi}{4}}^{-\frac{\pi}{6}}$

$= \left(-\csc\left(\frac{-\pi}{6}\right)\right) - \left(-\csc\left(\frac{-3\pi}{4}\right)\right)$

$= \left(-\left(\frac{-2}{1}\right)\right) - \left(-\left(\frac{-\sqrt{2}}{1}\right)\right) = 2-\sqrt{2}$

8.4 - Continued

26. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \csc^2 x \, dx$



$= -\cot x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= \left(-\cot \frac{\pi}{4}\right) - \left(-\cot \frac{\pi}{6}\right)$

$= -1 - \left(-\frac{\sqrt{3}}{1}\right) = -1 + \sqrt{3}$

27. $\int_{1}^{\sqrt{3}} \frac{6}{1+x^2} \, dx$

$= 6 \int_{1}^{\sqrt{3}} \frac{1}{1+x^2} \, dx$

$= 6 \left(\tan^{-1} x\right) \Big|_{1}^{\sqrt{3}}$

$= 6\left(\tan^{-1}\sqrt{3}\right) - 6\left(\tan^{-1} 1\right)$

$= 6\left(\frac{\pi}{3}\right) - 6\left(\frac{\pi}{4}\right) = 2\pi - \frac{3\pi}{2} = \frac{\pi}{2}$

28. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \csc \frac{1}{2}x \cot \frac{1}{2}x \, dx$ let $u = \frac{1}{2}x$ $du = \frac{1}{2} dx$

$= (2) - \csc \frac{1}{2}x \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \left(-2 \csc \frac{1}{2}\left(\frac{\pi}{2}\right)\right) - \left(-2 \csc \frac{1}{2}\left(\frac{\pi}{3}\right)\right)$

$= \left(-2 \csc \frac{\pi}{4}\right) - \left(-2 \csc \frac{\pi}{6}\right)$

$= \left(-2\left(\frac{\sqrt{2}}{1}\right)\right) - \left(-2\left(\frac{2}{1}\right)\right) = -2\sqrt{2} + 4$

29. $\int_{0}^{1} x(x^2+1)^5 \, dx$ let $u = x^2+1$ $du = 2x \, dx$

$= \frac{1}{2} \int_{0}^{1} (x^2+1)^5 \, 2x \, dx$

$= \frac{1}{2}\left(\frac{1}{6}(x^2+1)^6\right) \Big|_{0}^{1}$

$= \left(\frac{1}{12}(1^2+1)^6\right) - \left(\frac{1}{12}(0+1)^6\right) = \left(\frac{1}{12}(64)\right) - \left(\frac{1}{12}(1)\right) = \frac{21}{4}$

8.4- Continued

30. $\int_{-1}^{1} (x+1)e^{x^2+2x}\,dx$ Let $u = x^2+2x$ $du = (2x+2)\,dx$

$= \frac{1}{2}\int_{-1}^{1} e^{x^2+2x}\, 2(x+1)\,dx$

$= \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_{-1}^{1}$

$= \frac{1}{2}\left(e^{1+2}\right) - \frac{1}{2}\left(e^{1-2}\right)$

$= \frac{1}{2}e^3 - \frac{1}{2}e^{-1}$

31. $\int_{0}^{\frac{\pi}{2}} \cos^3 x \sin x\,dx$ Let $u = \cos x$ $du = -\sin x\,dx$

$= -1\int_{0}^{\frac{\pi}{2}} (\cos x)^3 (-1)\sin x\,dx$

$= -1\left(\frac{1}{4}(\cos x)^4\right)\Big|_{0}^{\frac{\pi}{2}}$

$= \left(-\frac{1}{4}\left(\cos\frac{\pi}{2}\right)^4\right) - \left(\frac{-1}{4}(\cos 0)^4\right)$

$= -\frac{1}{4}(0)^4 - \left(\frac{-1}{4}(1)\right) = \frac{1}{4}$

32. $\int_{-2}^{-1} \frac{1}{(2x+1)^4}\,dx$ Let $u = 2x+1$ $du = 2\,dx$

$= \frac{1}{2}\int_{-2}^{-1} (2x+1)^{-4}\, 2\,dx$

$= \frac{1}{2}\left(\frac{-1}{3}\right)(2x+1)^{-3}\Big|_{-2}^{-1}$

$= \left(\frac{-1}{6}(2(-1)+1)^{-3}\right) - \left(\frac{-1}{6}(2(-2)+1)^{-3}\right)$

$= \left(\frac{-1}{6}\left(\frac{-1}{1}\right)\right) - \left(\frac{-1}{6}\left(\frac{-1}{27}\right)\right)$

$= \left(\frac{1}{6}\right) - \left(\frac{1}{162}\right)$

$= \frac{13}{81}$

8.4 - Continued

33 $\int_0^5 \sqrt{1+3r}\, dr$ let $u = 1+3r$ $du = 3dr$

$= \frac{1}{3}\int_0^5 (1+3r)^{1/2} dr$

$= \frac{1}{3}\left(\frac{2}{3}\right)\left(1+3r\right)^{3/2}\Big|_0^5$

$= \left(\frac{2}{9}\left(1+3(5)\right)\right)^{3/2} - \left(\frac{2}{9}\left(1+3(0)\right)\right)^{3/2}$

$= \frac{2}{9}(16)^{3/2} - \frac{2}{9}(1)^{3/2} = \frac{128}{9} - \frac{2}{9} = \frac{126}{9} = 14$

34. $\int_{-3}^{-1} \frac{2x}{x^2+5} dx$ let $u = x^2+5$ $du = 2x\,dx$

$= \ln|x^2+5|\Big|_{-3}^{-1}$

$= \ln|(-1)^2+5| - \ln|(-3)^2+5|$

$= \ln 6 - \ln 14$

$= \ln\left(\frac{6}{14}\right)$

$= \ln \frac{3}{7}$

35. $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos x\, dx$ let $u = \sin x$ $du = \cos x\, dx$.

$= e^{\sin x}\Big|_0^{\frac{\pi}{2}}$

$= e^{\sin\frac{\pi}{2}} - e^0$

$= e^1 - 1$

36 $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \cos(\sin x)\, dx$ let $u = \sin x$ $du = \cos x\, dx$

$\sin(\sin x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \left(\sin\left(\sin\frac{\pi}{2}\right)\right) - \left(\sin\left(\sin\frac{\pi}{6}\right)\right)$

$= (\sin(1)) - \sin\left(\frac{1}{2}\right)$