8.3 Integration Using u Substitution P. 366 1-60

1. $\int e^{\sin x} \cos x \, dx$, let $u = \sin x$ $\quad \frac{du}{dx} = \cos x$

$= e^{\sin x} + C$ $\quad du = \cos x \, dx$

2. $\int x (2x^2+5)^8 \, dx$, let $u = 2x^2+5$ $\quad du = 4x\,dx$

$= \frac{1}{4} \int (2x^2+5)^8 \, 4x \, dx$

$= \frac{1}{4} \left(\frac{1}{9}\right) (2x^2+5)^9 + C$

$= \frac{1}{36} (2x^2+5)^9 + C$

3. $\int x^2 \cos 5x^3 \, dx$, let $u = 5x^3$ $\quad du = 15x^2 dx$

$= \frac{1}{15} \int \cos 5x^3 \, 15x^2 dx$

$= \frac{1}{15} \sin 5x^3 + C$

4. $\int \frac{1}{10x+7} \, dx$; let $u = 10x+7$ $\quad du = 10dx$

$= \frac{1}{10} \int (10x+7)^{-1} \, 10dx$

$= \left(\frac{1}{10}\right) \ln |10x+7| + C$

5. $\int \sqrt{5x-9} \, dx$, let $u = 5x-9$ $\quad du = 5dx$

$= \frac{1}{5} \int (5x-9)^{1/2} \, 5dx$

$= \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) (5x-9)^{3/2} + C$

$= \frac{2}{15} (5x-9)^{3/2} + C$

8.3- Continued

6. $\int \frac{\sin(\ln x)}{x}$, let $u = \ln x$ $\quad du = \frac{1}{x}\,dx$

$= -\cos(\ln x) + C$

7. $\int e^{6x}\,dx$, let $u = 6x$ $\quad du = 6\,dx$

$= \frac{1}{6}\int e^{6x}\,6dx$

$= \left(\frac{1}{6}\right)e^{6x} + C$

8. $\int \cos 4x\,dx$ $\quad$ Let $u = 4x$ $\quad du = 4dx$

$= \frac{1}{4}\int \cos 4x\,4dx$

$= \frac{1}{4}\sin 4x + C$

9. $\int \frac{1}{3x+8}\,dx$ $\quad$ let $u = 3x+8$ $\quad du = 3\,dx$

$= \frac{1}{3}\int (3x+8)^{-1}\,3dx$

$= \frac{1}{3}\ln|3x+8| + C$

10. $\int (6x-11)^8\,dx$ $\quad$ let $u = 6x-11$ $\quad du = 6dx$

$= \frac{1}{6}\int (6x-11)^8\,6dx$

$= \frac{1}{6}\left(\frac{1}{9}\right)(6x-11)^9 + C$

$= \frac{1}{54}(6x-11)^9 + C$

8.3 - Continued

11. $\int x(x^2-6)^{11}\,dx$ Let $u = x^2-6$ $du = 2x\,dx$

$= \frac{1}{2}\int (x^2-6)^{11}\,2x\,dx$

$= \frac{1}{2}\left(\frac{1}{12}\right)(x^2-6)^{12} + C$

$= \frac{1}{24}(x^2-6)^{12} + C$

12. $\int x^2 \sin x^3\,dx$ Let $u = x^3$ $du = 3x^2\,dx$

$= \frac{1}{3}\int \sin x^3\, 3x^2\,dx$

$= \left(\frac{1}{3}\right)\left(-\cos x^3\right) + C$

$= -\frac{1}{3}\cos x^3 + C$

13. $\int 3^{2x+1}\,dx$ Let $u = 2x+1$ $du = 2\,dx$

$= \frac{1}{2}\int 3^{2x+1}\,2\,dx$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\ln 3}\right)\left(3^{2x+1}\right) + C$

$= \frac{3^{2x+1}}{2\ln 3} + C$

14. $\int x\sqrt{2x^2-5}\,dx$ Let $u = 2x^2-5$ $du = 4x\,dx$

$= \frac{1}{4}\int \cancel{\sqrt{}}(2x^2-5)^{1/2}\,4x\,dx$

$= \frac{1}{4}\left(\frac{2}{3}\right)(2x^2-5)^{3/2} + C$

$= \frac{1}{6}(2x^2-5)^{3/2} + C$

8.3 - Continued

15. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ Let $u = \sqrt{x}$ $du = \frac{1}{2}x^{-1/2}dx$

$= 2\int \sin\sqrt{x}\ \frac{1}{2}x^{-1/2}dx$

$= 2(-\cos\sqrt{x}) + C$

$= -2\cos\sqrt{x} + C$

16. $\int \sqrt{x+2}\,dx$ Let $u = x+2$ $du = dx$

$= \frac{2}{3}(x+2)^{3/2} + C$

17. $\int \frac{5}{(x-2)^3}\,dx$ Let $u = x-2$ $du = dx$

$= 5\int (x-2)^{-3}dx$

$= 5\left(-\frac{1}{2}\right)(x-2)^{-2} + C$

$= -\frac{5}{2}(x-2)^{-2} + C$

18. $\int (3-t)^4\,dt$ Let $u = 3-t$ $du = -dt$

$= -1\int (3-t)^4(-1)dt$

$= -1\left(\frac{1}{5}\right)(3-t)^5 + C$

$= -\frac{1}{5}(3-t)^5 + C$

19. $\int \frac{x}{x^2-4}\,dx$ Let $u = x^2-4$ $du = 2x\,dx$

$= \frac{1}{2}\int (x^2-4)^{-1}\,2x\,dx$

$= \frac{1}{2}\ln|x^2-4| + C$

8.3- Continued

20 $\int \frac{3x+1}{\sqrt{3x^2+2x+1}}$ Let $u = 3x^2+2x+1$ $du = (6x+2)\,dx$

$du = 2(3x+1)\,dx$

$= \frac{1}{2}\int (3x^2+2x+1)^{-1/2}\, 2(3x+1)\,dx$

$= \left(\frac{1}{2}\right)\left(\frac{2}{1}\right)(3x^2+2x+1)^{1/2} + C$

$= (3x^2+2x+1)^{1/2} + C$

21. $\int \sin\theta\, e^{\cos\theta}\, d\theta$ Let $u = \cos\theta$ $du = -\sin\theta\, d\theta$

$= -1\int e^{\cos\theta}(-1)\sin\theta\, d\theta$

$= (-1)(e^{\cos\theta}) + C$

22. $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}}\, dx$ Let $u = \sqrt{x-1}$ $du = \frac{1}{2}(x-1)^{-1/2}\,dx$

$= 2\int e^{\sqrt{x-1}}\, \frac{1}{2}(x-1)^{-1/2}\,dx$

$= 2\,e^{\sqrt{x-1}} + C$

23. $\int (3x+7)^{4.2}\,dx$ Let $u = 3x+7$ $du = 3\,dx$

$= \frac{1}{3}\int (3x+7)^{4.2}\, 3\,dx$

$= \left(\frac{1}{3}\right)\left(\frac{1}{5.2}\right)(3x+7)^{5.2} + C$

$= \frac{5}{78}(3x+7)^{5.2} + C$

24. $\int \sqrt[4]{5-2x}\,dx$ Let $u = 5-2x$ $du = -2\,dx$

$= -\frac{1}{2}\int (5-2x)^{1/4}\, -2\,dx$

$= -\frac{1}{2}\left(\frac{4}{5}\right)(5-2x)^{5/4} + C = \frac{-2}{5}(5-2x)^{5/4} + C$

8.3 - Continued

25. $\int \sin \frac{1}{2} x \, dx$ Let $u = \frac{1}{2}x$ $du = \frac{1}{2}dx$

$= 2 \int \sin \frac{1}{2}x \, \frac{1}{2}dx$

$= 2\left(-\cos \frac{1}{2}x\right) + C$

$= -2 \cos \frac{1}{2}x + C$

26. $\int (r+2)^{20} \, dr$ Let $u = r+2$ $du = dr$

$= \frac{1}{21}(r+2)^{21} + C$

27. $\int \frac{x}{(x^2-1)^{11}} \, dx$ Let $u = x^2-1$ $du = 2x\,dx$

$= \frac{1}{2} \int (x^2-1)^{-11} \, 2x\,dx$

$= \left(\frac{1}{2}\right)\left(\frac{-1}{10}\right)(x^2-1)^{-10} + C$

$= \frac{-1}{20}(x^2-1)^{-10} + C$

28. $\int \sin x \, e^{\cos x} \, dx$ Let $u = \cos x$ $du = -\sin x \, dx$

$= -1 \int e^{\cos x} (-1) \sin x \, dx$

$= (-1)\, e^{\cos x} + C$

29. $\int (\sin 2t)^3 \cos 2t \, dt$ Let $u = \sin 2t$ $du = 2\cos 2t \, dt$

$\frac{1}{2} \int (\sin 2t)^3 \, 2\cos 2t \, dt$

$= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)(\sin 2t)^4 + C = \frac{1}{8} \sin^4 2t + C$

8.3 - Continued

30. $\int \sin x \sqrt{\cos x}\, dx$ let $u = \cos x$ $du = -\sin x\, dx$

$= -1 \int (\cos x)^{1/2} (-1) \sin x\, dx$

$= (-1)\left(\frac{2}{3}\right)(\cos x)^{3/2} + C$

$= -\frac{2}{3} (\cos x)^{3/2} + C$

31. $\int x^2 \sqrt[4]{x^3+4}\, dx$ Let $u = x^3+4$ $du = 3x^2 dx$

$= \frac{1}{3} \int (x^3+4)^{1/4}\, 3x^2 dx$

$= \frac{1}{3}\left(\frac{4}{5}\right)(x^3+4)^{5/4} + C$

$= \frac{4}{15} (x^3+4)^{5/4} + C$

32. $\int \frac{x+1}{x^2+2x-5}\, dx$ Let $u = x^2+2x-5$ $du = (2x+2)dx$

$= \frac{1}{2} \int (x^2+2x-5)^{-1}\, 2(x+1) dx$

$= \left(\frac{1}{2}\right) \ln |x^2+2x-5| + C$

33. $\int e^x (e^x+1)^4\, dx$ Let $u = e^x+1$ $du = e^x dx$

$= \frac{1}{5}(e^x+1)^5 + C$

34. $\int \frac{(\ln x)^4}{x}\, dx$ Let $u = \ln x$ $du = \frac{1}{x} dx$

$= \frac{1}{5} (\ln x)^5 + C$

8.3- Continued

35. $\int \cos^6 x \sin x \, dx$ let $u = \cos x$ $du = -\sin x \, dx$

$= -1 \int (\cos x)^6 \, -1 \sin x \, dx$

$= (-1) \frac{1}{7} (\cos x)^7 + C$

$= -\frac{1}{7} \cos^7 x + C$

36. $\int \frac{x+5}{x^2+10x-23} dx$ let $u = x^2+10x-23$ $dx = (2x+10) dx$

$= \frac{1}{2} \int (x^2+10x-23)^{-1} \, 2(x+5) dx$

$= \left(\frac{1}{2}\right) \ln|x^2+10x-23| + C$

37. $\int e^{4x} \cos(e^{4x}) dx$ let $u = e^{4x}$ $du = 4e^{4x} dx$

$= \frac{1}{4} \int \cos(e^{4x}) \, 4e^{4x} dx$

$= \left(\frac{1}{4}\right) \sin(e^{4x}) + C$

38. $\int x^4 \cos x^5 \, dx$ let $u = x^5$ $du = 5x^4 dx$

$= \frac{1}{5} \int \cos x^5 \, 5x^4 dx$

$= \left(\frac{1}{5}\right) \sin x^5 + C$

39. $\int x^3 \sin x^4 (\cos^5 x^4) dx$ let $u = \cos x^4$ $du = -\sin x^4 \cdot 4x^3 dx$
$= -4x^3 \sin x^4 dx$

$= -\frac{1}{4} \int (\cos x^4)^5 \, -4x^3 \sin x^4 dx$

$= -\frac{1}{4}\left(\frac{1}{6}\right)(\cos x^4)^6 = -\frac{1}{24} \cos^6(x^4) + C$

8.3 - Continued

40. $\int x^2 e^{-x^3}\,dx$ Let $u = -x^3$ $du = -3x^2\,dx$

$= -\frac{1}{3}\int e^{-x^3}(-3)x^2\,dx$

$= -\left(\frac{1}{3}\right)e^{-x^3} + C$

41. $\int e^{-\sin x}\cdot\cos x\,dx$ let $u = -\sin x$ $du = -\cos x\,dx$

$= -1\int e^{-\sin x}(-1)\cos x\,dx$

$= -1\,e^{-\sin x} + C$

42. $\int x\cos(x^2-1)\,dx$ Let $u = x^2-1$ $du = 2x\,dx$

$= \frac{1}{2}\int \cos(x^2-1)\,2x\,dx$

$= \left(\frac{1}{2}\right)\sin(x^2-1) + C$

43. $\int \frac{\sqrt{\ln x}}{x}\,dx$ Let $u = \ln x$ $du = \frac{1}{x}\,dx$

$= \frac{2}{3}(\ln x)^{3/2} + C$

44. $\int \sin x\cos x\,dx$ Let $u = \sin x$ $du = \cos x\,dx$

$= \frac{1}{2}(\sin x)^2 + C$

$= \frac{1}{2}\sin^2 x + C$

45. $\int \frac{\cos x}{\sin x}\,dx$ let $u = \sin x$ $du = \cos x\,dx$

$= \int (\sin x)^{-1}\cos x\,dx = \ln|\sin x| + C$

8.3 - Continued

46 $\int \frac{e^{1/x}}{x^2}\,dx$ Let $u = \frac{1}{x} = x^{-1}$ $du = -1x^{-2}\,dx$

$= -1 \int e^{1/x}(-1)x^{-2}\,dx$

$= (-1)(e^{\frac{1}{x}}) + C$

47. $\int \frac{x}{(x^2+1)^{5/2}}\,dx$ Let $u = x^2+1$ $du = 2x\,dx$

$= \frac{1}{2}\int (x^2+1)^{-5/2}\,2x\,dx$

$= \frac{1}{2}\left(\frac{-2}{3}\right)(x^2+1)^{-3/2} + C$

$= \frac{-1}{3}(x^2+1)^{-3/2} + C$

48 $\int 3x^2\cos x^3\,dx$ Let $u = x^3$ $du = 3x^2\,dx$

$= \sin x^3 + C$

49. $\int (x^3-5)(e^{x^4-20x})\,dx$ Let $u = x^4 - 20x$ $du = 4x^3 - 20$

$\frac{1}{4}\int e^{x^4-20x}\,4(x^3-5)\,dx$

$= \left(\frac{1}{4}\right)e^{x^4-20x} + C$

50. $\int (\cos 2t)\,e^{\sin 2t}\,dt$ Let $u = \sin 2t$ $du = 2\cos 2t\,dt$

$\frac{1}{2}\int e^{\sin 2t}\,2\cos 2t\,dt$

$= \frac{1}{2}e^{\sin 2t} + C$

51. $\int (1+\sin x)^4\cos x\,dx$ Let $u = 1+\sin x$ $du = \cos x\,dx$

$= \frac{1}{5}(1+\sin x)^5 + C$

8.3 Continued

52. $\int x^{1/3}\sqrt{x^{4/3}+1}\,dx$ let $u = x^{4/3}+1$ $du = \frac{4}{3}x^{1/3}dx$

$= \frac{3}{4}\int (x^{4/3}+1)^{1/2}\left(\frac{4}{3}\right)x^{1/3}dx$

$= \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(x^{4/3}+1\right)^{3/2} + C$

$= \frac{1}{2}\left(x^{4/3}+1\right)^{3/2} + C$

53. $\int \frac{\cos x}{1+\sin x}dx$ let $u = 1+\sin x$ $du = \cos x\,dx$

$= \int (1+\sin x)^{-1}\cos x\,dx$

$= \ln|1+\sin x| + C$

54. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx$ let $u = e^x - e^{-x}$ $du = (e^x+e^{-x})dx$

$= \int (e^x-e^{-x})^{-1}(e^x+e^{-x})\,dx$

$= \ln|e^x - e^{-x}| + C$

55. $\int \frac{e^x}{1+e^{2x}}dx$ let $u = e^x$ $du = e^x dx$

$= \int \frac{1}{1+e^{2x}}e^x dx$

$= \tan^{-1}(e^x) + C$

$\left(\begin{array}{l} y = \tan^{-1}u \\ y' = \frac{1}{1+u^2}\frac{du}{dx} \end{array} \right)$

56. $\int \frac{1}{\sqrt{1-25x^2}}dx$ $y = \sin^{-1}x$ let $u = 5x$ $du = 5dx$

$y' = \frac{1}{\sqrt{1-x^2}}$

$= \frac{1}{5}\int \frac{1}{\sqrt{1-25x^2}}5\,dx$

$= \frac{1}{5}\sin^{-1}5x + C$

8.3 Continued

57. $\int x^2 \sec^2(x^3)\,dx$ Let $u = x^3$ $du = 3x^2\,dx$

$= \frac{1}{3}\int \sec^2 x^3\, 3x^2\,dx$

$= \left(\frac{1}{3}\right) \tan x^3 + C$

58. $\int \cot^3 x \csc^2 x\,dx$ Let $u = \cot x$ $du = -\csc^2 x\,dx$

$= -\int (\cot x)^3 \csc^2 x\,(-1)\,dx$

$= -\frac{1}{4}(\cot x)^4 + C$

$= -\frac{1}{4}\cot^4 x + C$

59. $\int \sec^3 2x \tan 2x\,dx$ Let $u = \sec 2x$ $du = 2\sec 2x \tan 2x\,dx$

$= \frac{1}{2}\int (\sec 2x)^2\, 2\sec 2x \tan 2x\,dx$

$= \frac{1}{2}\left(\frac{1}{3}\right)(\sec 2x)^3 + C$

$= \frac{1}{6}\sec^3 2x + C$

(?) 60. $\int \frac{2}{x^2+25}\,dx$ Let $u = \frac{1}{5}x$ $du = \frac{1}{5}dx$

$\frac{1}{1+\left(\frac{1}{5}x\right)^2}\cdot\frac{1}{5} = \frac{1}{1+\frac{1}{25}x^2}\cdot\left(\frac{1}{5}\right) = \frac{1}{\frac{25+x^2}{25}}\cdot\frac{1}{5} = \frac{25}{x^2+25}\left(\frac{1}{5}\right)$

$= \frac{5}{x^2+25}$

$= \frac{2}{5}\tan^{-1}\left(\frac{1}{5}x\right) + C$

61. $\int e^x \csc^2 e^x \cot e^x\,dx$ Let $u = e^x$ $du = e^x\,dx$

$= -\frac{1}{2}\cot^2 e^x + C$

62. $\int \frac{\cos x}{1+\sin^2 x}\,dx$ let $u = \sin x$ $du = \cos x\,dx$

$= \tan^{-1}(\sin x) + C$