8.4 Logarithmic and Exponential Equations

In this section we are going to learn how to solve **logarithmic and exponential equations.**

A **logarithmic equation** is an equation containing the logarithm of a value.

For example $2log_x = log36$

Equality Statements for Exponential and Logarithmic Equations

The following equality statements are useful when solving an exponential equation or a logarithmic equation.

Given $c, L, R > 0$ and $c \neq 1$,

- if $\log_c L = \log_c R$, then $L = R$
- if $L = R$, then $\log_c L = \log_c R$

$$2^4 = 2^x$$

$$\log_3 17 = \log_3 x$$

Solving Logarithmic Equations

Example 1: a) Solve $\underbrace{log_7 x + log_7 4} = log_7 12$

$$\log_7(4x) = \log_7 12$$

$$4x = 12$$

$$\boxed{x = 3}$$

b) Solve: $log_2(x-6) = 3 - log_2(x-4)$

$$\log_2(x-6) + \log_2(x-4) = 3$$

$$\log_2(x-6)(x-4) = 3$$

$$2^3 = (x-6)(x-4)$$

$$8 = x^2 - 10x + 24$$

$$0 = x^2 - 10x + 16$$

$$0 = (x-8)(x-2)$$

$\boxed{x=8}$ ✓

OR

$\boxed{\cancel{x=2}}$

c) Solve: $log_3(x^2 - 8x)^5 = 10$

$$\frac{\cancel{5}\log_3(x^2 - 8x)}{\cancel{5}} = \frac{10}{5}$$

$$\log_3(x^2 - 8x) = 2$$

$$3^2 = x^2 - 8x$$

$$0 = x^2 - 8x - 9$$

$$0 = (x-9)(x+1)$$

$$x = 9 \qquad x = -1$$

$$(-1)^2 - 8(-1)$$

$$1 + 8$$

$$= 9$$

Solving Exponential Equations Using Logarithms

Example 2: Solve rounding your answer to $\boxed{\text{2 decimal places.}}$

$a)\ 2^x = 2500$

$$\log 2^x = \log 2500$$

$$\frac{x \cancel{\log 2}}{\cancel{\log 2}} = \frac{\log 2500}{\log 2}$$

$$x = 11.29$$

$b)\ 5^{x-3} = 1700$

$$\log 5^{x-3} = \log 1700$$

$$\frac{(x-3)\cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 1700}{\log 5}$$

$$x-3 = \left(\frac{\log 1700}{\log 5}\right)$$

$$x = \left(\frac{\log 1700}{\log 5}\right) + 3$$

$$x = 7.62$$

$6^{5.05} = 8^{4.35}$

c) $6^{3x+1} = 8^{x+3}$

$\log 6^{3x+1} = \log 8^{x+3}$

$(3x+1)\log 6 = (x+3)\log 8$

$x3\log 6 + \log 6 = x\log 8 + 3\log 8$

$x3\log 6 - x\log 8 = 3\log 8 - \log 6$

$x(\cancel{3\log 6 - \log 8}) = \dfrac{(3\log 8 - \log 6)}{(3\log 6 - \log 8)}$

$X = 1.35$

$$4^{x-3} = 7^{x+2}$$

$$(x-3)\log 4 = (x+2)\log 7$$

$$(x-3)(.6021) = (x+2)(.8451)$$

$$.6021x - 1.8063 = .8451x + 1.6902$$

$$\frac{\cancel{-.243}x}{\cancel{-.243}} = \frac{3.4932}{-.243}$$

$$x = -14.38$$

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Example 6

Determine how long \$1000 needs to be invested in an account that earns 8.3% compounded semi-annually before it increases in value to \$1490.

$$A = P\left(1 + \frac{i}{n}\right)^{nt}$$

$P = 1000$

$n = 2$

$A = 1490$

$i = .083$

$t = ?$

$$\frac{1490}{1000} = \frac{\cancel{1000}\left(1 + \frac{.083}{2}\right)^{2t}}{\cancel{1000}}$$

$$1.490 = (1.0415)^{2t}$$

$$\log 1.490 = \log (1.0415)^{2t}$$

$$\frac{\log 1.490}{2\log 1.0415} = \frac{\cancel{2}t\,\cancel{(\log 1.0415)}}{\cancel{2\log 1.0415}}$$

$$4.90\,\text{yrs} = t$$

Application: Example #4:

Palaeontologists can estimate the size of a dinosaur from incomplete skeletal remains. For a carnivorous dinosaur, the relationship between the length, s, in metres, of the skull and the body mass, m, in kilograms, can be expressed using the logarithmic equation $3.6022 \log s = \log m - 3.4444$. Determine the body mass, to the nearest kilogram, of an Albertosaurus with a skull length of 0.78 m.

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$$3.6022 \log .78 = \log m - 3.4444$$

$$\underbrace{3.4444 + 3.6022 \log .78} = \log m$$

$$3.0557 = \log_{10} m$$

$$10^{3.0557} = m$$

$$1136.84\ kg = m$$

Your Turn

To the nearest hundredth of a metre, what was the skull length of a Tyrannosaurus rex with an estimated body mass of 5500 kg?

Application: Exponential Decay Example #5:

Solve a Problem Involving Exponential Growth and Decay

When an animal dies, the amount of radioactive carbon-14 (C-14) in its bones decreases. Archaeologists use this fact to determine the age of a fossil based on the amount of C-14 remaining.
The half-life of C-14 is 5730 years.

Buffalo skull display, Head-Smashed-In buffalo Jump Visitor Centre, near Fort McLeod, Alberta

Head-Smashed-In Buffalo Jump in southwestern Alberta is recognized as the best example of a buffalo jump in North America. The oldest bones unearthed at the site had 49.5% of the C-14 left. How old were the bones when they were found?

$$\boxed{m(t) = m_0\left(\frac{1}{2}\right)^{\frac{t}{h}}}$$

$m_0 = 100$

$m(t) = 49.5$

$h = 5730$

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$m_0 = 100$

$m(t) = 49.5$

$h = 5730$

$$\frac{49.5}{100} = \frac{100\left(\frac{1}{2}\right)^{\frac{t}{5730}}}{100}$$

$$.495 = \left(\frac{1}{2}\right)^{t/5730}$$

$$\frac{5730\left(\log .495\right)}{\log 1/2} = \frac{\frac{t}{\cancel{5730}}\log\frac{1}{2}}{\log 1/2}$$

$$5813\ \text{yrs} = t$$

Your Turn

The rate at which an organism duplicates is called its doubling period. The general equation is $N(t) = N_0(2)^{\frac{t}{d}}$, where N is the number present after time t, N_0 is the original number, and d is the doubling period. *E. coli* is a rod-shaped bacterium commonly found in the intestinal tract of warm-blooded animals. Some strains of *E. coli* can cause serious food poisoning in humans. Suppose a biologist originally estimates the number of *E. coli* bacteria in a culture to be 1000. After 90 min, the estimated count is 19 500 bacteria. What is the doubling period of the *E. coli* bacteria, to the nearest minute?

$N_0 = 1000$

$N(t) = 19500$

$t = 90$

$d = ?$

$$\frac{19500}{1000} = \frac{1000\left(2^{90/d}\right)}{1000}$$

$$19.5 = 2^{90/d}$$

$$d \log 19.5 = \frac{90}{\cancel{d}} \log 2$$

$$d = \frac{90 \log 2}{\log 19.5}$$

$$d = 21 \text{ mins}$$

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