8.4 Definite Integration

Definite integration is used in many applications of calculus. Today we will learn how to evaluate a definite integral and what a definite integral really means from a graphical sense. In future math classes you will have the opportunity to see where definite integrals apply.

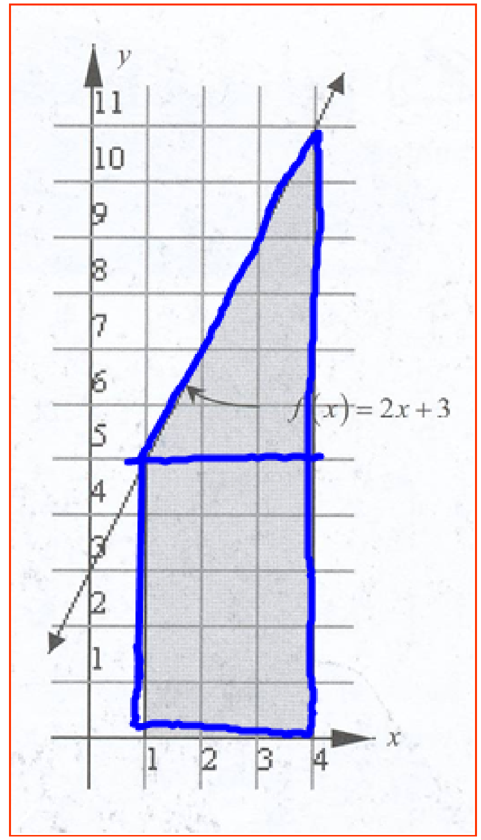
Say we are asked to evaluate the following:

$$\int_{1}^{4}(2x+3)dx$$

What is different between this integral and the one's we did before?

If we were to graph (2x+3) we would get the following:

Calculate the area of the shaded region.



How does this compare to the answer for $\int_1^4 (2x+3)dx$?

$= (3)(5) + \frac{1}{2}(3)(6)$

$= 15 + 9 = 24$

Based on the previous result we can conclude:

$$\int_a^b f(x) = \text{area under the graph of } f(x) \text{ from } a \text{ to } b.$$

$$\int_1^4 (2x+3)\,dx$$

$$= x^2 + 3x \Big|_1^4$$

$$= \left[(4)^2 + 3(4)\right] - \left[(1)^2 + 3(1)\right]$$

$$= 28 - (4) = 24$$

In general to evaluate a definite integral we do the following:

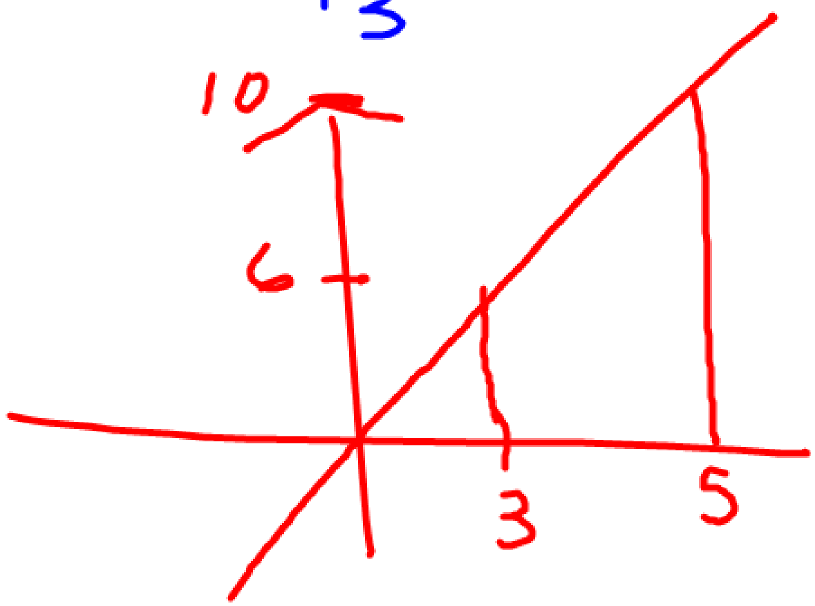
$$\int_a^b f(x)dx = F(b) - F(a), \text{ where } F \text{ is the integral of } f.$$

F

Ex.1 Evaluate the following definite integrals:

$$\text{a) } \int_3^5 2x\,dx$$

$$= x^2 \Big|_3^5 = (5)^2 - (3)^2 = 25 - 9 = 16$$



$$\text{b) } \int_1^3 \left(x^2 + 2x\right) dx$$

$$= \frac{x^3}{3} + x^2 \Big|_1^3$$

$$= \left[\frac{(3)^3}{3} + (3)^2\right] - \left[\frac{1}{3} + 1\right]$$

$$= 18 - \frac{4}{3}$$

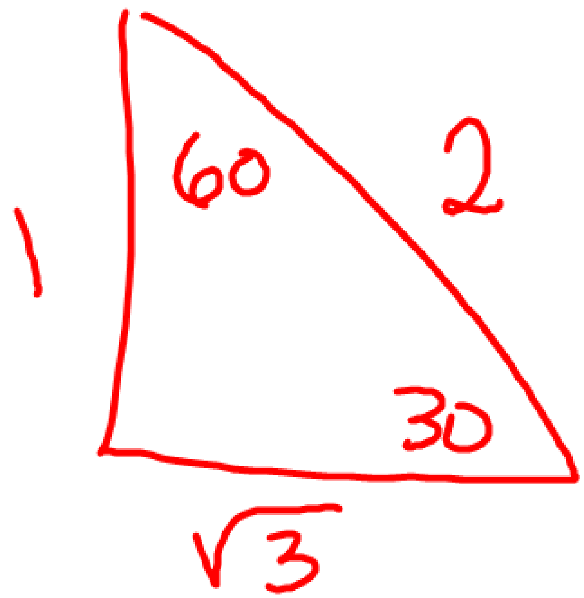
$$= \frac{54}{3} - \frac{4}{3} = \frac{50}{3}$$

c) $\displaystyle\int_{\pi/3}^{\pi/6} \cos x\,dx$

$$\left.\sin x\right|_{\pi/3}^{\pi/6}$$

$$= \sin \frac{\pi}{6} - \sin \frac{\pi}{3}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$$



d) $\displaystyle\int_{\pi/30}^{\pi/10} \sin 5x\,dx$

$$= -\frac{\cos 5x}{5}\Bigg|_{\pi/30}^{\pi/10}$$

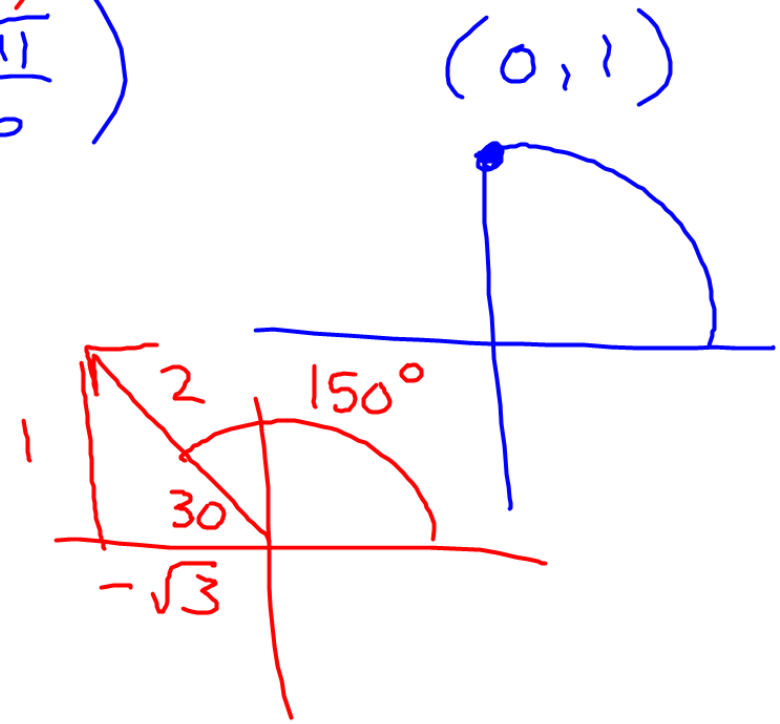
$$= -\frac{1}{5}\left(\cos 5x \Big|_{\pi/30}^{\pi/10}\right)$$

$$= -\frac{1}{5}\left(\left(\cos 5\left(\frac{\pi}{10}\right)\right) - \cos 5\left(\frac{\pi}{30}\right)\right]$$

$$= -\frac{1}{5}\left(\overset{0}{\cancel{\cos\frac{\pi}{2}}} - \overset{(-\sqrt{3}/2)}{\cos\frac{5\pi}{6}}\right)$$

$$= -\frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{\sqrt{3}}{10}$$



f) $\int_5^{10} \frac{1}{s}\,ds$

$$= \ln|s| \Big|_5^{10}$$

$$= \ln 10 - \ln 5$$

$$= \ln\left(\frac{10}{5}\right)$$

$$= \ln 2$$

http://archives.math.utk.edu/visual.calculus/4/ftc.10/index.html

Assignment
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2,4,5,7,8,10,12,15,16,18,19

7 $\displaystyle\int_{-4}^{-2} \frac{1}{x^2}\,dx = \int_{-4}^{-2} x^{-2}\,dx$

$= -x^{-1} \Big|_{-4}^{-2}$

$$-\frac{1}{x}\Big|_{-4}^{-2}$$

$$= -\frac{1}{-2} - \left(-\frac{1}{-4}\right)$$

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

8. $$\int_1^3 \frac{w^2+1}{w}\,dw$$

$$\int_1^3 \left(\frac{w^2}{w} + \frac{1}{w}\right) dw = \int_1^3 \left(w + \frac{1}{w}\right) dw$$

$$= \frac{w^2}{2} + \ln|w| \Big|_1^3$$

$$\left(\frac{(3)^2}{2} + \ln 3\right) - \left(\frac{(1)^2}{2} + \ln 1\right)$$

$$= \frac{9}{2} + \ln 3 - \frac{1}{2} - \cancel{\ln 1} = 0$$

$$= 4 + \ln 3$$

15

$$\int_{\pi/8}^{\pi/4} \sin 4x \, dx$$

S
C
−S
−C
S

$$= -\frac{\cos 4x}{4} \Bigg|_{\pi/8}^{\pi/4}$$

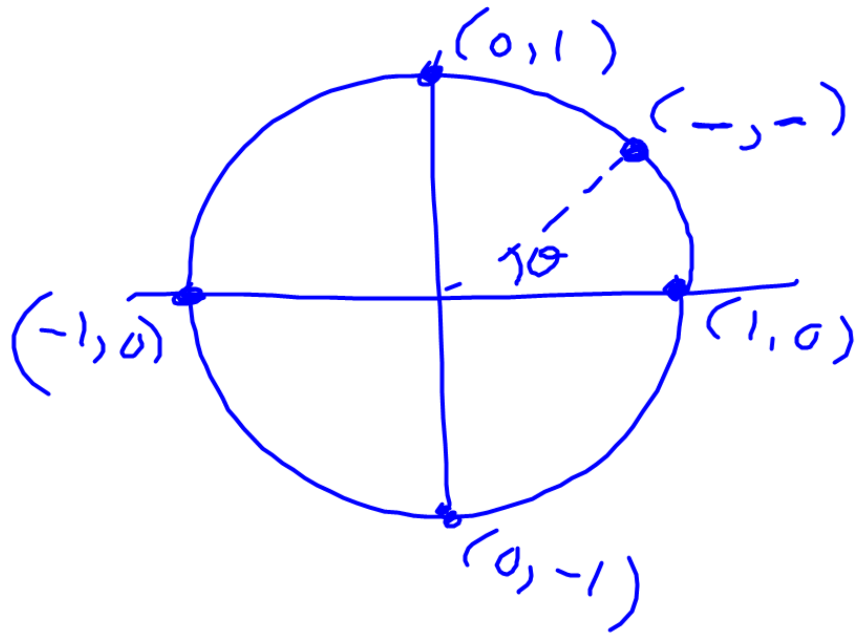
$$= -\frac{1}{4}\left[\cos 4x \Big|_{\pi/8}^{\pi/4}\right]$$

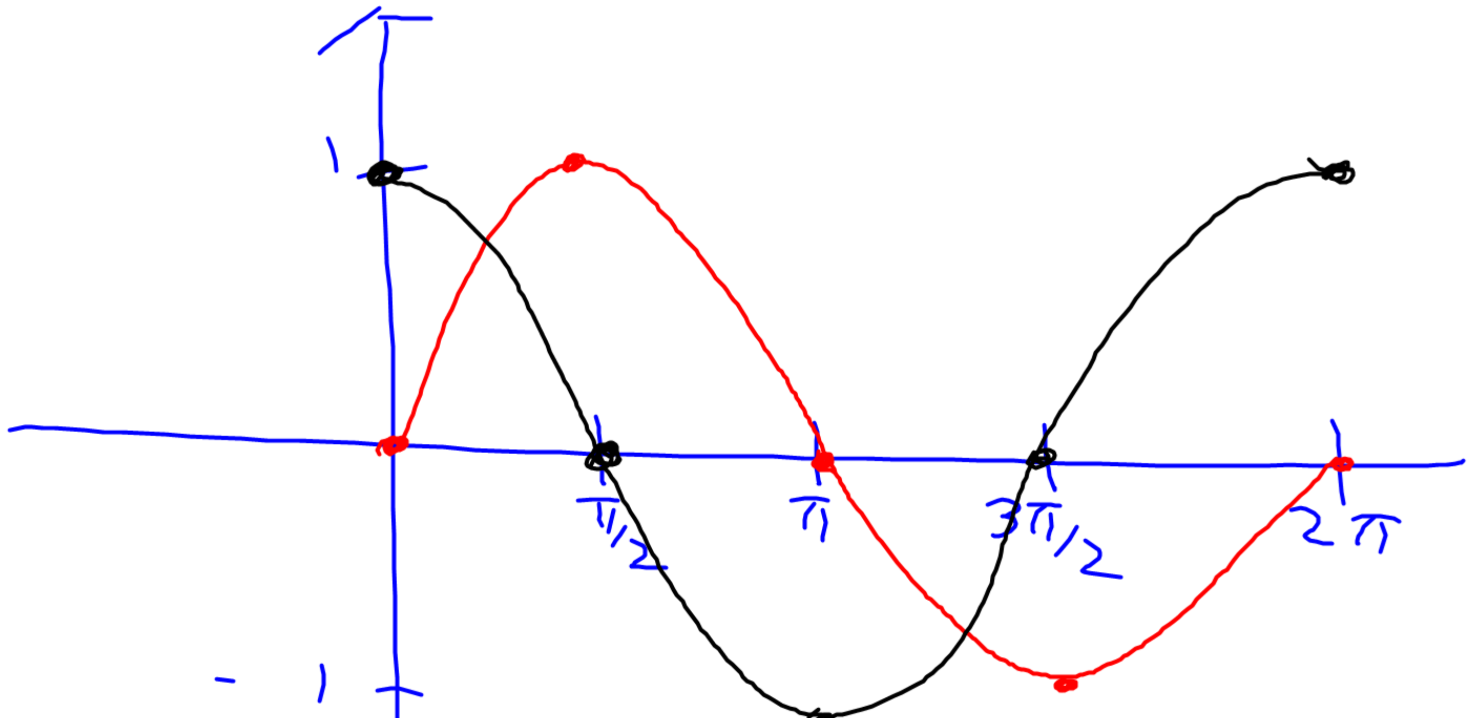
$$= -\frac{1}{4}\left[\cos 4\left(\pi/4\right) - \cos 4\left(\pi/8\right)\right]$$

$$= -\frac{1}{4}\left[\cos \pi - \cos \pi/2\right]$$

$$= -\frac{1}{4}\left[-1 - 0\right]$$

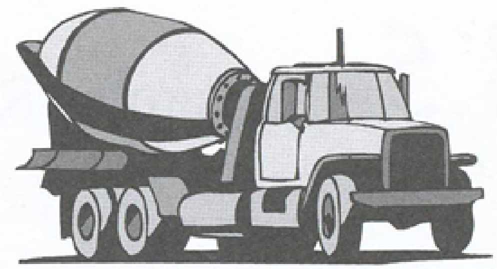
$$= 1/4$$





1
-1
π/2
π
3π/2
2π

39. If $p(t)$ is the rate of depreciation of a vehicle t years after being purchased, then $\int_a^b p(t)dt$ gives the total amount the automobile has depreciated between the a^{th} and b^{th} years. Suppose the rate of depreciation of a concrete truck can be modelled by the function $p(t) = 1200(8-t)$ for $0 \le t \le 7$.



(a) At what rate is the truck depreciating after 1 year?
(b) At what rate is the truck depreciating after 7 years?
(c) By what amount did the truck depreciate between $t = 0$ and $t = 3$?
(d) By what amount did the truck depreciate between $t = 3$ and $t = 6$?