

8.3 Integration By U-Substitution

Say we were asked to find the antiderivative of the following:

$$\int (x+2)^5 dx$$

We could expand and integrate each term, but that would take a long time!

$$\text{let } u = x + 2$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$= \frac{(x+2)^6}{6} + C$$

$$= \int u^5 du$$
$$= \frac{u^6}{6} + C$$

Example 1: Find the following indefinite integrals.

$$\text{a) } \int \sqrt{4x-1} dx$$

$$\text{let } u = 4x - 1$$

$$du = 4 dx$$

$$\frac{1}{4} du = dx$$

$$\int u^{1/2} \left(\frac{1}{4} du \right)$$

$$= \frac{1}{4} \int u^{1/2} du$$

$$= \frac{1}{4} \left[\frac{2}{3} u^{3/2} + C \right]$$

$$= \frac{1}{6} (4x-1)^{3/2} + C$$

$$\text{b) } \int (2x^3 - 1)^5 \underbrace{x^2 dx}$$

$$\text{let } u = 2x^3 - 1$$

$$du = 6x^2 dx$$

$$\frac{1}{6} du = x^2 dx$$

$$\begin{aligned} \frac{1}{6} \int u^5 du &= \frac{1}{6} \left[\frac{u^6}{6} + C \right] \\ &= \frac{u^6}{36} + C = \frac{(2x^3 - 1)^6}{36} + C \end{aligned}$$

$$\text{c) } \int \frac{4x^2}{(x^3 - 1)^3} dx$$

$$\text{let } u = x^3 - 1$$

$$\frac{4}{3} du = 4x^2 dx$$

$$\frac{4}{3} \int \frac{1}{u^3} du$$
$$\frac{4}{3} \int u^{-3} du$$

$$= \frac{4}{3} \left[\frac{u^{-2}}{-2} + C \right]$$
$$= -\frac{2}{3} u^{-2} + C$$
$$= -\frac{2}{3} (x^3 - 1)^{-2} + C$$

$$\text{d) } \int (x^2 + 2x - 3)^2 (x + 1) dx$$

Trigonometric Integrands

Example 2: Find the following indefinite integrals.

a) $\int \cos(7x + 5) dx$

$$\text{c) } \int \sin^4 x \cos x dx$$

2003 MC Question

8. $\int x^2 \cos(x^3) dx =$

(A) $-\frac{1}{3} \sin(x^3) + C$

(B) $\frac{1}{3} \sin(x^3) + C$

(C) $-\frac{x^3}{3} \sin(x^3) + C$

(D) $\frac{x^3}{3} \sin(x^3) + C$

(E) $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$

let $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3} \int \cos u du$

$= \frac{1}{3} [\sin u + C]$

$= \frac{1}{3} \sin(x^3) + C$

U-Substitution To Evaluate Definite Integrals

Example 3: Evaluate the following definite integrals.

$$\text{a) } \int_0^1 x(x^2 + 1)^5 dx$$

$$\text{let } u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

* Watch limits

$$= \frac{1}{2} \int_1^2 u^5 du$$

$$= \frac{1}{2} \left[\frac{u^6}{6} \right]_1^2$$

$$= \frac{1}{12} \left[u^6 \right]_1^2 = \frac{1}{12} [2^6 - 1^6] = \frac{63}{12} = \frac{21}{4}$$

$$\text{b) } \int_1^3 \frac{(x+1)}{x^2 + 2x + 5} dx$$

Assignment Page 366

#'s 2,5,7,10,11,14,19,28

Page 370

#'s 29, 33, 35