8.3 Laws of Logarithms

Log_{10}

ln

log_e

Laws of Logarithms

Product Law of Logarithms

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

For example: $log_5(xy) = log_5 x + log_5 y$

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

For example: $log_5\left(\frac{x}{y}\right) = log_5 x - log_5 y$

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^P = P \log_c M$$

For example: $log_5 x^3 = 3log_5 x$

Example 1

Write each expression in terms of individual logarithms of *x*,*y*, and *z*.

a) $log_6 \frac{x}{y}$

$= \log_6 x - \log_6 y$

b) $log_5 \sqrt{xy}$

$= \log_5 (xy)^{1/2}$

$= \frac{1}{2}\left[\log_5 (xy)\right]$

$= \frac{1}{2}\left[\log_5 x + \log_5 y\right]$

c) $log_3 \frac{9}{\sqrt[3]{x^2}}$

d) $log_7 \frac{x^5 y}{\sqrt{z}}$

c) $\log_3 \frac{9}{\sqrt[3]{x^2}}$

$= \log_3 \frac{9}{x^{2/3}}$

$= \log_3 9 - \log_3 x^{2/3}$

$= \log_3 9 - \left(\frac{2}{3}\right)\log_3 x$

c) $\log_3 \frac{9}{\sqrt[3]{x^2}}$

$= \log_3 \frac{9}{x^{2/3}}$

$= \log_3 9 - \log_3 x^{2/3}$

$= \log_3 9 - \left(\frac{2}{3}\right)\log_3 x$

d) $\log_7 \frac{x^5 y}{\sqrt{z}}$

$= \log_7 \frac{x^5 \cdot y}{z^{1/2}}$

$= \log_7 x^5 + \log_7 y - \log_7 z^{1/2}$

$= 5 \log_7 x + \log_7 y - \frac{1}{2} \log_7 z$

$\log_5 \frac{1000}{(4)(2)} = \log_5 125$

Example 2

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_3 9\sqrt{3}$

b) $(\log_5 1000 - \log_5 4) - \log_5 2$

c) $2 \log_3 6 - \frac{1}{2} \log_3 64 + \log_3 2$

a) $\log_3 9 + \log_3 \sqrt{3}$

$= \log_3 3^2 + \log_3 3^{1/2}$

$= 2 + \frac{1}{2}$

$= \frac{5}{2}$

$= \log_5\left(\frac{1000}{4}\right) - \log_5 2$

$= \log_5 250 - \log_5 2$

$= \log_5\left(\frac{250}{2}\right)$

$= \log_5 125$

$= \log_5 5^3 = 3$

c) $\log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$= \log_3 36 - \log_3 8 + \log_3 2$

$= \log_3\left(\frac{36 \cdot 2}{8}\right)$

$= \log_3\left(\frac{72}{8}\right)$

$= \log_3 9 = \log_3 3^2 = 2$

Example 3

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $4 \log_3 x - \frac{1}{2}(\log_3 x + 5 \log_3 x)$

b) $\log_2 (x^2 - 9) - \log_2 (x^2 - x - 6)$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

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