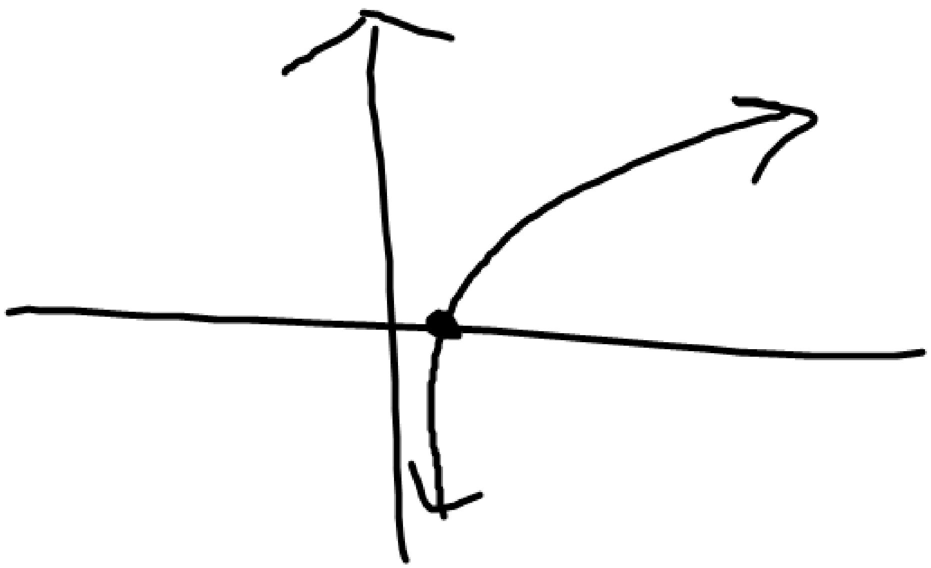
8.2 Transformations of Logarithmic Functions



The graph of the logarithmic function $y = a \log_c (b(x - h)) + k$ can be obtained by transforming the graph of $y = \log_c x$. The table below uses mapping notation to show how each parameter affects the point (x, y) on the graph of $y = \log_c x$.

$$y = a \log_c (b(x-h)) + k$$

Parameter	Transformation
a	$(x, y) \rightarrow (x, ay)$
b	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
h	$(x, y) \rightarrow (x + h, y)$
k	$(x, y) \rightarrow (x, y + k)$

These transformations are no different than all the other transformations we have done this year.

Base $y = \log_3 x$

$3^y = x$

x	y
$\frac{1}{3}$	-1
1	0
3	1
9	2

Example 1

a) Use transformations to sketch the graph of the function $y = \log_3 (x + 9) + 2$.

b) Identify the following characteristics of the graph of the function.

i) the equation of the asymptote

ii) the domain and range

iii) the y-intercept, if it exists

iv) the x-intercept, if it exists

Start by sketching $y = log_3 x$

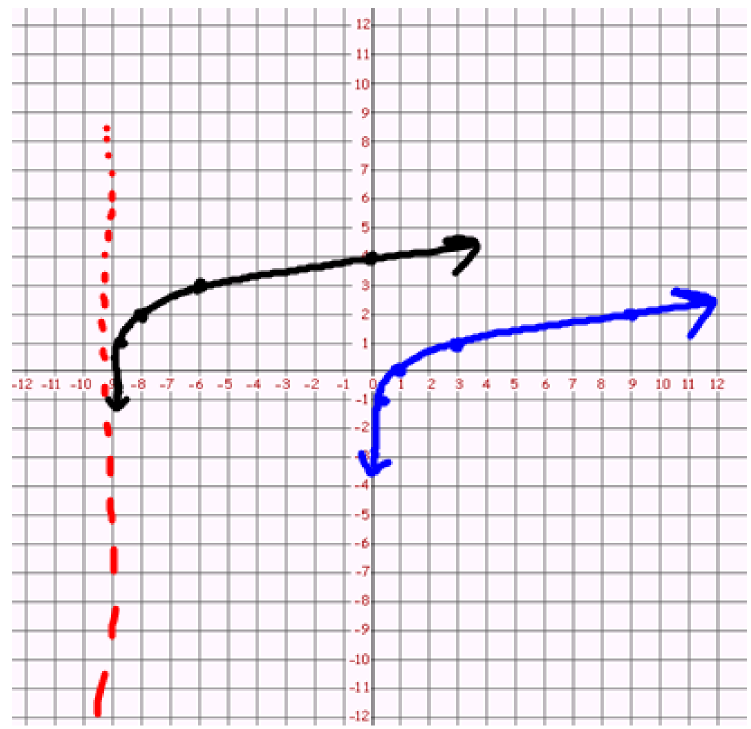
$a = 1$ $b = 1$

$h = -9$ $K = 2$

$\left(\frac{x}{b} + h, ay + k\right)$

$(x - 9, y + 2)$

x	y
$-8\frac{2}{3}$	1
-8	2
-6	3
0	4



b) i) $x = -9$

ii) D: $x > -9$

R: $y \in R$

iii) <u>y int</u>

$(0, 4)$

iv) <u>x int</u>

let $y = 0$

$0 = \log_3(x+9) + 2$

$-2 = \log_3(x+9)$

$3^{-2} = x + 9$

$\frac{1}{9} = x + 9$

$x = -8\frac{8}{9}$

Your Turn

a) Use transformations to sketch the graph of the function $y = 2\log_3(-x + 1)$.

b) Identify the following characteristics.

i) the equation of the asymptote

ii) the domain and range

iii) the y-intercept, if it exists

iv) the x-intercept, if it exists

$3^y = x$

x	y
$\frac{1}{3}$	-1
1	0
3	1
9	2

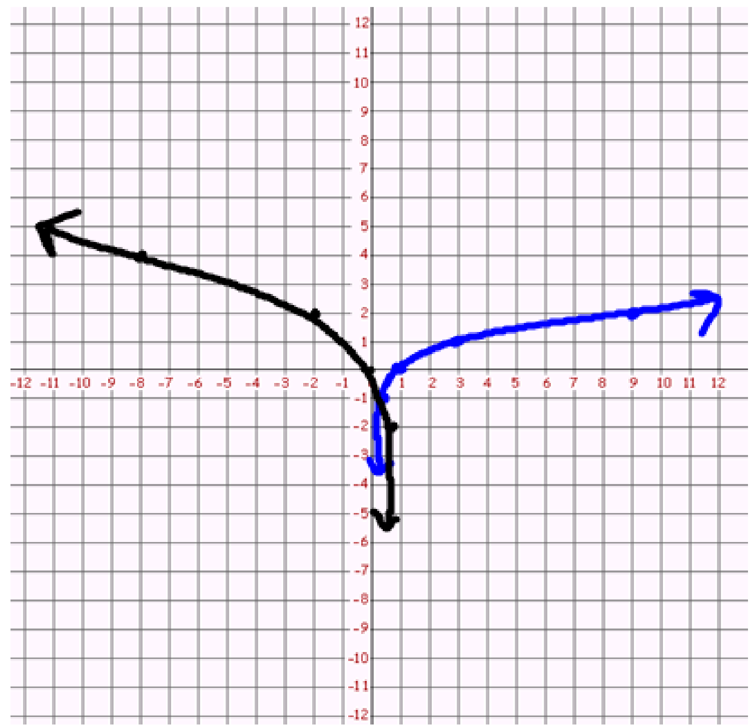
Start by sketching $y = log_3 x$

$y = 2\log_3(-(x-1))$

$a = 2$ $b = -1$ $h = 1$

$\left(\frac{x}{-1} + 1, 2y\right)$

x	y
$2/3$	-2
0	0
-2	2
-8	4



i) Asymptote

$x = 1$

ii) D: $x < 1$

R: $y \in R$

iii) <u>y int</u>

$(0,0)$

<u>x int</u>

$0 = 2\log_3(-x+1)$

$3^0 = 2(-x+1)$

$\frac{1}{2} = -x+1$

$x = 1/2$

Graph $y = 3\log_2(2x-4)+1$

Identify: a) x intercept b) y intercept NONE c) equation of vertical asymptote

$x = 2$

$y = 3\log_2(2(x-2))+1$

$y = \log_2 x$

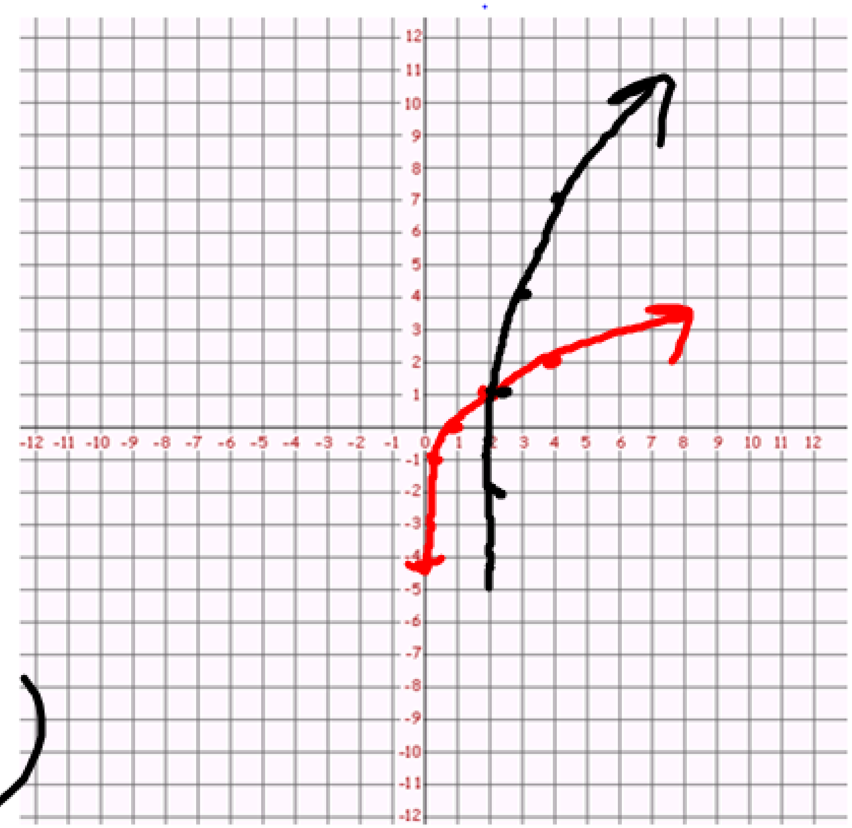
$2^y = x$

x	y
1/2	-1
1	0
2	1
4	2

$a = 3$
$b = 2$
$h = 2$
$k = 1$

$\left(\frac{x}{2}+2,\ 3y+1\right)$

x	y
2.25	-2
2.5	1
3	4
4	7



$$0 = 3\log_2(2x-4)+1$$

$$-1 = 3\log_2(2x-4)$$

$$-\frac{1}{3} = \log_2(2x-4)$$

$$\left(2^{-1/3}\right) = 2x-4$$

$$\left(\frac{1}{2}\right)^{1/3} = 2x-4$$

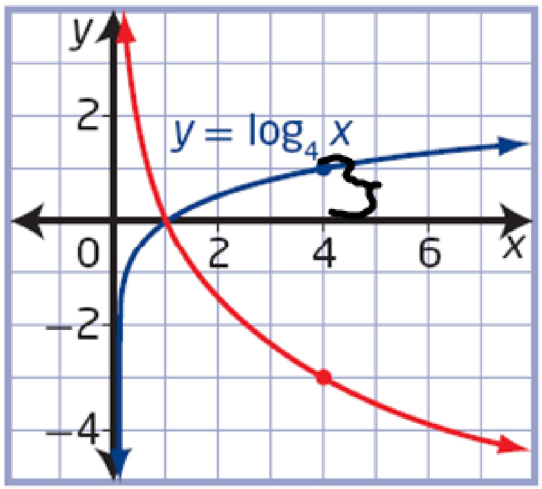
$$.79 = 2x-4$$

$$4.79 = 2x$$

$$2.4 = x$$

Your Turn

The red graph can be generated by stretching and reflecting the graph of $y = \log_4 x$. Write the equation that describes the red graph.



$a = -3$

$y = -3\log_4 x$

Application

Your Turn

There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, F, of flower species that a butterfly feeds on and the number, B, of butterflies ~~observed can be modelled by~~ the function $\boxed{F = -2.641 + 8.958 \log B.}$

Predict the number of butterfly observations in a region with 25 flower species.

F # flower species

B # butterflies

$F = 25$

$B = ?$

$$25 = -2.641 + 8.958 \log B$$

$$\frac{27.641}{8.958} = \frac{\cancel{8.958}(\log B)}{\cancel{8.958}}$$

$$3.086 = \log_{10} B$$

$$10^{3.086} = B$$

$$1219 \simeq B$$

Key Ideas

- To represent real-life situations, you may need to transform the basic logarithmic function $y = \log_b x$ by applying reflections, stretches, and translations. These transformations should be performed in the same manner as those applied to any other function.

- The effects of the parameters a, b, h, and k in $y = a \log_c (b(x - h)) + k$ on the graph of the logarithmic function $y = \log_c x$ are shown below.

Vertically stretch by a factor of $|a|$ about the x-axis. Reflect in the x-axis if $a < 0$.

Vertically translate k units.

$$y = a \log_c (b(x - h)) + k$$

Horizontally stretch by a factor of $\left|\frac{1}{b}\right|$ about the y-axis. Reflect in the y-axis if $b < 0$.

Horizontally translate h units.

- Only parameter h changes the vertical asymptote and the domain. None of the parameters change the range.

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#'s 1,3,5,6,7,9,10,13,14