Unit #8 Integration

8.2 Integration By Sight

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

Up to this point in the course we have been largely concerned with finding the derivative of a given function.

Today we are going to shift our focus to the reverse process.

of differentiation

* This reverse process is called many things: anti-derivative, integration, or finding the indefinite integral. *

Say we are asked to find a function, say $f(x)$, such that $f'(x) = 2x + 7$.

The notation we use is:

$$\boxed{\int (2x^{1} + 7)\,dx} = \frac{\cancel{2}x^2}{\cancel{2}} + 7x + C$$

$$= x^2 + 7x + C$$

integral

Example 1 Given $f'(x)$ find $f(x)$ for each of the following:

a) $f'(x) = 4x^3 + 6x - 10$

$$\int (4x^3 + 6x^1 - 10)\,dx = x^4 + 3x^2 - 10x + C$$

b) $f'(x) = 9x^2 - 4x + 2$

$$f(x) = \int (9x^2 - 4x + 2)\,dx$$

$$= 3x^3 - 2x^2 + 2x + C$$

Example 2: Find $f(x)$ if $f'(x) = 6x^{\frac{3}{4}} + 12x^{-\frac{2}{3}}$

$$f(x) = \int \left(6x^{3/4} + 12x^{-2/3}\right) dx$$

$$= \left(\frac{4}{7}\right) 6x^{7/4} + 3(12)x^{1/3} + C$$

$$= \frac{24}{7}x^{7/4} + 36x^{1/3} + C$$

Basic Integration Rules

$$\int c\,dx = cx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\boxed{\begin{aligned}&\int cf(x)\,dx = c\int f(x)\,dx\\&\int e^{x}\,dx = e^{x} + C\\&\int e^{kx}\,dx = \frac{e^{kx}}{k} + C\\&\int a^{kx}\,dx = \frac{a^{kx}}{k\ln a} + C\end{aligned}}$$

?

Example 2: Find $f(x)$ if $f'(x) = \cos x - e^x$

$$f(x) = \int (\cos x - e^x)\, dx$$

$$= \sin x - e^x + C$$

Example 3: Evaluate each integral.

a) $\int x^{-\frac{5}{6}} dx$

b) $\int \sqrt[5]{x^4}\,dx$

$$\int x^{4/5}\,dx$$

$$= \frac{5}{9}x^{9/5} + C$$

* c) $\int (\cos u - \cos 2u)\,du$

$$= \sin u - \frac{\sin 2u}{2} + C$$

$$= \sin u - \frac{1}{2}\sin 2u + C$$

$$\cos u - (\cos 2u)$$

d) $\int \left(e^{3x} - e^{-4x}\right)dx$

$$= \frac{e^{3x}}{3} - \frac{e^{-4x}}{-4} + C$$

$$= \frac{1}{3}e^{3x} + \frac{1}{4}e^{-4x} + C$$

e) $\int \left(x^3 - 4x^2 + 7x - \frac{1}{x} \right) dx$

$$= \frac{x^4}{4} - \frac{4}{3}x^3 + \frac{7}{2}x^2 - \ln|x| + c$$

Sometimes we need to simplify before integrating!

Example 4: Evaluate each integral.

a) $\int x^4(2x-3)dx$

$$= \int (2x^5 - 3x^4)\,dx$$

$$= \frac{1}{3}x^6 - \frac{3}{5}x^5 + C$$

b) $\int \left(x^2 + 3\right)^2 dx$

$*$ c) $\int \frac{2x^2 - 6x}{3x^3} dx$

$$= \int \left(\frac{2x^2}{3x^3} - \frac{6x}{3x^3} \right) dx$$

$$= \int \left(\frac{2}{3} \cdot \frac{1}{x} - 2x^{-2} \right) dx$$

$$= \frac{2}{3} \ln|x| - \frac{2x^{-1}}{-1} + C$$

$$= \frac{2}{3} \ln|x| + 2x^{-1} + C$$

Initial Value Problems

Find the function $f(x)$ that passes through the point (2,3) and has the derivative $f'(x) = 2x + 7$.

$$f(x) = \int (2x + 7)\,dx$$

$$f(x) = x^2 + 7x + C$$

$$3 = (2)^2 + 7(2) + C$$

$$3 = 4 + 14 + C$$

$$-15 = C$$

$$f(x) = x^2 + 7x - 15$$

Assignment

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Oral Exercises

#’ 1-29 odd

Written Exercises

#’s 12,13,15,17,20,21,
23,25,27,29,31,33,36,38