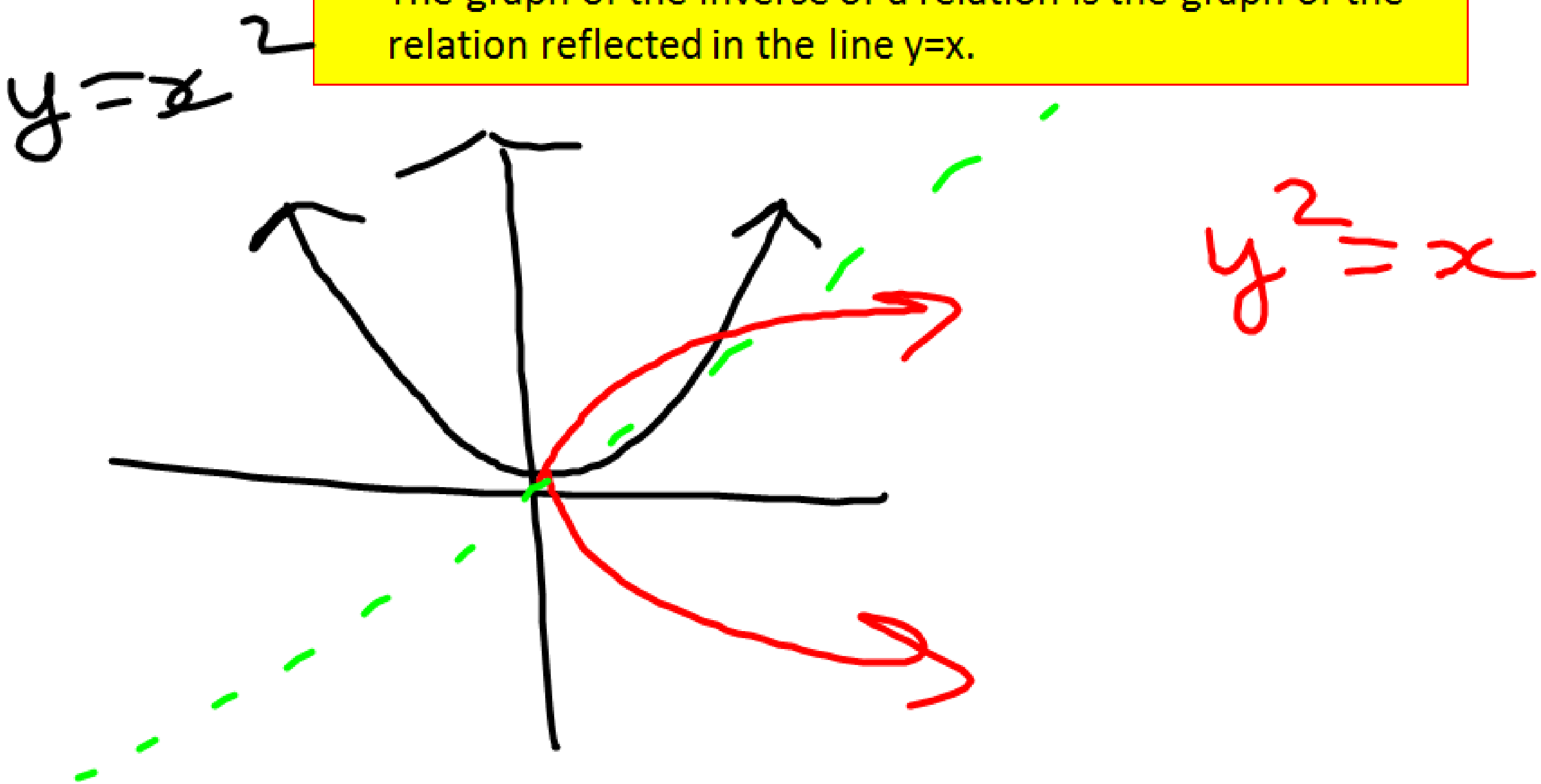
Chapter 8 Logarithmic Functions

8.1 Understanding Logarithms

Recall: In section 1.4 we studied inverse functions. Inverse functions have the following properties:

- You can find the inverse of a relation by interchanging the x and y coordinate.
- The graph of the inverse of a relation is the graph of the relation reflected in the line y=x.



For the exponential function $y=c^x$, the inverse is $x=c^y$.

This inverse $x=c^y$ is also known as the **logarithmic function.**

It is written in the form $y = log_c x$, where c is a positive number other than 1.

$$y = \log_c x$$

$$x = c^y$$

We know what the graph of $y = 3^x$ looks like.

x	y
0	1
1	3
2	9
-1	1/3

8.1 Graphs

Find the inverse equation of $y = 3^x$.
Graph the inverse.

$x = 3^y$

x	y
1	0
3	1
9	2
$\frac{1}{3}$	-1

$y = \log_3 x$

How else could I represent this inverse equation?

Logarithmic Form | **Exponential Form**

exponent

$log_c x = y$ $\qquad$ $c^y = x$

base

Since our number system is based on the number 10, logarithms with a base of 10 are called common logarithms.

When writing common logarithms you do not need to write the base, it is understood to be 10.
$y = log7$ is the same as $y = log_{10}7$

$$3^4 = 81$$

$$x = c^y$$

$$y = \log_c x$$

$$4 = \log_3 81$$

$$2^5 = 32$$

$$5 = \log_2 32$$

Ex. 1 Evaluate the following logarithms:

a) $log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$x = 5$

b) $log_7 1 = x$

$7^x = 7^0$

$x = 0$

c) $log_9 \sqrt[5]{81} = x$

$9^x = \sqrt[5]{81}$

$9^x = (81)^{1/5}$

$9^x = (9^2)^{1/5}$

$9^x = 9^{2/5}$

$x = 2/5$

d) $log 1000000 = x$

$10^x = 1\,000\,000$

$10^x = 10^6$

$x = 6$

e) ~~[illegible]~~

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$$2^x = 2^5$$

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b) $log_7 1 = x$

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$$9^x = \sqrt[5]{81}$$
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$$9^x = (9^2)^{1/5}$$
$$9^x = 9^{2/5}$$

$x = 2/5$

d) $log 1000000 = x$

$$10^x = 1000000$$
$$10^x = 10^6$$

$x = 6$

e) ~~[illegible]~~

Ex. 1 Evaluate the following logarithms:

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$x = 2/5$

d) $\log 1000000 = x$

$10^x = 1\,000\,000$

$10^x = 10^6$

$x = 6$

e) [illegible]

$$\log_3 9\sqrt{3} = x$$

$$3^x = 9\sqrt{3}$$

$$3^x = (3^2)(3^{1/2})$$

$$3^x = 3^{5/2}$$

$$x = 5/2$$

$$\log_2 32$$

$$\log_2 2^5$$

If $c > 0$ and $c \neq 1$, then the following statements are true:

- * $log_c 1 = 0$
- * $log_c c = 1$
- * $log_c c^x = x$
- $c^{log_c x} = x \; if \; x > 0$

$c^0 = 1$

$3^4 = 81$

$4 = log_3 81$

$c^{log_c x} = x$

$log_c x = log_c x$

$\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$x = 5$

$\log_2 32$ — 5

$\log_2 2$

$\log_2 16$ — 4

Ex. 2 Determine the value of x:

a) $log_4 x = -2$

$4^{-2} = x$

$\left(\frac{1}{4}\right)^2 = x = \frac{1}{16}$

b) $log_{16} x = -\frac{1}{4}$

$16^{-1/4} = x$

$\left(\frac{1}{16}\right)^{1/4} = x$

$\frac{1}{2} = x$

c) $log_x 9 = \frac{2}{3}$

$\left(x^{2/3}\right)^3 = (9)^3$

$x^2 = 729$

$x = \pm 27$

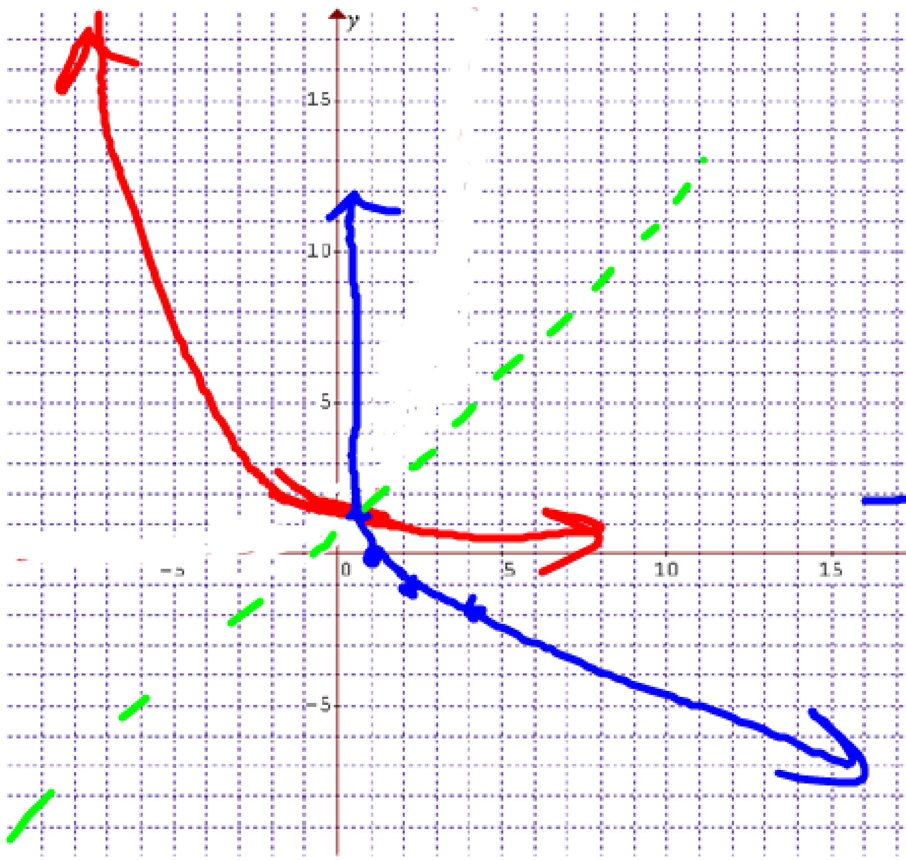
$x = 27$

Ex. 3 a) Write the inverse of ~~$y = 2^x$~~ $y = \left(\frac{1}{2}\right)^x$

b) The graph of f(x) is given below, sketch the inverse on the same graph and identify the following characteristics for the inverse:

- Domain and range
- x intercept (if it exists)
- y intercept (if it exists)
- The equations of any asymptotes

D: $x \in R$
R: $y > 0$



x int
NONE

y int
(0, 1)

HA
$y = 0$

D: $x > 0$
R: $y \in R$

VA
$x = 0$

y int
NONE

x int
(1, 0)

An Application of Logarithms

Naikoon Provincial Park, Haida Gwaii

In 1935, American seismologist Charles R. Richter developed a scale formula for measuring the magnitude of earthquakes. The Richter magnitude, M, of an earthquake is defined as $M = \log \frac{A}{A_0}$, where A is the amplitude of the ground motion, usually in microns, measured by a sensitive seismometer, and A_0 is the amplitude, corrected for the distance to the actual earthquake, that would be expected for a "standard" earthquake.

a) In 1946, an earthquake struck Vancouver Island off the coast of British Columbia. It had an amplitude that was $10^{7.3}$ times A_0. What was the earthquake's magnitude on the Richter scale?

b) The strongest recorded earthquake in Canada struck Haida Gwaii, off the coast of British Columbia, in 1949. It had a Richter reading of 8.1. How many times as great as A_0 was its amplitude?

c) Compare the seismic shaking of the 1949 Haida Gwaii earthquake with that of the earthquake that struck Vancouver Island in 1946.

a) $M = \log \frac{A}{A_0}$

$m = \log \frac{10^{7.3} A_0}{A_0}$

$m = \log_{10} 10^{7.3}$

$10^m = 10^{7.3}$

$m = 7.3$

b)

$$M = \log \frac{A}{A_0}$$

$$8.1 = \log_{10} \frac{\boxed{A}}{A_0}$$

$$10^{8.1} = \frac{A}{A_0}$$

$$10^{8.1} A_0 = A$$

Your Turn

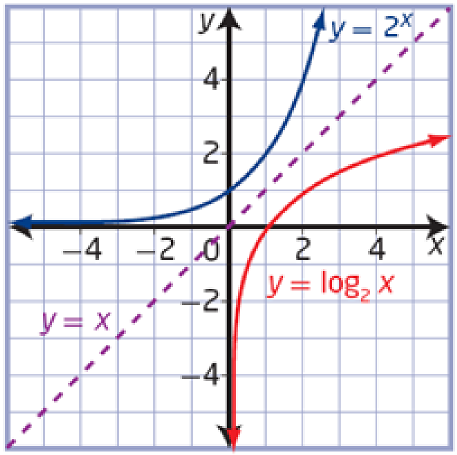
The largest measured earthquake struck Chile in 1960. It measured 9.5 on the Richter scale. How many times as great was the seismic shaking of the Chilean earthquake than the 1949 Haida Gwaii earthquake, which measured 8.1 on the Richter scale?

Key Ideas

- A logarithm is an exponent.
- Equations in exponential form can be written in logarithmic form and vice versa.

Exponential Form	**Logarithmic Form**
$x = c^y$	$y = \log_c x$

- The inverse of the exponential function $y = c^x$, $c > 0$, $c \neq 1$, is $x = c^y$ or, in logarithmic form, $y = \log_c x$. Conversely, the inverse of the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$, is $x = \log_c y$ or, in exponential form, $y = c^x$.
- The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown.



- For the logarithmic function $y = \log_c x$, $c > 0$, $c \neq 1$,
  - the domain is $\{x \mid x > 0, x \in R\}$
  - the range is $\{y \mid y \in R\}$
  - the x-intercept is 1
  - the vertical asymptote is $x = 0$, or the y-axis
- A common logarithm has base 10. It is not necessary to write the base for common logarithms:

$$\log_{10} x = \log x$$

Assignment

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