Unit 7 · Differentiating Transcendental Functions

7.1 Logarithmic Functions P. 303 1-7, 8a-g, 9a-f, 12, 13

1. a) $\log_9 81$
$= \log_9 9^2$
$= 2$

b) $\log_3 81$
$= \log_3 3^4$
$= 4$

c) $\log 1000$
$= \log_{10} 10^3$
$= 3$

d) $\log_2 (\frac{1}{4})$
$= \log_2 2^{-2}$
$= -2$

e) $\log_7 7$
$= \log_7 7^1$
$= 1$

F) $\log_6 1$
$= \log_6 6^0$
$= 0$

g) $\log_8 8^{13}$
$= 13$

h) $\log_{11} 11^{-20}$
$= -20$

i) $\ln 1 = x$
$\log_e 1 = x$
$e^x = 1$
$x = 0$

j) $\ln e = x$
$\log_e e = x$
$\log_e e^1 = x$
$1 = x$

k) $\ln e^5 = x$
$\log_e e^5 = x$
$5 = x$

l) $\ln e^{-7} = x$
$\log_e e^{-7} = x$
$-7 = x$

m) $\ln\left(\frac{1}{e}\right) = x$
$\log_e e^{-1} = x$
$-1 = x$

n) $\ln\left(\frac{1}{e^3}\right) = x$
$\log_e e^{-3} = x$
$-3 = x$

2 a) $\log_3 12 + \log_3 10$
$= \log_3 120$

b) $\ln 50 - \ln 5$
$= \ln 10$

c) $2\log_3 11$
$= \log_3 11^2$
$= \log_3 121$

d) $-2\ln 3$
$= \ln 3^{-2}$
$= \ln \frac{1}{9}$

e) $\log_9 6 + \log_9 7 - \log_9 21$
$= \log_9 \frac{42}{21}$
$= \log_9 2$

F) $\ln 3 + \ln 4 + \ln 5 - \ln 6$
$= \ln \frac{3 \cdot 4 \cdot 5}{6}$
$= \ln 10$

g) $3\log_5 4 - 4\log_5 2$
$= \log_5 4^3 - \log_5 2^4$
$= \log_5 64 - \log_5 16$
$= \log_5 \frac{64}{16}$
$= \log_5 4$

h) $5\ln 2 - 2\ln 4$
$= \ln 2^5 - \ln 4^2$
$= \ln 32 - \ln 16$
$= \ln \frac{32}{16}$
$= \ln 2$

i) $\ln 12 - \ln 6 - \ln 2 - \ln 3$
$= \ln \frac{12}{6 \cdot 2 \cdot 3}$
$= \ln (\frac{1}{3})$

7.1 - Continued

2 j) $3\log_{14} a - \log_{14} b - \log_{14} c$

$= \log_{14}\left(\frac{a^3}{bc}\right)$

k) $-2\ln t + 5\ln w - \ln r + 3\ln s$

$= \ln t^{-2} + \ln w^5 - \ln r + \ln s^3$

$= \ln\left(\frac{w^5 s^3}{t^2 r}\right)$

l) $\frac{2}{3}\log_7 x - \frac{3}{4}\log_7 y$

$= \log_7 x^{2/3} - \log_7 y^{3/4}$

$= \log_7\left(\frac{x^{2/3}}{y^{3/4}}\right)$

3. a) $\log_2(x^{10})$

$= 10\log_2 x$

b) $\ln y^{-7}$

$= -7\ln y$

c) $\log_3 \sqrt{x}$

$= \frac{1}{2}\log_3 x$

d) $\ln \sqrt[5]{x}$

$= \frac{1}{5}\ln x$

e) $\log_6\left(\frac{1}{x^3}\right)$

$= -3\log_6 x$

f) $\ln \sqrt[5]{x^4}$

$= \frac{4}{5}\ln x$

g) $\log_5\left(\frac{x+2}{x-2}\right)$

$= \log_5(x+2) - \log_5(x-2)$

h) $\ln\left[\frac{x}{(x+1)(x+2)}\right]$

$= \ln x - \ln(x+1) - \ln(x+2)$

i) $\log_8\left[x^3(x+2)\right]$

$= \log_8 x^3 + \log_8(x+2)$

$= 3\log_8 x + \log_8(x+2)$

j) $\ln\left[\sqrt{x+1}\,(2x-1)\right]$

$= \ln\sqrt{x+1} + \ln(2x-1)$

$= \frac{1}{2}\ln(x+1) + \ln(2x-1)$

k) $\log_3\sqrt{\frac{x}{x+4}}$

$= \log_3 x^{1/2} - \log_3(x+4)^{1/2}$

$= \frac{1}{2}\log_3 x - \frac{1}{2}\log_3(x+4)$

l) $\ln\sqrt[3]{\frac{x^2}{2x+1}}$

$= \ln x^{2/3} - \ln(2x+1)^{1/3}$

$= \frac{2}{3}\ln x - \frac{1}{3}\ln(2x+1)$

7.1 - Continued

4. a) $8\log 12$
$= 8.63345$

b) $\ln 603$
$= 6.40192$

c) $\log_3 603$
$= 5.82728$

d) $\log_3 e$
$= \dfrac{\log e}{\log 3}$
$= 0.91024$

e) $-3\ln 50$
$= -11.73607$

f) $\log_7\left(\frac{5}{9}\right)$
$= \dfrac{\log \frac{5}{9}}{\log 7}$
$= -0.24153$

g) $\ln\left(\frac{1}{20}\right)$
$= \ln 1 - \ln 20$
$= -2.99573$

h) e^3
$= 20.08554$

i) $\dfrac{3\pi}{e}$
$= 3.46718$

j) $\ln\left(\dfrac{e^3}{\pi}\right)$
$= 1.85527$

5. a) $\log_5 x = 3$
$5^3 = x$
$125 = x$

b) $\ln x = 1$
$\log_e x = 1$
$e^1 = x$
$e = x$

c) $\ln(x-1) = 2$
$\log_e(x-1) = 2$
$e^2 = x-1$
$e^2+1 = x$

d) $\ln(2x)+1 = 0$
$\log_e(2x) = -1$
$e^{-1} = 2x$
$\dfrac{1}{2e} = x$

6 a) $y = \log_2 x$; $x = 4$
$y' = \frac{1}{x}\log_2 e \cdot (1)$

$f'(4) = \frac{1}{4}\log_2 e$
$f'(4) = \dfrac{1\log e}{4\log 2}$
$= 0.36067$

b) $y = \log_5 x$; $x = 0.5$
$y' = \frac{1}{x}\log_5 e\ (1)$

$f'(0.5) = \frac{1}{0.5}\log_5 e$
$= \dfrac{1\log e}{0.5\log 5}$
$= 1.24267$

7.1 - Continued

6 c) $y = \log x \,:\, x = 10$

$y' = \frac{1}{x} \log e \,(1)$

$F'(10) = \frac{1}{10} \log e$

$= 0.04343$

d) $y = \log_3 \sqrt{x} \,:\, x = 9$

$y' = \frac{1}{\sqrt{x}} \log_3 e \left(\tfrac{1}{2}\right)\left(x^{-1/2}\right)$

$F'(9) = \frac{1}{\sqrt{9}} \log_3 e \, \frac{1}{2\sqrt{9}}$

$= \frac{1}{18} \log_3 e$

$= \frac{\log e}{18 \log 3}$

$= 0.05057$

e) $y = \ln x \,:\, x = 4$

$y' = \frac{1}{x}(1)$

$F'(4) = \frac{1}{4} = 0.25$

F) $y = \ln x \,:\, x = 0.5$

$y' = \frac{1}{x}(1)$

$f'(0.5) = \frac{1}{0.5} = 2$

g) $y = \ln x \,:\, x = 10$

$y' = \frac{1}{x}(1)$

$f'(10) = \frac{1}{10} = 0.1$

h) $y = \ln \sqrt{x} \,;\, x = 9$

$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2}(x)^{-1/2}$

$f'(9) = \frac{1}{\sqrt{9}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{9}}$

$= \frac{1}{18} = 0.05556$

7 a) $y = \log_4 (5x+6)$

$y' = \frac{1}{5x+6} \log_4 e \,(5)$

$y' = \frac{5 \log_4 e}{5x+6}$

b) $y = 8 \cdot \log_6 (x^2 - 5x)$

$y = \log_6 (x^2 - 5x)^8$

$y' = \frac{1}{(x^2-5x)^8} \log_6 e \,(8)(x^2-5x)^7(2x$

$y' = \frac{8(2x-5)\log_6 e}{x^2 - 5x}$

7.1 - Continued

7 c) $y = \log_{10}\left(\frac{x-1}{x+1}\right)$

$y' = \frac{x+1}{x-1} \log_{10} e \left(\frac{1(x+1)-(x-1)(1)}{(x+1)^2}\right)$

$y' = \left(\frac{x+1}{x-1}\right) \log e \ \frac{x+1-x+1}{(x+1)^2}$

$y' = \frac{2\log e}{(x-1)(x+1)}$

d) $y = \log_{12}(2x^3+5)^{10}$

$y' = \frac{1}{(2x^3+5)^{10}} \log_{12} e \ (10)(2x^3+5)^9(6$

$y' = \frac{60x^2 \log_{12} e}{(2x^3+5)}$

8 a) $f(x) = \ln(5x)$

$f'(x) = \frac{1}{5x}(5)$

$f'(x) = \frac{1}{x}$

b) $f(x) = \ln(2\pi x)$

$f'(x) = \frac{1}{2\pi x}(2\pi)$

$f'(x) = \frac{1}{x}$

c) $f(x) = 6\ln(4x)$

$f'(x) = 6 \ \frac{1}{4x} \cdot (4)$

$f'(x) = \frac{24}{4x}$

$f'(x) = \frac{6}{x}$

d) $f(x) = -2\ln(10x)$

$f'(x) = -2\left(\frac{1}{10x}\right)(10)$

$f'(x) = \frac{-20}{10x}$

$f'(x) = \frac{-2}{x}$

e) $f(x) = \frac{3}{4}\ln\left(\frac{2x}{3}\right)$

$f'(x) \ \frac{3}{4}\left(\frac{1}{\frac{2x}{3}}\right)\left(\frac{2}{3}\right)$

$f'(x) = \left(\frac{3}{4}\right)\left(\frac{3}{2x}\right)\left(\frac{2}{3}\right)$

$f'(x) = \frac{3}{4x}$

f) $f(x) = 4\ln(2x+7)$

$f'(x) = 4 \ \frac{1}{(2x+7)} \ (2)$

$= \frac{8}{2x+7}$

g) $f(x) = -2\ln(6-5x)$

$f'(x) = -2 \ \frac{1}{6-5x} \ (-5)$

$f'(x) = \frac{10}{6-5x}$

7.1 P. 302

9 a) $f(x) = x^3 \ln x$

$f'(x) = 3x^2 \ln x + x^3\left(\frac{1}{x}\right)(1)$

$= x^2(3\ln x + 1)$

b) $f(x) = (x^2-4)\ln x$

$f'(x) = 2x \ln x + (x^2-4)\left(\frac{1}{x}\right)(1)$

$= \frac{2x^2 \ln x + x^2 - 4}{x}$

c) $f(x) = \frac{2x}{\ln x}$ $\frac{f'}{g} - \frac{f}{g'}$

$= \frac{(\ln x)(2) - \left(\frac{1}{x}\right)(2x)}{(\ln x)^2}$

$= \frac{2\ln x - 2}{(\ln x)^2}$

$= \frac{2(\ln x - 1)}{(\ln x)^2}$

d) $f(x) = (x^2-1)\ln(x+1)$

$f'(x) = (2x)(\ln(x+1)) + (x^2-1)\left(\frac{1}{x+1}\right)(1)$

$= (2x)(\ln(x+1)) + (x-1)$

e) $f(x) = 2x^3 \ln(1-x^3)$

$f'(x) = 6x^2 \ln(1-x^3) + 2x^3 \frac{1}{1-x^3}(-3x^2)$

$= 6x^2\left(\ln(1-x^3) - \frac{x^3}{1-x^3}\right)$

$= 6x^2\left[\frac{(1-x^3)\ln(1-x^3) - x^3}{1-x^3}\right]$

f) $f(x) = (x-1)^4 \ln\left[(x-1)^4\right]$

$f'(x) = 4(x-1)^3 \ln\left[(x-1)^4\right] + (x-1)^4 \frac{1}{(x-1)^4} 4(x-1)^3]$

$= 4(x-1)^3\left[\ln\left[(x-1)^4\right] + 1\right]$

13. $f(x) = x\ln x$ $x = e$

$f'(x) = (1)\ln x + x\left(\frac{1}{x}\right)(1)$

$f'(x) = \ln x + 1$

$f'(e) = \ln(e) + 1$

$= 1 + 1$

$= 2$

$y = 2x - e$

pt → $f(e) = e\ln e = e(1) = e$

12. $f(x) = 2x + \ln x$ $x = 1$

$f'(x) = 2 + \frac{1}{x}(1)$

$f'(x) = 2 + 1$

$f'(x) = 3$

pt → $f(1) = 2(1) + \ln 1$ $(\overset{y_1}{1}, \overset{x_1}{2})$

$= 2 + 0 = 2$

or $y - y_1 = m(x-$

$y = mx + b$

$2 = 3(1) + b$

$-1 = b$

$y = 3x - 1$