7.5 Derivatives of Reciprocal Trigonometric Functions

Recall:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

Let's develop the reciprocal trigonometric derivatives

$y = \csc x$

$$y = \frac{1}{\sin x} = (\sin x)^{-1}$$

$$y' = -1(\sin x)^{-2} \cdot \cos x$$

$$y' = \frac{-\cos x}{(\sin x)^2} = \frac{-1\cos x}{(\sin x)(\sin x)}$$

$$y' = -\csc x \cot x$$

$$y = \csc u$$

$$\frac{dy}{dx} = -\csc u \cot u \cdot \frac{du}{dx}$$

Ex. 1 Find the derivative of the following:

a) $y = \csc(x^2 + 2)$

$y' = (-\csc(x^2+2)\cot(x^2+2))\,2x$

$y' = -2x\csc(x^2+2)\cot(x^2+2)$

~~$y' = -2x\csc\cot(x^2+2)$~~

$$y = \sec x$$

$$y = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$y' = -(\cos x)^{-2} \cdot (-\sin x)$$

$$= \frac{1 \sin x}{(\cos x)(\cos x)}$$

$$= \sec x \tan x$$

$$y = \sec u$$

$$\frac{dy}{dx} = \sec u \tan u \cdot \frac{du}{dx}$$

$b)\ y = \sec^2(2x)$

$y = [\sec(2x)]^2$

$y' = 2[\sec 2x]'(\sec 2x \tan 2x)\ 2$

$= 4\sec^2 2x \tan 2x$

$$= -\csc^2 x$$

$$y = \cot x$$

$$y = \frac{\cos x}{\sin x}$$

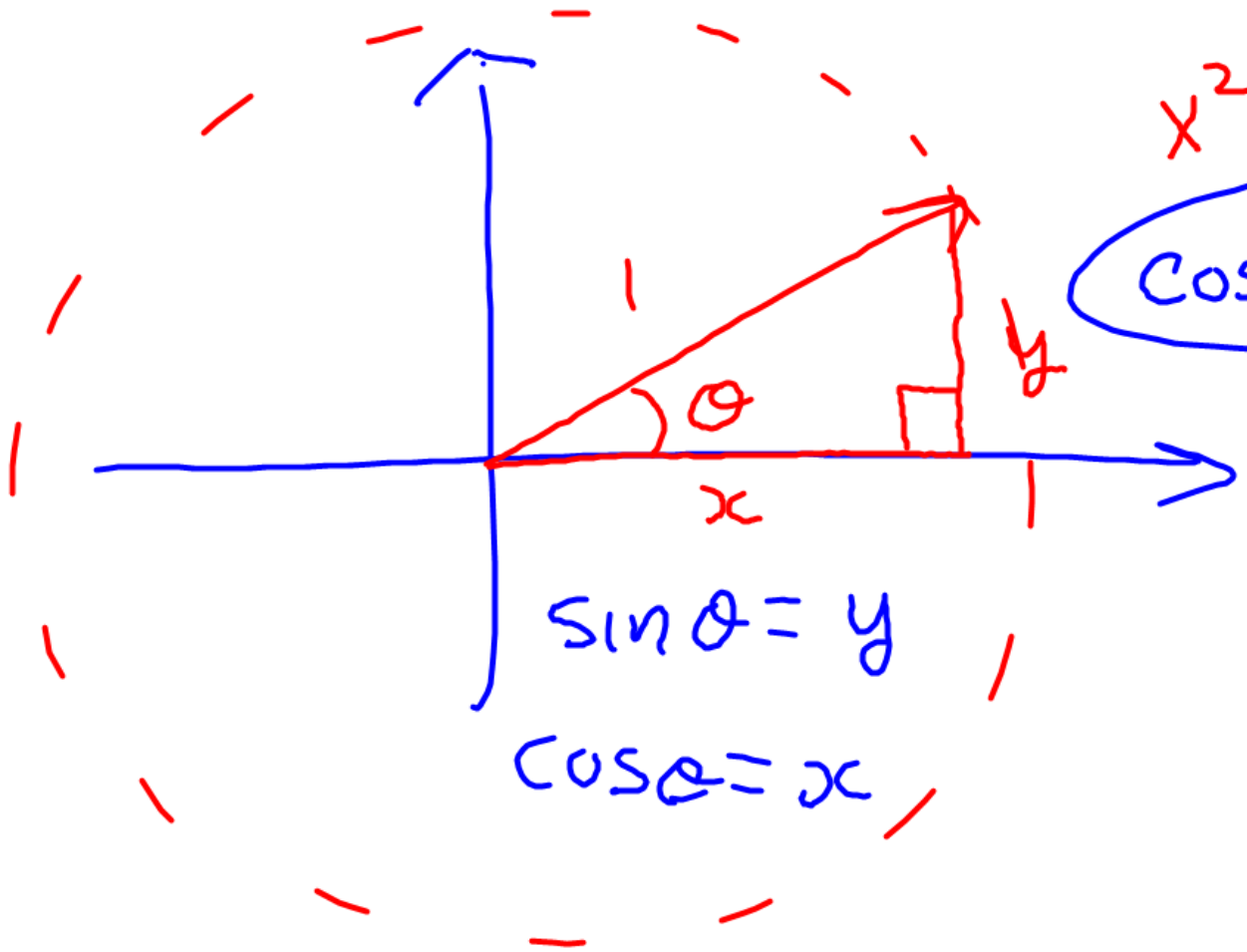
$$y' = \frac{(\sin x)(-\sin x) - \cos x(\cos x)}{(\sin x)^2}$$

$$y' = \frac{-\sin^2 x - \cos^2 x}{(\sin x)^2}$$

$$= \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

$$y = \cot u$$

$$\frac{dy}{dx} = -\csc^2 u \cdot \frac{du}{dx}$$



$x^2 + y^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\sin\theta = y$

$\cos\theta = x$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{\cos^2\theta}{\sin^2\theta} \quad \frac{\sin^2\theta}{\sin^2\theta} \quad \frac{1}{\sin^2\theta}$$

$$\cot^2\theta + 1 = \csc^2\theta$$

c) $y = \cot(3x^2 - 4x)$

$y' = [-\csc^2(3x^2 - 4x)](6x - 4)$

$= -2(3x - 2)\csc^2(3x^2 - 4x)$

Derivatives of Reciprocal Trig Functions

$$\frac{d}{dx}cscu = -\text{cscucotu}\frac{du}{dx}$$

$$\frac{d}{dx}secu = \text{secutanu}\frac{du}{dx}$$

$$\frac{d}{dx}cotu = -csc^2u\frac{du}{dx}$$

d) $y = e^{3x}\csc 5x$

$y' = e^{3x}(-\csc 5x \cot 5x)\cdot 5 + \csc 5x\, e^{3x}\cdot 3$

$y' = -5e^{3x}\csc 5x \cot 5x + 3e^{3x}\csc 5x$

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$y' = e^{3x}\csc 5x(-5\cot 5x + 3)$

e) $y = \dfrac{2x}{\cot 3x}$

$$y' = \frac{\cot 3x\,(2) - 2x\left(-\csc^2 3x\right)\cdot 3}{(\cot 3x)^2}$$

$$y' = \frac{2\cot 3x + 6x\csc^2 3x}{\cot^2 3x}$$

$$= \frac{2(\cot 3x + 3x\csc^2 3x)}{\cot^2 3x}$$

f) $x^3 + 2y = \sec(4y^3) + 9$

$$3x^2 + 2\frac{dy}{dx} = \sec(4y^3)\tan(4y^3)12y^2\frac{dy}{dx}$$

$$3x^2 = 12y^2\sec(4y^3)\tan(4y^3)\frac{dy}{dx} - 2\frac{dy}{dx}$$

$$3x^2 = \frac{dy}{dx}\left(12y^2\sec(4y^3)\tan(4y^3) - 2\right)$$

$$\frac{3x^2}{12y^2\sec(4y^3)\tan(4y^3) - 2} = \frac{dy}{dx}$$

Assignment
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36, 40, 41, 44