

## **7.4 Derivatives of Sine and Cosine Functions**

**Today we are going to revisit derivatives and look at derivatives of trigonometric functions.**

**Speaking of derivatives, what is the derivative of as cow?**



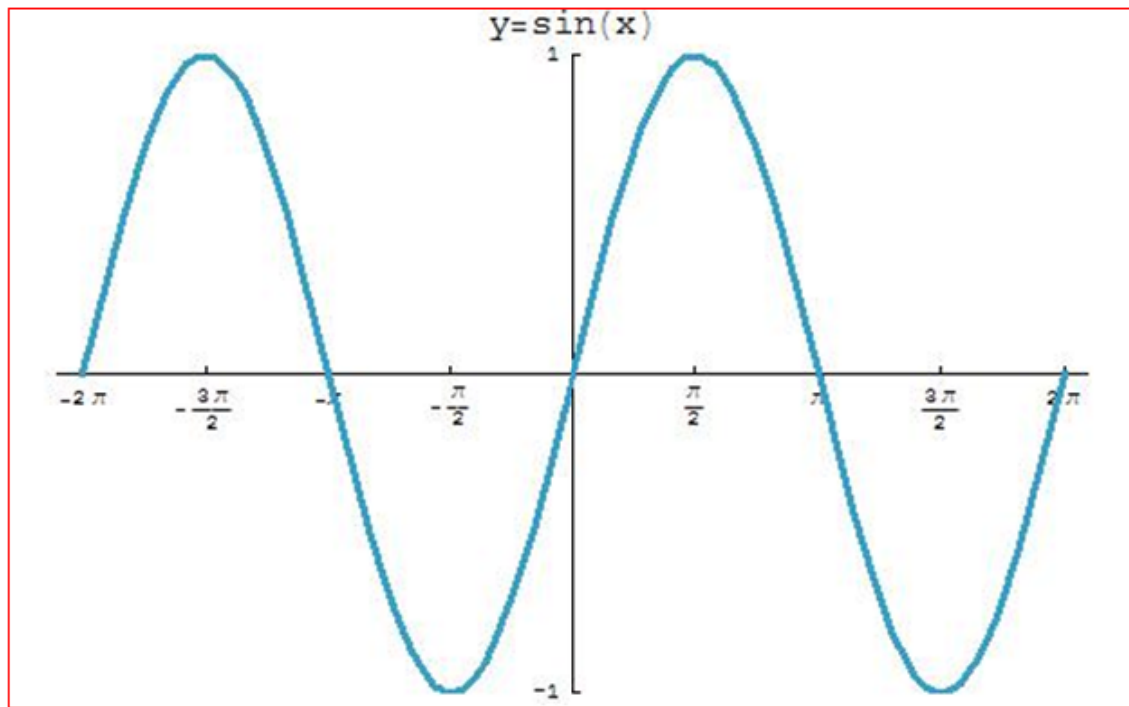
**Today we are going to revisit derivatives and look at derivatives of trigonometric functions.**

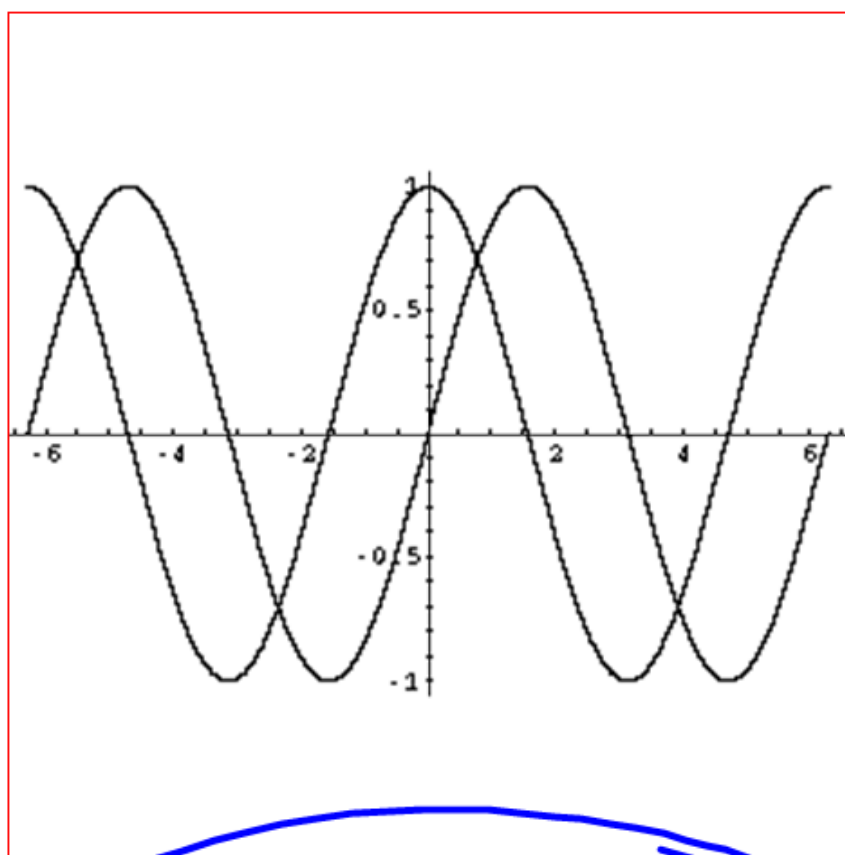
**Speaking of derivatives, what is the derivative of as cow?**



**Prime Rib!**

# Lets revisit the sine curve





$$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

Similarly it can be shown that:

$$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

Ex. 1 Differentiate the following:

$$a) y = \sin 3x$$

$$b) y = \cos(x^2 + 2)$$

composite functions  
"chain rule"

$$\begin{aligned} y' &= \cos 3x \cdot 3 \\ &= 3 \cos 3x \end{aligned}$$

$$\begin{aligned} y' &= -\sin(x^2 + 2) \cdot 2x \\ &= -2x \sin(x^2 + 2) \end{aligned}$$

$$c) f(x) = \sin(x^3)$$

$$\begin{aligned} f'(x) &= \cos(x^3) \cdot 3x^2 \\ &= 3x^2 \cos(x^3) \end{aligned}$$

$$d) f(x) = \sin^3 x$$

$$f(x) = \sin^3(2x)$$

$$f(x) = [\sin(2x)]^3$$

$$f'(x) = 3[\sin(2x)]^2 \cos 2x \cdot 2$$

$$= 6 \sin^2(2x) \cos 2x$$

~~$$6 \sin^2 \cos 2x$$~~



$$e) y = \cos^3(x^2 - 5)$$

$$y = (\cos(x^2 - 5))^3$$

$$y' = 3(\cos(x^2 - 5))^2(-\sin(x^2 - 5)) \cdot 2x$$

$$= -6x \cos^2(x^2 - 5) \sin(x^2 - 5)$$

$$f) y = x^2 \cos(4x + 2)$$

$$y' = x^2 (-\sin(4x + 2) \cdot 4) + \cos(4x + 2) \cdot 2x$$

$$y' = -4x^2 \sin(4x + 2) + 2x \cos(4x + 2)$$

$$y' = 2x (-2x \sin(4x + 2) + \cos(4x + 2))$$

\*  $g) y = \cos(\sin 2x)$

??  
 $\cos(3x)$

$$y' = -\sin(\sin 2x) \cdot (\cos 2x) 2$$

$$y' = -2 \cos 2x \sin(\sin 2x)$$

~~i)  $y = \sin^3(x^2 + 1) \cos^2(3x)$~~

$$\text{h) } y = \frac{e^{\cos x}}{e^{\sin x}}$$

$$\begin{aligned}
 y' &= \frac{\cancel{e^{\sin x}} \cdot e^{\cos x} \cdot (-\sin x) - e^{\cos x} \cancel{e^{\sin x}} \cdot \cos x}{(e^{\sin x})^2} \\
 &= \frac{-e^{\cos x} (\sin x) - \cos x e^{\cos x}}{e^{\sin x}} \\
 &= \frac{e^{\cos x} (-\sin x - \cos x)}{e^{\sin x}}
 \end{aligned}$$

Assignment

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8, 11, 15, 16, 20, 21, 28, 30, 35, 37, 39,  
41, 43, 44, 56

Knowing our derivatives for sine and cosine, we can derive the derivatives for the remaining trigonometric functions.

$$y = \tan x$$

$$\frac{1 + \cancel{B}}{\cancel{B}}$$

$$y = \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos x (\cos x) - \sin x (-\sin x)}{\cos^2 x}$$

$$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx} \tan u = \sec^2 u \cdot \frac{du}{dx}$$



**Example 2 Differentiate the following:**

a)  $y = \tan^2(2x)$

$$y = (\tan 2x)^2$$

$$y' = 2(\tan 2x)^1 \sec^2 2x \cdot 2$$

$$y' = 4 \tan 2x \cdot \sec^2 2x$$

$$\text{b) } y = \tan(\sin 3x^2)$$

Assignment

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