7.2 Exponential Functions

5a) $y = 7^x$

$y' = 7^x \ln 7$

b) $y = 3^{2x+5}$

$y' = 3^{2x+5}(2)\ln 3$

$y' = 2(3^{2x+5})\ln 3$

c) $y = \left(\frac{2}{3}\right)^{x^3-4x}$

$y' = \left(\frac{2}{3}\right)^{x^3-4x} \ln\left(\frac{2}{3}\right)(3x^2-4)$

d) $y = 4(2^{x^3})$

$y' = 4(2^{x^3} \cdot 3x^2)\ln 2$

$y' = 12x^2(2^{x^3})\ln 2$

e) $y = 6(5^{-3x})$

$y' = 6(5^{-3x})\cdot(-3)\cdot\ln 5$

$y' = -18(5^{-3x})\ln 5$

f) $y = (2x^4)(5^{3-x})$

$y' = (2x^4)(5^{3-x})(-1)(\ln 5) + 5^{3-x}(8x^3)$

$y' = 5^{3-x}\left[-2x^4\ln 5 + 8x^3\right]$

$y' = 2x^3[5^{3-x}][4 - x\ln 5]$

g) $y = \frac{3^x}{x^2}$

$y' = \frac{x^2[3^x\ln 3] - 3^x(2x)}{x^4}$

$y' = \frac{3^x[x^2\ln 3 - 2x]}{x^4} = \frac{x\,3^x[x\ln 3 - 2]}{x^4} = \frac{3^x[x\ln 3 - 2]}{x^3}$

h) $y = (2^x)(3^{x^2})$

$y' = (2^x)(3^{x^2} \cdot 2x \cdot \ln 3) + 3^{x^2}(2^x)\ln 2$

$= 2^x \cdot 3^{x^2}(2x \ln 3 + \ln 2)$

6. $y = e^{8x}$

$y' = e^{8x} \cdot 8$

$y' = 8e^{8x}$

7. $y = e^{-4x}$

$y' = e^{-4x} \cdot (-4)$

$y' = -4e^{-4x}$

8. $y = 3e^{5x}$

$y' = 3 \cdot e^{5x} \cdot 5$

$y' = 15e^{5x}$

9. $y = -2e^{-4x}$

$y' = -2e^{-4x} \cdot (-4)$

$y' = 8e^{-4x}$

10. $y = 6e^{2-x^2}$

$y' = 6e^{2-x^2}(-2x)$

$y' = -12x\, e^{2-x^2}$

11. $y = e^{x^2-5x+6}$

$y' = e^{x^2-5x+6} \cdot (2x-5)$

12. $y = e^{-2x^3}$

$y' = e^{-2x^3} \cdot (-6x^2)$

$y' = -6x^2 e^{-2x^3}$

13. $y = 10e^{4x-2x^2}$

$y' = 10e^{4x-2x^2}(4-4x)$

$y' = 40(1-x)e^{4x-2x^2}$

14. $y = e^{(x-4)^3}$

$y' = e^{(x-4)^3} \cdot 3(x-4)^2$

15. $y = -5e^{-(3x+1)^2}$

$y' = -5e^{-(3x+1)^2} \cdot -2(3x+1)^1(3)$

$y' = 30(3x+1)e^{-(3x+1)^2}$

16. $y = e^{2/x}$

$y' = e^{2/x} \cdot -2x^{-2}$

$y' = \frac{-2e^{2/x}}{x^2}$

17. $y = 11e^{-4/x^2}$

$y' = 11e^{-4/x^2} \cdot 8x^{-3}$

$y' = \frac{88e^{-4/x^2}}{x^3}$

18. $y = e^{\frac{4}{(x+3)}}$

$y' = e^{\frac{4}{(x+3)}} \cdot -4(x+3)^{-2}$

$y' = \frac{-4e^{\frac{4}{x+3}}}{(x+3)^2}$

19. $y = 9e^{\frac{-2}{(x^3+4)}}$

$y' = 9e^{\frac{-2}{(x^3+4)}} \cdot 2(x^3+4)^{-2} \cdot 3x^2$

$y' = \frac{54x^2 e^{\frac{-2}{(x^3+4)}}}{(x^3+4)^2}$

20. $y = e^{\ln 7x}$

$y' = e^{\ln 7x} \cdot \frac{1}{7x} \cdot 7$

$y' = 7x \cdot \frac{1}{7x} \cdot 7$

$y' = 7$

~~21. $y = e^{\ln(2x^3 - x^2)}$~~

~~$y' = e^{\ln(2x^3-x^2)} \cdot \frac{1}{(2x^3-x^2)} \cdot (6x^2-2x)$~~

21. $y = e^{\ln(2x^3-x^2)}$

$y' = e^{\ln(2x^3-x^2)} \cdot \frac{1}{(2x^3-x^2)} \cdot (6x^2-2x)$

$y' = 2x(3x-1)$

22. $y = e^{4\ln x}$

$y = e^{\ln x^4}$

$y = x^4$

$y' = 4x^3$

23. $y = e^{-3\ln(2x-1)}$

$y = e^{\ln(2x-1)^{-3}}$

$y = (2x-1)^{-3}$

$y' = -3(2x-1)^{-4}(2)$

$y' = \dfrac{-6}{(2x-1)^4}$

24. $y = x e^x$

$y' = x e^x + e^x(1)$

$y' = e^x(x+1)$

25. $y = 2x^2 e^x$

$y' = 2x^2 e^x + e^x(4x)$

$y' = 2xe^x(x+2)$

26. $y = (x+2)e^{2x}$

$y' = (x+2)(e^{2x}\cdot 2) + e^{2x}$

$y' = e^{2x}(2(x+2)+1)$

$= e^{2x}(2x+5)$

27. $y = (x+2)e^{(x+2)^2}$

$y' = (x+2)e^{(x+2)^2}\cdot(2(x+2)) + e^{(x+2)^2}$

$y' = e^{(x+2)^2}[(x+2)(2x+4)+1]$

$y' = e^{(x+2)^2}[2x^2+4x+4x+8+1]$

$y' = e^{(x+2)^2}[2x^2+8x+9]$

28. $y = (2x^3+3)e^{2x^3+3}$

$y' = (2x^3+3)e^{2x^3+3}(6x^2) + e^{2x^3+3}(6x^2)$

$y' = e^{2x^3+3}6x^2[2x^3+3+1]$

$= e^{2x^3+3}\cdot 6x^2[2x^3+4]$

$= e^{2x^3+3}\cdot 12x^2[x^3+2]$

29. $y = \frac{x}{e^x}$

$y' = \frac{e^x(1) - x e^x}{(e^x)^2}$

$y' = \frac{1 - x}{e^x}$

30. $y = \frac{e^{2x}}{2x^3}$

$y' = \frac{2x^3(e^{2x} \cdot 2) - e^{2x}(6x^2)}{(2x^3)^2}$

$y' = \frac{4x^3 e^{2x} - 6x^2 e^{2x}}{4x^6}$

$y' = \frac{e^{2x} 2x^2 (2x - 3)}{4x^6}$

$y' = \frac{e^{2x}(2x-3)}{2x^4}$

31. $y = \frac{e^{4x+3}}{4x+3}$

$y' = \frac{(4x+3)(e^{4x+3} \cdot 4) - e^{4x+3}(4)}{(4x+3)^2}$

$y' = \frac{4e^{4x+3}[4x+3-1]}{(4x+3)^2}$

$y' = \frac{4e^{4x+3}[4x+2]}{(4x+3)^2}$

$= \frac{8e^{4x+3}[2x+1]}{(4x+3)^2}$

32. $y = 6e^{\sqrt{x}}$

$y = 6e^{x^{1/2}}$

$y' = 6e^{x^{1/2}} \cdot \frac{1}{2}x^{-1/2}$

$= \frac{3e^{x^{1/2}}}{x^{1/2}}$

$= \frac{3e^{\sqrt{x}}}{\sqrt{x}}$

33. $y = -4e^{\sqrt[4]{x^3}}$

$y = -4e^{x^{3/4}}$

$y' = -4e^{x^{3/4}} \cdot \frac{3}{4}x^{-1/4}$

$y' = \frac{-3e^{x^{3/4}}}{x^{1/4}}$

34. ~~$y = 4e^{\sqrt[4]{x^3}}$~~

34. $y = \pi x e^{\pi x}$

$y' = \pi x e^{\pi x} \cdot \pi + e^{\pi x} \cdot \pi$

$y' = \pi^2 x e^{\pi x} + \pi e^{\pi x}$

$y' = \pi e^{\pi x}[\pi x + 1]$

35. $y = \frac{1}{3}x^3 e^{\ln 6x}$

$y = \frac{1}{3}x^3(6x)$

$y = \frac{1}{3}6x^4$

$y = 2x^4$

$y' = 8x^3$

42. $y = \ln(e^{3x} + 2)$

$$y' = \frac{1}{e^{3x}+2} \cdot e^{3x} \cdot 3$$

$$y' = \frac{3e^{3x}}{e^{3x}+2}$$

43. $y = e^x \ln x$

$$y' = e^x \cdot \frac{1}{x} + \ln x \cdot e^x$$

$$y' = e^x\left(\frac{1}{x} + \ln x\right)$$

$$= e^x\left(\frac{1 + x\ln x}{x}\right)$$

44. $y = \ln\left(\frac{e^x+1}{e^x-1}\right)$

$$y' = \frac{1}{\frac{e^x+1}{e^x-1}} \cdot \left[\frac{(e^x-1)(e^x) - (e^x+1)(e^x)}{(e^x-1)^2}\right]$$

$$y' = \left(\frac{e^x-1}{e^x+1}\right)\left[\frac{(e^x)^2 - e^x - (e^x)^2 - e^x}{(e^x-1)^2}\right]$$

$$= \frac{-2e^x}{(e^x+1)(e^x-1)} = \frac{-2e^x}{e^{2x}-1}$$

45. $y = x\ln(xe^x)$

$$y' = x\left(\frac{1}{x\,e^x} \cdot (xe^x + e^x)\right) + \ln(xe^x)$$

$$= \frac{x e^x(x+1)}{x e^x} + \ln(x\cdot e^x)$$

$$= x+1 + \ln x + \ln e^x$$

$$= x+1 + \ln x + x$$

$$= 2x+1+\ln x$$