7.2 Derivatives of Exponential Functions.

$y = \log_3(x^4)$

$y' = \frac{1}{x^4} \cdot 4x^3 \cdot \log_3 e$

$y' = \frac{4\log_3 e}{x}$

$y = \ln(x^4)$

$y' = \frac{1}{x^4} \cdot 4x^3$

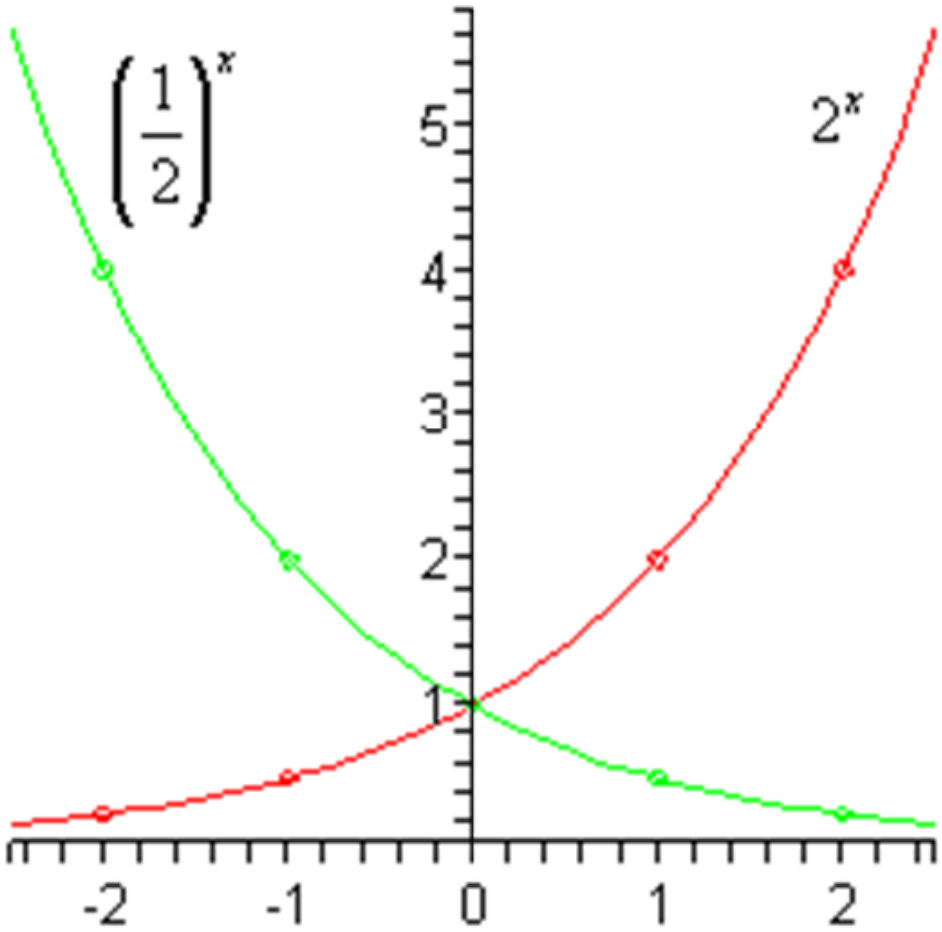
$y' = \frac{4}{x}$

Recall from Math B30 **exponential functions** are of the form:

$y = b^x$

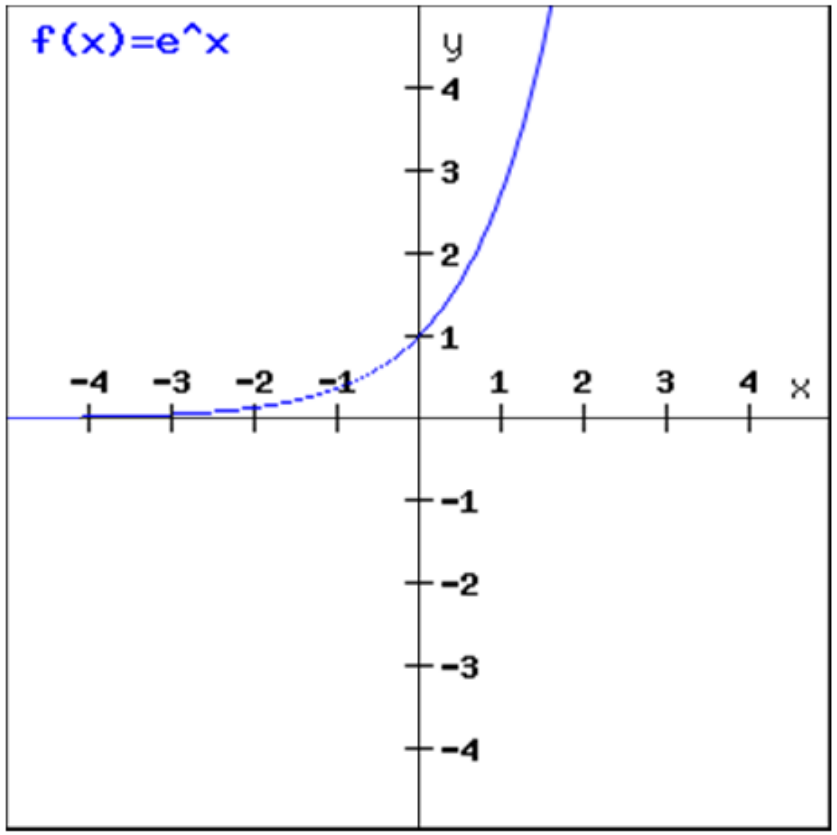
where b the base is any positve number except 1

$$y = 2^x \qquad y = \left(\frac{1}{2}\right)^x$$


$\left(\frac{1}{2}\right)^x$
2^x
5
4
3
2
1
-2
-1
0
1
2

In calculus we deal with a special number "e" which is a constant.

$$e = 2.718281828$$



$f(x) = e^{4x^3}$

$f(x) = 3^{4x}$

When we differentiate exponential functions, we will be dealing with two kinds those involving base "**e**" and those involving base "**a**"

Base "a" Exponential Derivatives

Ex.1 Differentiate the following:

$a)\ y = 5^{3x}$

$y' = 5^{3x} \cdot 3 \cdot \ln 5$

itself

derv. exp.

natural log base

In general we have :

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(a^{u}\right)=a^{u}\cdot\frac{du}{dx}\cdot\ln a$$

$b)\ f(x)=17^{4x^2}$

$f'(x)= 17^{4x^2} \cdot 8x \cdot \ln 17$

c) $f(x) = (3x^4)(20^{2-x})$

$f'(x) = 3x^4(20^{2-x})(-1)\ln 20 + 20^{2-x}(12x^3)$

GCF

$f'(x) = 20^{2-x}(3x^3)[-x\ln 20 + 4]$

Base "e" Exponential Derivatives

Ex.1 Differentiate the following:

$a)\ y = e^{3x}$

$y' = e^{3x} \cdot 3 \cdot (\ln e)$ $= 1$

$y' = 3e^{3x}$

In general we have :

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(e^{u}\right)=e^{u}\cdot\frac{du}{dx}$$

$$b)\ f(x) = e^{4x^2+3x}$$

$$f' = e^{4x^2+3x} \cdot (8x+3)$$

$$c)\ y = \left(e^{4x+1}\right)^3$$

$$y' = 3\left(e^{4x+1}\right)^2 \cdot \left(e^{4x+1}\right)' \cdot 4$$

$$y' = 12\left(e^{4x+1}\right)^3$$

* d) $y = e^{\ln\left(x^2+2x\right)}$ $= x^2+2x$

$2^3 = 8$

$\ln(x^2+2x) = \log_e y$ $\quad 3 = \log_2 8$

$\ln(x^2+2x) = \ln y$

$y = x^2+2x$

$y' = 2x+2$

If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

$a)\ 1$

$b)\ \dfrac{e^{2x}(1-2x)}{2x^2}$

$c)\ e^{2x}$

$d)\ \dfrac{e^{2x}(2x+1)}{x^2}$

$e)\ \dfrac{e^{2x}(2x-1)}{2x^2}$

$f' = \dfrac{2x(2e^{2x})-e^{2x}(2)}{(2x)^2}$

$f' = \dfrac{4xe^{2x}-2e^{2x}}{4x^2}$

$f' = \dfrac{2e^{2x}(2x-1)}{4x^2}$

Assignment
Page 311
#'s 5,6,8,10,13,17,20,22,24, 25,29,31,35,42,43,48