$y = \log_e x$

Unit #7 Differentiating Transcendental Functions

7.1 Derivatives of Logarithmic Functions

$y = \log_2 x$

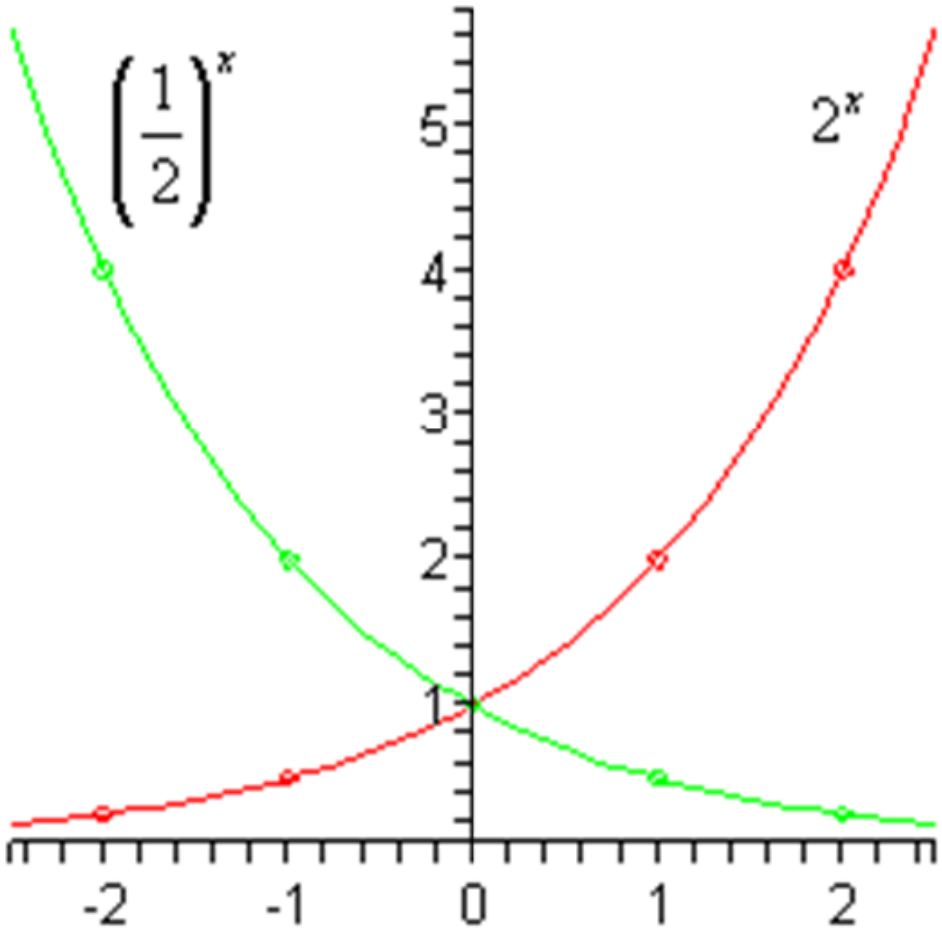
$y = \ln x$

Recall from Pre-Calc 30 **exponential functions** are of the form:

$y = b^x$

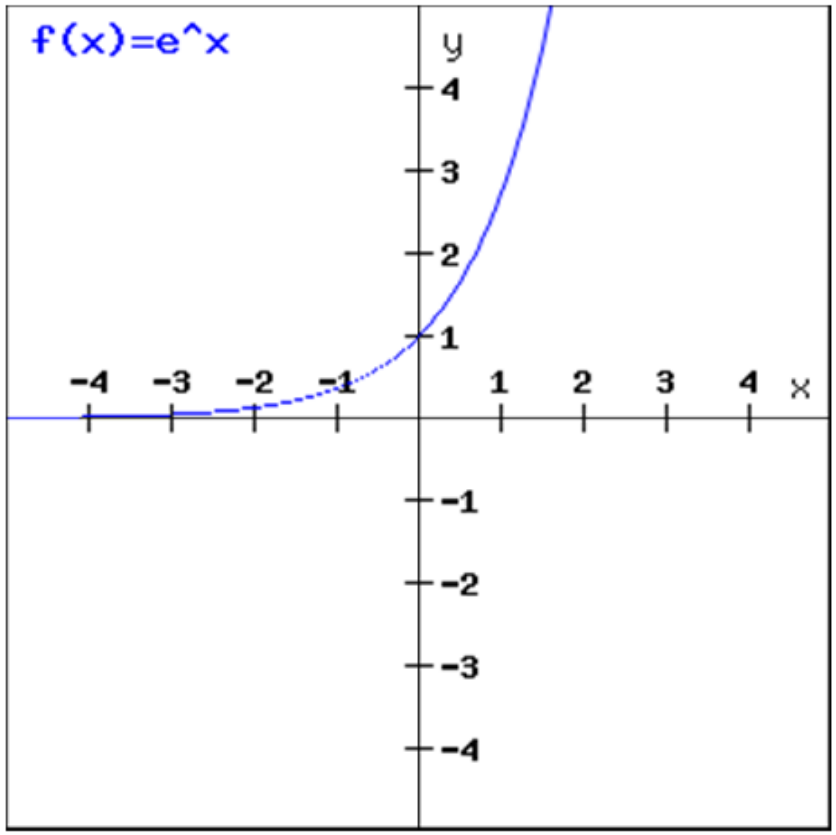
where b the base is any positve number except 1

$$y = 2^x \qquad y = \left(\frac{1}{2}\right)^x$$

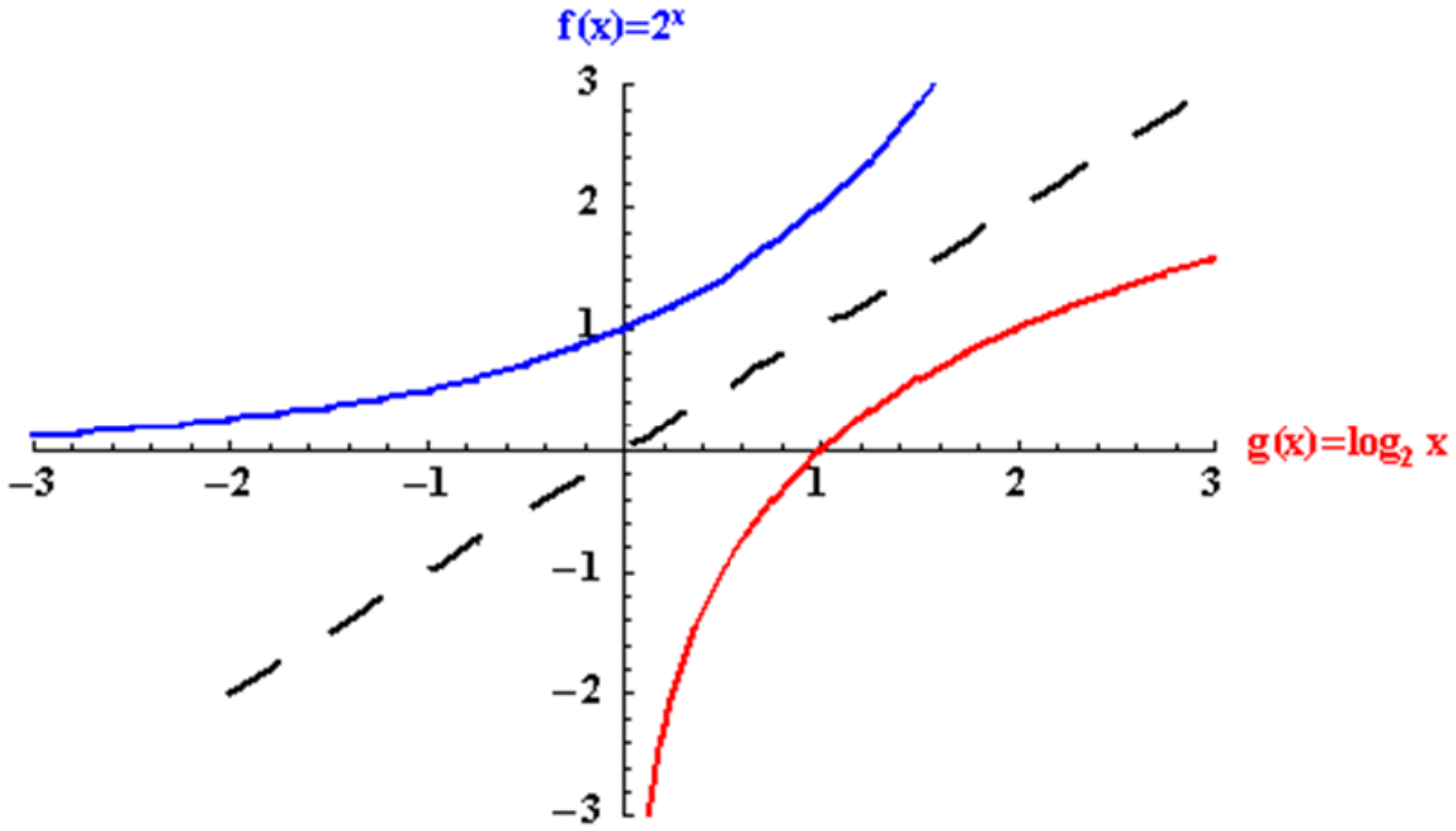

$\left(\frac{1}{2}\right)^x$
2^x
5
4
3
2
1
-2
-1
0
1
2

In calculus we deal with a special number "e" which is a constant.

$$e = 2.718281828$$



Recall from Pre-Calc 30 that logarithmic functions are inverses of exponential functions.



Today we will be learning how to differentiate two types of logarithmic functions.

Common Logarithms

$$y = \log_3 2x$$

$$y = \log_{\frac{1}{2}} \left(x^2\right)$$

$$y = \log\left(3x^3\right)$$

What is the base for this logarithm?

Common Logarithms

Example 1: Differentiate the following:

a) $y = \log_7(10x^3)$

$$y' = \frac{1}{10x^3} \cdot 30x^2 \cdot \log_7 e$$

$$y' = \frac{3\log_7 e}{x} \quad \Big| \quad \frac{3}{x \ln 7}$$

In general we have :

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\log_b u\right) = \frac{1}{u} \cdot \frac{du}{dx} \cdot \log_b e$$

b) $y = \log_5\left(\frac{3x}{x^2-4}\right)$

$$y' = \left(\frac{x^2-4}{3x}\right)\left[\frac{(x^2-4)(3)-3x(2x)}{(x^2-4)^2}\right]\log_5 e$$

$3x^2-12-6x^2$

$$y' = \frac{(-3x^2-12)\log_5 e}{3x(x^2-4)} = -\frac{3(x^2+4)\log_5 e}{3x(x^2-4)}$$

Natural Logarithms

$$y = \ln(3x)$$

This is the same as ⟶ $y = \log_e(3x)$

Instead of writing $\log_e(x)$, we just $\ln(x)$.

You may want to commit to memory:

$\ln e = 1$ $\quad$ $\ln 1 = 0$

Example 2: Differentiate the following:

a) $y = \ln(7x^2 - 3x)$

$$y' = \frac{1}{7x^2 - 3x} \cdot (14x - 3)$$

$$y' = \frac{14x - 3}{7x^2 - 3x}$$

In general we have :

$$\frac{\mathrm{d}}{\mathrm{dx}}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$

b) $y = 3\ln\left(x^2 - 4x + 1\right)$

$$y' = 3\left(\frac{1}{x^2 - 4x + 1} \cdot (2x - 4)\right)$$

$$y' = \frac{6(x - 2)}{x^2 - 4x + 1}$$

c) $y = x^6 \ln(3x+7)$

$$y' = x^6\left(\frac{1}{3x+7}\cdot 3\right) + \ln(3x+7)\cdot 6x^5$$

$$y' = \frac{3x^6}{3x+7} + 6x^5\ln(3x+7)$$

$$y' = 3x^5\left(\frac{x}{3x+7} + 2\ln(3x+7)\right)$$

d) $f(x) = \ln\left(\frac{x-4}{3x+1}\right)$

$$f'(x) = \left(\frac{3x+1}{x-4}\right)\left(\frac{(3x+1)(1) - (x-4)3}{(3x+1)^2}\right)$$

$3x+1-3x+12$

$$f' = \frac{13}{(x-4)(3x+1)}$$

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#" 7 a,d, 8 a,d,e,h,k,q,r, 9 a,c,e,g,i, 10, 12