Chapter 7 Exponential Functions

7.1 Characteristics of Exponential Functions

The Penny Story

The Penny Story begins with this question – *If I took a penny and each day double its value, how much money would you have at the end of one month?*

To be more clear, one penny doubling yields 2 pennies after the 1st day, 4 pennies on the 2nd day , 8 pennies on the 3rd day, 16 pennies on the 4th day, 32 pennies on the 5th day and 64 pennies on the 6th day. By the end of 1 week, in just 7 days, your penny has doubled again and again to where its value is now $1.28. With that information, I will ask you again, "If I took a penny and each day double its value, how much money would you have at the end of one month?" **Take a moment to think of a number yourself.**

Is it $10?

Is it $1,000?

Is it $100,000?

Day	Value
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64
8	$1.28
9	$2.56
10	$5.12
11	$10.24
12	$20.48
13	$40.96
14	$81.92
15	$163.84
16	$327.68
17	$655.36
18	$1,310.72
19	$2,621.44
20	$5,242.88
21	$10,485.76
22	$20,971.52
23	$41,943.04
24	$83,886.08
25	$167,772.16
26	$335,544.32
27	$671,088.64
28	$1,342,177.28
29	$2,684,354.56
30	$5,368,709.12

Exponential functions are of the form:

$\boxed{y = c^x}$

where c is a constant ($c > 0$ and $c \neq 1$) and x is a variable

$$y = 2^x$$

$c > 1$

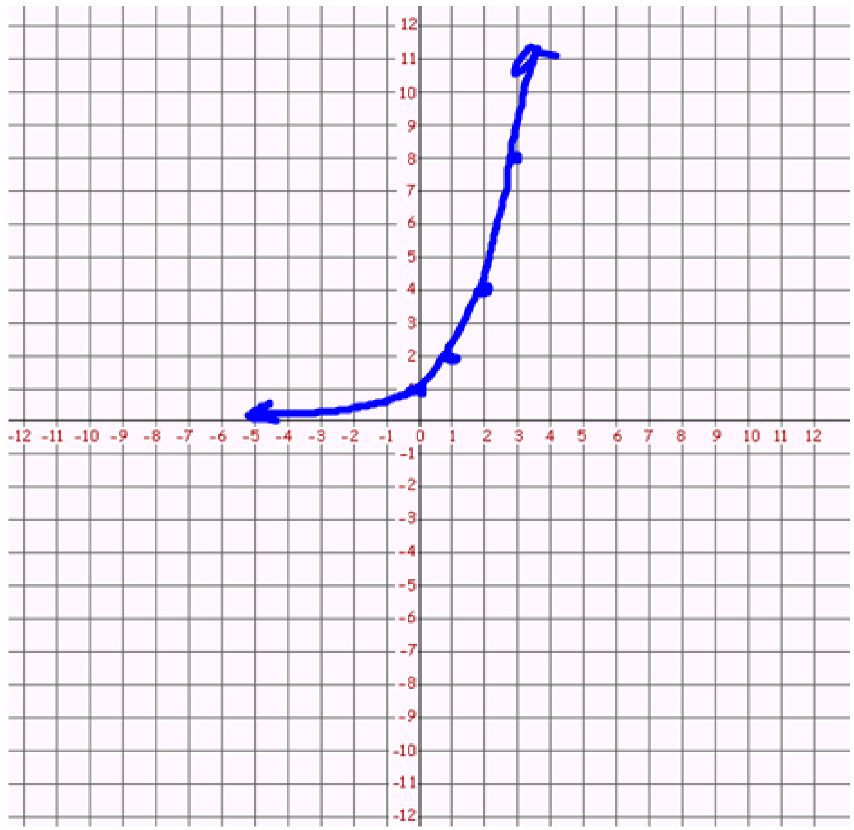
$$y = \left(\frac{1}{3}\right)^x$$

$0 < c < 1$

Let's graph using a table of values:

$$y = 2^x$$

x	y
0	1
1	2
2	4
3	8



a) Identify the domain and range.

D: $x \in \mathbb{R}$ R: $y > 0$

b) Identify the x and y intercepts.

No x int / y int (0, 1)

c) Identify whether function is increasing or decreasing.

Increasing

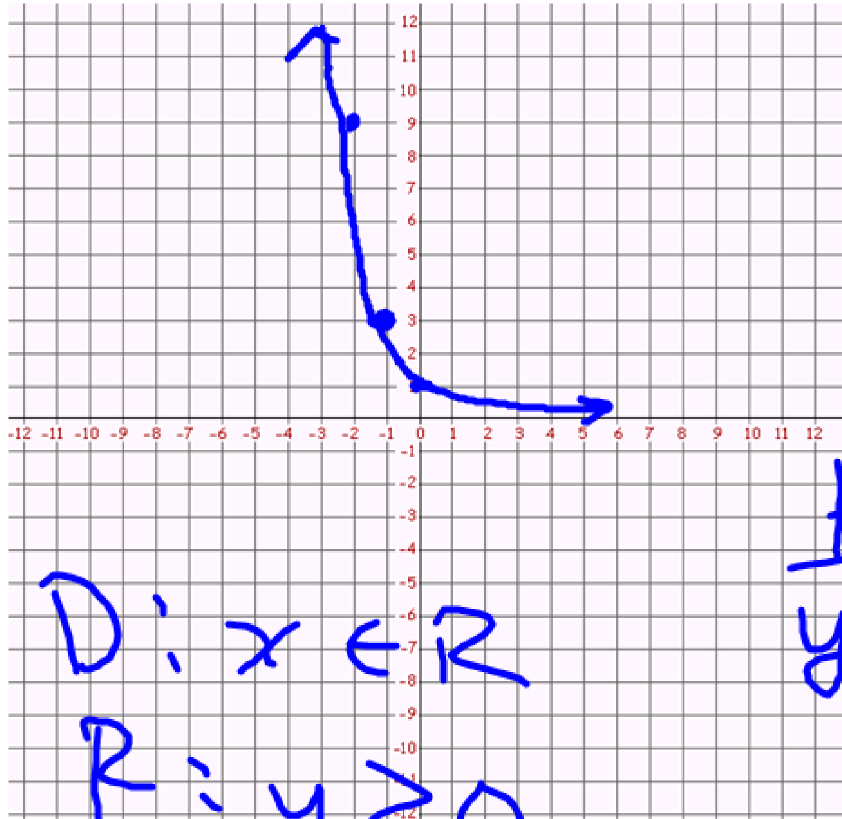
d) Identify the equation of the horizontal asymptote.

$y = 0$

Let's graph using a table of values:

$$y = \left(\frac{1}{3}\right)^x$$

x	y
0	1
-1	3
-2	9



D: $x \in R$

R: $y > 0$

HA

$y = 0$

x int

None

y int

(0,1)

graph decreasing

7.3 Solving Exponential Functions

Recall the an **exponential equation** is one that has a variable in the exponent.

For example:

$$3^{x-5} = 27$$

Today we are going to learn how to solve exponential equations

To solve exponential equations we often need to rewrite our equation so that the bases on both sides are the same.

Example 1 : Write each expression as a power with a base of 2

a) 4^3

$= (2^2)^3$

$= 2^6$

b) $\frac{1}{8}$

$= \frac{1}{2^3}$

$= 2^{-3}$

c) $8^{\frac{2}{3}}\left(\sqrt{16}\right)^3$

$= \left(2^3\right)^{2/3}\left(4\right)^3$

$= \left(2^3\right)^{2/3}\left(2^2\right)^3$

$= \left(2^2\right)\left(2^6\right)$

$= 2^8$

To solve exponential equations we can use the following property:

Equal Powers and Bases Property of Exponents

If b is a positive real number other than 1, then if $b^m = b^n$, then $m = n$.

Example 2 : Solve

$$2^{4x} = 4^{x+3}$$

$$2^{4x} = (2^2)^{x+3}$$

$$2^{4x} = 2^{2x+6}$$

$$4x = 2x + 6$$

$$2x = 6$$

$$x = 3$$

Example 3: Solve

$$9^{4x} = 27^{x-1}$$

$$(3^2)^{4x} = (3^3)^{x-1}$$

$$3^{8x} = 3^{3x-3}$$

$$8x = 3x - 3$$

$$5x = -3$$

$$x = -3/5$$

Example 4 : Solve

$$\sqrt{2}^{6x} = 32^{x-1}$$

$$\left(2^{1/2}\right)^{6x} = \left(2^5\right)^{x-1}$$

$$2^{3x} = 2^{5x-5}$$

$$3x = 5x - 5$$

$$-2x = -5$$

$$x = 5/2$$

Example 4 : Solve

$$2^x = 7$$

We run into a bit of a problem with this question.

In chapter 8 we will learn a way to solve this type of problem, but for now we will use technology for assistance.

$2^x = 7$

$2^{2.7} = 6.49$

$2^{2.8} = 6.96$

$2^{2.82} = 7.06$

$2^{2.81} = 7.01$

An application of exponential equations are those dealing with compound interest.

The formula for compound interest is $A=P(1+i)^n$, where A is the amount of money at the end of the investment, P is the principal amount deposited, i is the interest rate per compounding period expressed as a decimal and n is the number of compounding periods.

Compound Interest

$$A = P\left(1 + \frac{i}{n}\right)^{nt}$$

A = final amount

P = principal

t = time

i = interest rate

n = # compounds

Example 5

John invests \$5000 into an account that pays 3.5% compounded quarterly. How much will his investment be worth after 7 years?

$P = 5000$

$i = .035$

$n = 4$

$t = 7$

$$A = 5000\left(1 + \frac{.035}{4}\right)^{4(7)}$$

$$A = 5000(1.00875)^{28}$$

$$A = \$6381.30$$

Example 6

Determine how long \$1000 needs to be invested in an account that earns 8.3% compounded semi-annually before it increases in value to \$1490.

$P = 1000$

$A = 1490$

$i = .083$

$n = 2$

$t = ?$

$$A = P\left(1 + \frac{i}{n}\right)^{nt}$$

$$1490 = 1000\left(1 + \frac{.083}{2}\right)^{2t}$$

$$1.49 = (1.0415)^{2t}$$

$$1.50 = (1.0415)^{10}$$

$$t = 5$$

THE MAGIC OF COMPOUND INTEREST AND AN RRSP

The "Time Value" of Money

Individual A

Opens RRSP at 12%—Invests $2,000† a year for six years, then stops.

Individual B

Spends $2,000 a year on himself for six years, then opens RRSP at 12%. Invests $2,000† a year for the next 37 years.

Look at age 65 — Individual A, who only deposited $12,000 has accumulated nearly as much money as Individual B, who deposited $74,000!

Start early — let time work for **you!**

†$2,000 use for example only; contribution may be more or less.

	Example A		Example B	
Age	Payment	Accumulation End of Year	Payment	Accumulation End of Year
22	$2,000	$ 2,240	0	0
23	2,000	4,749	0	0
24	2,000	7,559	0	0
25	2,000	10,706	0	0
26	2,000	14,230	0	0
27	2,000	18,178	0	0
28	0	20,359	$2,000	$ 2,240
29	0	22,803	2,000	4,749
30	0	25,539	2,000	7,559
31	0	28,603	2,000	10,706
32	0	32,036	2,000	14,230
33	0	35,880	2,000	18,178
34	0	40,186	2,000	22,599
35	0	45,008	2,000	27,551
36	0	50,409	2,000	33,097
37	0	56,458	2,000	39,309
38	0	63,233	2,000	46,266
39	0	70,821	2,000	54,058
40	0	79,320	2,000	62,785
41	0	88,838	2,000	72,559
42	0	99,499	2,000	83,507
43	0	111,438	2,000	95,767
44	0	124,811	2,000	109,499
45	0	139,788	2,000	124,879
46	0	156,563	2,000	142,105
47	0	175,351	2,000	161,397
48	0	196,393	2,000	183,005
49	0	219,960	2,000	207,206
50	0	246,355	2,000	234,310
51	0	275,917	2,000	264,668
52	0	309,028	2,000	298,668
53	0	346,111	2,000	336,748
54	0	387,644	2,000	379,398
55	0	434,161	2,000	427,166
56	0	486,261	2,000	480,665
57	0	544,612	2,000	540,585
58	0	609,966	2,000	607,695
59	0	683,162	2,000	682,859
60	0	765,141	2,000	767,042
61	0	856,958	2,000	861,327
62	0	959,793	2,000	966,926
63	0	1,074,968	2,000	1,085,197
64	0	1,203,964	2,000	1,217,661
65	0	1,348,440	0	1,363,780

Assignment Page 364

#'s 1, 2, 3, 4, 11, ~~12ab~~, 13ab