6.5 Related Rates - Part Two Page 290 1-20

1.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dr}{dt} = 25 \frac{cm}{s}$

$r = 200$

$\frac{dA}{dt} = 2\pi (200)(25)$

$\frac{dA}{dt} = 31415.93 \frac{cm^2}{s}$

or $10{,}000\pi \frac{cm^2}{s}$

2.

80 m/min

13

$\frac{dx}{dt} = 0$ (no change in till machine width)

$A = xy$

$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

$\frac{dA}{dt} = (13)(80) + y(0)$

$\frac{dA}{dt} = 1040 \frac{m^2}{min}$

3.

$\frac{dr}{dt} = -2 \frac{mm}{day}$

$r = 2.5$ cm

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi(-2)$

$\frac{dC}{dt} = -4\pi$ or -12.57

Decreasing by 4π or 12.57 mm/day

6.5 - continued

4.

$A = xy$

$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$

$\frac{dA}{dt} = (10)(22) + (30)(-15)$

$\frac{dA}{dt} = 220 - 450$

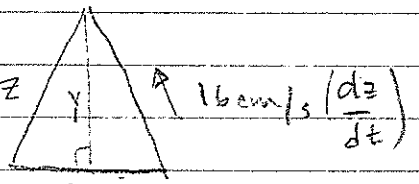
$\frac{dA}{dt} = -230\ cm^2/s$

$x = 30$ (length)

$y = 22$ (width)

Shrinking at a rate of $230\ cm^2/s$.

5.



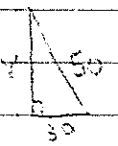
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(60)y$

$A = 30y$

$z = 50\ cm$

$x = 30$



$50^2 = y^2 + 30^2$

$y = 40$

$y^2 = z^2 - x^2$

$y^2 = z^2 - 30^2$

$y = \sqrt{z^2 - 900}$

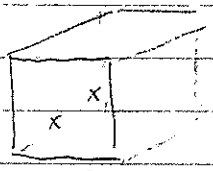
$A = 30\sqrt{z^2-900}$

$\frac{dA}{dt} = 30\left(\frac{1}{2}\right)(z^2-900)^{-1/2}(2z)\frac{dz}{dt}$

$\frac{dA}{dt} = 30\left(\frac{1}{2}\right)(50^2-900)^{-1/2}(2(50))(16)$

$\frac{dA}{dt} = 600\ cm^2/s$

6.



$\frac{dV}{dt} = -15\ mm^3/s$

$x = 30\ mm$

$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$-15 = 3(30)^2\frac{dx}{dt}$

$-0.0056\ \frac{mm}{s} = \frac{dx}{dt}$

Shrinking at $-0.0056\ \frac{mm}{s}$ or $\frac{1}{180}\ \frac{mm}{s}$

6.5 - Continued.

7.

radius → $\frac{dr}{dt} = \frac{3\ cm}{s}$

$r = 20$

a) $A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi (20)(3)$

$\frac{dA}{dt} = 480\pi \frac{cm^2}{s}$ or $1507.96 \frac{cm^2}{s}$

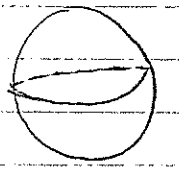
b) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (20)^2 (3)$

$\frac{dV}{dt} = 4800\pi \frac{cm^3}{s}$ or $15079.64 \frac{cm^3}{s}$

8.



radius → $\frac{dr}{dt} = \frac{2\ cm}{s}$

$r = 8\ cm$

$V = \frac{4}{3}\pi r^3$

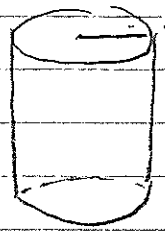
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (8)^2 (2)$

$\frac{dV}{dt} = 512\pi \frac{cm^3}{s}$ or $1608.50 \frac{cm^3}{s}$

6.5- Continued

9.



$\frac{dh}{dt} = 0.4 \frac{cm}{s}$

$V = \pi r^2 h$

$V = \pi (5)^2 h$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

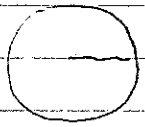
$\frac{dV}{dt} = 25\pi (0.4)$

$\frac{dV}{dt} = 10\pi \frac{cm^3}{s}$

or

$31.41 \frac{cm^3}{s}$

10.



$\frac{dA}{dt} = 128\pi \frac{cm^2}{s}$

$r = 4$

Surface Area → $A = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$128\pi = 8\pi (4) \frac{dr}{dt}$

$4 \frac{cm}{s} = \frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (4)^2 (4)$

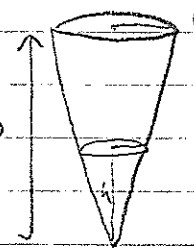
$\frac{dV}{dt} = 256\pi \frac{cm^3}{s}$

or

$804.25 \frac{cm^3}{s}$

65- Continued

11.



$\frac{dv}{dt} = 50 \frac{cm^3}{s}$

$r = 6$ cm
$h = 30$ cm
$d = 10$ cm (needed)

$\frac{r}{h} = \frac{6}{30}$

$\frac{r}{h} = \frac{1}{5}$

$5r = h$

$r = \frac{1}{5}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{5}h\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{25}h^2\right)(h)$

$V = \frac{1}{75}\pi h^3$

$\frac{dV}{dt} = \frac{\pi}{25} h^2 \frac{dh}{dt}$

$50 = \frac{\pi}{25}(10)^2 \frac{dh}{dt}$

$50 = 4\pi \frac{dh}{dt}$

$\frac{25}{2\pi} \frac{cm}{s} \frac{dh}{dt}$

or

$3.98 \frac{cm}{s}$

12.

$\frac{dr}{dt} = 0.8$ m/min

$r = 3h$

$h = \frac{1}{3}r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{1}{3}r\right)$

$V = \frac{1}{9}\pi r^3$

$\frac{dv}{dt} = \frac{1}{3}\pi r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = \frac{1}{3}\pi (3)^2 (0.8)$

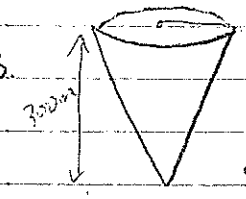
$\frac{dv}{dt} = \frac{12\pi}{5} \frac{m^3}{m}$

or

$7.54 \frac{m^3}{m}$

6.5 Continued

13.



$\frac{dV}{dt} = \frac{20 L}{s} = \frac{20000 cm^3}{s}$

$\frac{r}{h} = \frac{100 cm}{300 cm}$ $\quad h = 150 cm$

$\frac{r}{h} = \frac{1}{3}$

$3r = h$

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (\frac{1}{3}h)^2 (h)$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

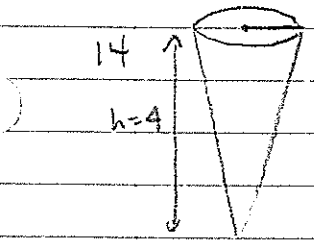
$20000 = \frac{1}{9}\pi (150)^2 \frac{dh}{dt}$

$\frac{8 \ cm}{\pi \ s} = \frac{dh}{dt}$

or

$2.55 \frac{cm}{s}$

14



$\frac{dv}{dt} = ?$

$h = 3$ or $300cm$ $\quad \frac{dh}{dt} = -0.05 \frac{cm}{s}$

$\frac{r}{h} = \frac{2}{4}$

$\frac{r}{h} = \frac{1}{2}$

$2r = h$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (\frac{1}{2}h)^2 h$

$V = \frac{1}{12}\pi h^3$

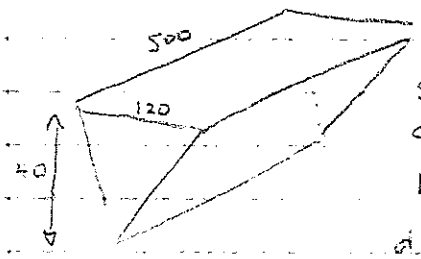
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi (300)(-0.05)$

$\frac{dV}{dt}$ $\quad 1125\pi \frac{cm^2}{s}$ or $3534.29 \frac{cm^2}{s}$

6.5 Continued

15.



$\frac{dV}{dt} = 6500 \text{ cm}^3/\text{s}$

$h = 5$

$\frac{dh}{dt} = ?$

$\frac{dl}{dt} = 0$ (constant) ← doesn't change

$b = 120 = 3h$

$h = 40 = \frac{1}{3}b$

$l = 500$

$V = \frac{1}{2}bhl$

$= \frac{1}{2}(3h)(h)l$

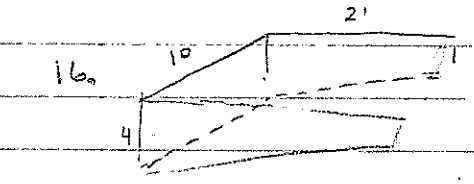
$V = \frac{3}{2}h^2 l$

$\frac{dV}{dt} = 3hl\frac{dh}{dt} + \frac{3}{2}h^2\frac{dl}{dt}$

$6500 = 3(5)(500)\left(\frac{dh}{dt}\right) + \frac{3}{2}(5)^2(0)$

$\frac{dh}{dt} = \frac{13}{15}$ or 0.87 cm/s.

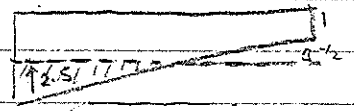
16.

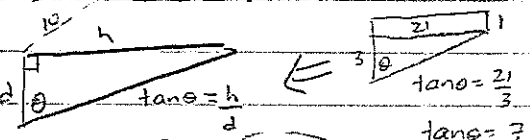


$\frac{dV}{dt} = 1.5 \text{ m}^3/\text{min}$

when $d = 2.5$ m

$\frac{d}{dt}$





$\tan\theta = 7$

$7 = \frac{h}{d}$ $h = 7d$

area of triangle (width of pool)

$V = Al$

$= (\frac{1}{2}dh)(10)$

$= \frac{1}{2}d(7d)(10)$

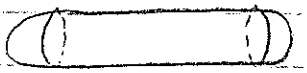
$V = 35d^2$

$\frac{dV}{dt} = 70d\frac{dd}{dt}$

$1.5 = 70(2.5)\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{6}{7}$ or 0.86 cm/min

17.



radius cylinder → $\frac{dr}{dt} = 3 \text{ cm/s}$

height cylinder → $\frac{dh}{dt} = -2 \text{ cm/s}$

$r = 9$

$h = 14$

cylinder sphere

$V = \pi r^2 h + \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 2\pi r h\frac{dr}{dt} + \pi r^2\frac{dh}{dt} + 4\pi r^2\frac{dr}{dt}$

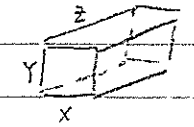
$= 2\pi(9)(14)(3) + \pi(81)(-2) + 4\pi(81)(3)$

$= 1566\pi$ or $4919.73 \text{ cm}^3/\text{s}$

Increasing at a rate of $1566\pi \text{ cm}^3/\text{s}$.

6.5- Continued

18



$\frac{dz}{dt} = 6\frac{cm}{s}$

$\frac{dx}{dt} = -5\frac{cm}{s}$

$\frac{dy}{dt} = 3\frac{cm}{s}$

$z = 14$

$x = 12$

$y = 10$

$V = l \times w \times h$

$V = z \cdot x \cdot y$

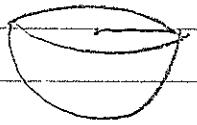
$\frac{dv}{dt} = \frac{dz}{dt}(x)(y) + \frac{dx}{dt}(z)(y) + \frac{dy}{dt}(z)(x)$

$\frac{dv}{dt} = (6)(12)(10) + (-5)(14)(10) + (3)(14)(12)$

$\frac{dv}{dt} = 720 - 700 + 504$

$\frac{dv}{dt} = \frac{524\ cm^3}{s}$ increasing at a rate of $524\frac{cm^3}{s}$

19.



$r = 20$

$\frac{dV}{dt} = -36\frac{cm^3}{s}$

$h = 4cm$

$\frac{dr}{dt} = 0$ (Constant)

$V = \pi h^2 r - \frac{1}{3}\pi h^3$

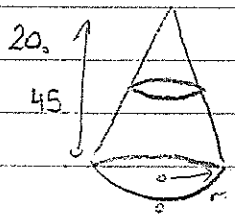
$\frac{dv}{dt} = 2\pi hr\frac{dh}{dt} + \pi h^2\frac{dr}{dt}^{=0} - \pi h^2\frac{dh}{dt}$

$\frac{dv}{dt} = \frac{dh}{dt}(2\pi hr - \pi h^2)$

$-36 = \frac{dh}{dt}(2\pi(4)(20) - \pi(4)^2)$

$\frac{dh}{dt} = \frac{-1}{4\pi}$ or $0.08\ cm/s$

20.



$\frac{dV}{dt} = -200\frac{cm^3}{s}$

$h = 45$

$r = 21$

$H \to 45$ } measurements as whole

$R \to 21$ }

$\frac{r}{h} = \frac{21}{45}$

$\frac{r}{h} = \frac{7}{15}$

$15r = 7h$

$r = \frac{7}{15}h$

whole ↓ part ↓

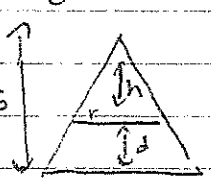
$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(21)^2(45) - \frac{1}{3}\pi\left(\frac{7}{15}h\right)^2 h$

$V = 6615\pi - \frac{49\pi h^3}{675}$

$V = 6615\pi - \frac{49\pi(45-d)^3}{675}$

$d = 24$



$h = 45 - d$

$\frac{dV}{dt} = -\frac{49\pi}{675}(3)(45-d)^2(-1)\frac{dd}{dt}$

$-200 = \frac{49\pi(45-24)^2}{225}\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{-5000}{441}$ or $0.66\ cm/s$