6.2 Distance, Velocity, Acceleration P. 271 1-7

1. a) $s(t) = t^3 + 4t^2 + 5t + 9$

$v(t) = s'(t) = 3t^2 + 8t + 5$ ← velocity

$a(t) = s''(t) = 6t + 8$ ← acceleration

b) $s(t) = t^2\sqrt{t+1}$

$v(t) = 2t(t+1)^{1/2} + t^2\left(\frac{1}{2}\right)(t+1)^{-1/2}$

$= \frac{1}{2}(t+1)^{-1/2}\left[4t(t+1) + t^2\right]$

$= \frac{1}{2}(t+1)^{-1/2}\left[4t^2 + 4t + t^2\right]$

$= \frac{1}{2}t(t+1)^{-1/2}(5t+4) \Rightarrow \left(\frac{5}{2}t^2 + 2t\right)(t+1)^{-1/2}$

c) $a(t) = (5t+2)(t+1)^{-1/2} + \frac{1}{2}t(5t+4)\left(-\frac{1}{2}\right)(t+1)^{-3/2}$

$= \frac{1}{4}(t+1)^{-3/2}\left[(5t+2)(t+1) - \frac{1}{4}t(5t+4)\right]$

$= \frac{1}{4}(t+1)^{3/2}\left[4(5t^2+7t+2) - t(5t+4)\right]$

$= \frac{1}{4}(t+1)^{-3/2}\left[20t^2 + 28t + 8 - 5t^2 - 4t\right]$

$= \frac{1}{4}(t+1)^{-3/2}\left[15t^2 + 24t + 8\right]$

c) $s(t) = \dfrac{3t^2}{t+2}$

$v(t) = \dfrac{(6t)(t+2) - (3t^2)(1)}{(t+2)^2}$

$= \dfrac{6t^2 + 12t - 3t^2}{(t+2)^2}$

$= \dfrac{3t^2 + 12t}{(t+2)^2}$

$= \dfrac{3t(t+4)}{(t+2)^2}$

$a(t) = \dfrac{(6t+12)(t+2)^2 - 3t(t+4)(2)(t+2)}{(t+2)^4}$

$= \dfrac{6\left[(t+2)(t+2) - t(t+4)\right]}{(t+2)^3}$

$= \dfrac{6\left[t^2 + 4t + 4 - t^2 - 4t\right]}{(t+2)^3}$

$= \dfrac{6(4)}{(t+2)^3} = \dfrac{24}{(t+2)^3}$

6.2 - Continued

2 $s(t) = t^3 + 6t^2 + 9t$

$v(t) = 3t^2 + 12t + 9$

$a(t) = 6t + 12$

a) Initial position

$s(0) = 0^3 + 6(0)^2 + 9(0)$

$= 0$

b) $v(t) = 3t^2 + 12t + 9$ $\quad a(t) = 6t + 12$

c) average velocity $\Rightarrow \frac{s(6) - s(2)}{6 - 2}$

$= \frac{456 - 50}{4} = 109 \text{ m/s}$

d) instantaneous velocity $\Rightarrow$ $t = 2 \Rightarrow v(2) = 3(2)^2 + 12(2) + 9 = 45 \text{ m/s}$

$v(4) = 3(4)^2 + 12(4) + 9 = 105 \text{ m/s}$

$v(6) = 3(6)^2 + 12(6) + 9 = 189 \text{ m/s}$

e) average instantaneous velocities $\frac{v(6) + v(2)}{2} = \frac{189 + 45}{2} = 117 \text{ m/s}$

average velocity $\Rightarrow$ 109 m/s

$\therefore$ NO

f) Instantaneous velocity at $t = 4$ is 105 m/s

ave instant. velocities $\frac{v(6) + v(2)}{2} = 117 \text{ m/s}$ } NO

g) in. velocity at $t = 4$ is 105 m/s } NO

average velocity is 109 m/s

h) $v(t) = 3t^2 + 12t + 9$

$144 = 3t^2 + 12t + 9$

$0 = 3t^2 + 12t - 135$

$= 3t^2 + 27t - 15t - 135$

$= 3t(t + 9) - 15(t + 9)$

$= (3t - 15)(t + 9)$

$t = 5$ $\quad t = -9$

To find height

$s(5) = 5^3 + 6(5)^2 + 9(5)$

$= 320 \text{ m}$ height

i) $a(5) = 6(5) + 12$

$= 42 \text{ m/s}^2$

6.2 Continued

2 j) $30 = 6t + 12$

$18 = 6t$

$3 = t$

"x" → time

"y" → $(3)^3 + 6(3)^2 + 9(3)$

$= 27 + 54 + 27$

$= 108$ m ← height

K) ave acceleration $\frac{v(6) - v(2)}{6 - 2}$

$= \frac{189 - 45}{4} = 36 \text{ m/s}^2$

3. $s(t) = t^3 - 9t^2 + 24t \quad t \geq 0$

$v(t) = 3t^2 - 18t + 24$

$a(t) = 6t - 18$

a) $v(1) = 3(1)^2 - 18(1) + 24 = 9 \text{ m/s}$

$v(3) = 3(3)^2 - 18(3) + 24 = -3 \text{ m/s}$

b) $v(t) = 3t^2 - 18t + 24$

$= 3(t^2 - 6t + 8)$

$= 3(t - 4)(t - 2)$

[+ + + + + + + + + + + + 3

[- - - - - - - - 0 + + + t-4

[- - - 0 + + + + + + + t-2

0 + + + 2 - - - - 4 + + +

b) increasing (moving to right)

$[0, 2) \cup (4, \infty)$

c) decreasing (moving to left)

$(2, 4)$

d) $s(0) = (0)^3 - 9(0)^2 + 24(0) = 0$ Distance travelled use roots

$s(2) = 2^3 - 9(2)^2 + 24(2) = 20$

$s(4) = 4^3 - 9(4)^2 + 24(4) = 16$

$s(5) = 5^3 - 9(5)^2 + 24(5) = 20$

Between $t = 0$ and $t = 2$ object moved $|20 - 0| = 20$ right $= +20$

Between $t = 2$ and $t = 4$ object moved $|16 - 20| = 4$ left $= +4$

Between $t = 4$ and $t = 5$ object moved $|20 - 16| = 4$ right $= +4$

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6.2 Continued

4. $h(t) = 72 - 4.9t^2$

$v(t) = -9.8t$

$a(t) = -9.8$

a) $v(3) = -9.8(3)$

$= -29.4 \text{ m/s}$

b) average velocity $\Rightarrow \frac{h(3) - h(1)}{3-1}$

$= \frac{27.9 - 67.1}{2} = -19.6 \text{ m/s}$

c) $h(t) = 72 - 4.9t^2$ ← hit the ground

$0 = 72 - 4.9t^2$

$14.6939 = t^2$

$t = 3.8 \text{ s}$

d) Acceleration of Ball $\Rightarrow -9.8 \text{ m/s}^2$

5. $h(t) = -5t^2 + 100t + 8$

$v(t) = -10t + 100$

$a(t) = -10$

a) $v(t) = -10t + 100$

$v(0) = -10(0) + 100$

$v(0) = 100 \text{ m/s}$

b) $v(3) = -10(3) + 100$

$= 70 \text{ m/s}$

c) $v(t) = -10(t - 10)$ ← find the root.

$t = 10 \text{ s}$

d) $h(10) = -5(10)^2 + 100(10) + 8$

$= 508 \text{ m}$

e) $h(t) = -5t^2 + 100t + 8$ ← hit the ground

$0 = -5t^2 + 100t + 8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-100 \pm \sqrt{100^2 - 4(-5)(8)}}{2(-5)}$

$= \frac{-100 \pm \sqrt{10160}}{-10} = \frac{-0.07968}{\text{or } 20.08 \text{ s}}$

f) at $t = 20.08 \text{ s}$

$v(20.08)$

$= -10(20.08) + 100$

$= -100.8 \text{ m/s}$

6.2 Continued

6. $s(t) = 3t^2$

$v(t) = 6t$

$a(t) = 6$

a) $s(10) = 3(10)^2$

$= 300\text{ m}$

b) $v(6) = 6(6)$

$= 36\text{ m/s}$

c) $192 = 3t^2$ $\quad v(8) = 6(8)$

$64 = t^2$ $\quad = 48\text{ m/s}$

$8 = t$

d) $60 = 6t$

$t = 10\text{ s}$

7. $s(t) = 33 - 32t + 8t^2$

$v(t) = -32 + 16t$

$a(t) = 16$

a) $s(0) = 33 - 32(0) + 8(0)^2$

$= 33\text{ m}$

b) $v(0) = -32 + 16(0)$

$= -32\text{ m/s}$

c) $0 = -32 + 16t$ ← when she stopped

$-16t = -32$

$t = 2\text{ s}$

d) $s(2) = 33 - 32(2) + 8(2)^2$

$= 1\text{ m}$