6.3 Proving Identities

To prove that an identity is true for all permissible values, it is necessary to express both sides of the identity in equivalent forms. One or both sides of the identity must be algebraically manipulated into an equivalent form to match the other side.

You cannot perform operations across the equal sign when proving a potential identity. Simplify the expressions on each side of the identity independently.

Example Prove the following:

$$1 - sin^2x = sinxcosxcotx$$

$\cos^2 x$ | $\cancel{\sin x}\cos x \dfrac{\cos}{\cancel{\sin x}}$

$\cos^2 x$

QED

Example: Prove the following: $\frac{tanx cosx}{cscx} = 1 - cos^2x$

$\left(\frac{\sin x}{\cancel{\cos x}}\right)\frac{\cancel{\cos x}}{1}\ \frac{\sin x}{1}$

$\sin^2 x$

$\sin^2 x$

QED

Example   Prove the following:

$$\frac{sinx - sinxcos^2x}{sin^2x} = sinx$$

$$\frac{\sin x\,(\underline{1-\cos^2 x})}{\sin^2 x}$$

$$\frac{\sin x\,(\cancel{\sin^2 x})}{\cancel{\sin^2 x}}$$

$$\sin x$$

QED

Example: Prove the following:

$$\frac{1}{sinx-1} \cdot \frac{sinx+1}{sinx+1} + \frac{1}{sinx+1} \cdot \frac{(sinx-1)}{(sinx-1)} = \frac{-2tanx}{cosx}$$

$$\frac{sinx+1+sinx-1}{(sinx+1)(sinx-1)}$$

$$\frac{2sinx}{sin^2x-1}$$

$$\frac{2sinx}{-cos^2x}$$

$$\frac{-2\left(\frac{sinx}{cosx}\right)}{\cancel{cosx}} \quad \frac{1}{cosx}$$

$$\frac{-2sinx}{cos^2x}$$

QED

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#'s 1b,c,d, 2, 3

Sometimes we need to be aware of our double angle identities to prove an identity.

Example: Prove that $\frac{1}{cotx} = \frac{1-\boxed{cos2x}}{sin2x}$

$$\frac{\sin x}{\cos x}$$

$$\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$$

$$\frac{\cancel{1-1+}\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cos x}$$

$$\frac{\sin x}{\cos x}$$

QED

Your Turn

Prove that $\frac{\sin 2x}{\cos 2x + 1} = \tan x$ is an identity for all permissible values of x.

$$\frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x \cancel{- 1 + 1}}$$

$$\frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x}$$

QED

Sometimes we need to convert our identity in terms of $sinx$ or $cosx$, or we may need to have to multiply by a conjugate.

Example 3: Prove that $\frac{1-cosx}{sinx} = \frac{sinx}{1+cosx}$

$$\frac{sinx}{(1+cosx)} \cdot \frac{(1-cosx)}{(1-cosx)}$$

$$\frac{(\sin x)(1-\cos x)}{1-\cos^2 x}$$

$$\frac{\cancel{\sin x}(1-\cos x)}{\sin^{\cancel{2}} x}$$

Your Turn

Prove that $\frac{1}{1+\sin x} = \frac{\sec x - \sin x \sec x}{\cos x}$ is an identity for all permissible values of x.

$$\frac{1}{1+\sin x}$$

$$\frac{\sec x(1-\sin x)}{\cos x}$$

$$\frac{(1-\sin x)}{\cos^2 x}\cdot\frac{(1+\sin x)}{(1+\sin x)}$$

$$\frac{\cancel{1-\sin^2 x}\ 1}{\cancel{\cos^2 x}(1+\sin x)}$$

Example 4: Prove that $\cot x - \csc x = \frac{\cos 2x - \cos x}{\sin 2x + \sin x}$

$$\frac{\cos x}{\sin x} - \frac{1}{\sin x}$$

$$\frac{\cos x - 1}{\sin x}$$

QNED

$$\frac{2\cos^2 x - 1 - \cos x}{2\sin x\cos x + \sin x}$$

$$\frac{2\cos^2 x - \cos x - 1}{\sin x(2\cos x + 1)}$$

$$\frac{\cancel{(2\cos x + 1)}(\cos x - 1)}{\sin x(2\cancel{\cos x + 1})}$$

Your Turn

Prove the identity $\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1} = \dfrac{\sin^2 x \cos x + \cos^3 x}{2\sin x + 1}$ for all permissible values of x.

$$\frac{2\sin x\cos x - \cos x}{(2\sin x - 1)(2\sin x + 1)}$$

$$\frac{\cos x(\cancel{2\sin x - 1})}{(\cancel{2\sin x - 1})(2\sin x + 1)}$$

$$\frac{\cos x(\cancel{\sin^2 x + \cos^2 x})}{2\sin x + 1}$$

QEDFS

Key Ideas

- Verifying an identity using a specific value validates that it is true for that value only. Proving an identity is done algebraically and validates the identity for all permissible values of the variable.
- To prove a trigonometric identity algebraically, separately simplify both sides of the identity into identical expressions.
- It is usually easier to make a complicated expression simpler than it is to make a simple expression more complicated.
- Some strategies that may help you prove identities include:
  - Use known identities to make substitutions.
  - If quadratics are present, the Pythagorean identity or one of its alternate forms can often be used.
  - Rewrite the expression using sine and cosine only.
  - Multiply the numerator and the denominator by the conjugate of an expression.
  - Factor to simplify expressions.

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#'s 7,10 (just prove algebraically) 11, 15