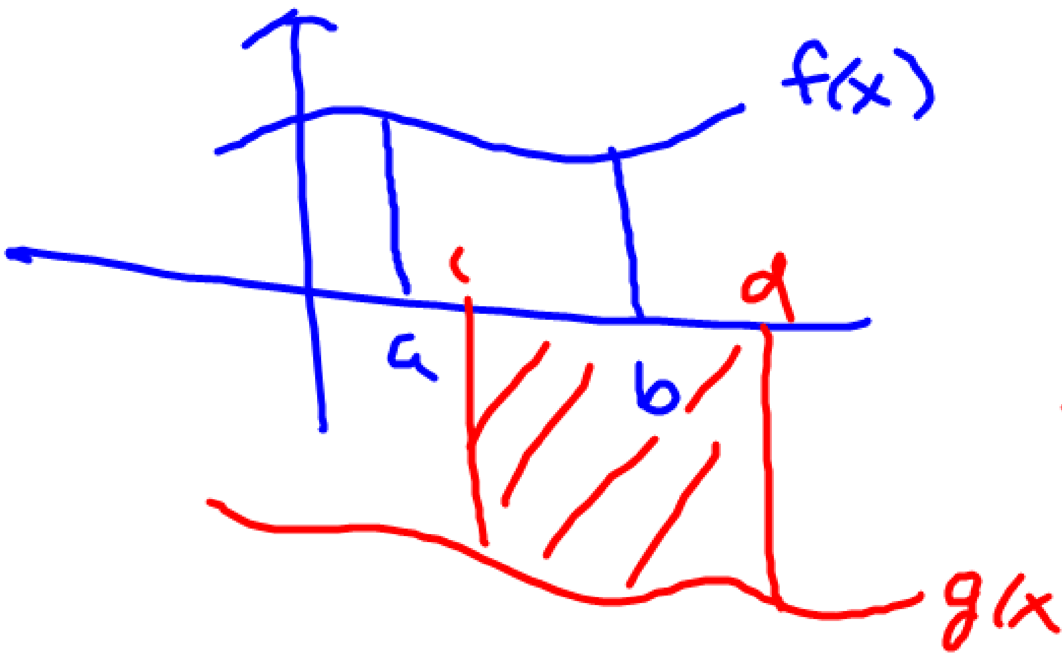
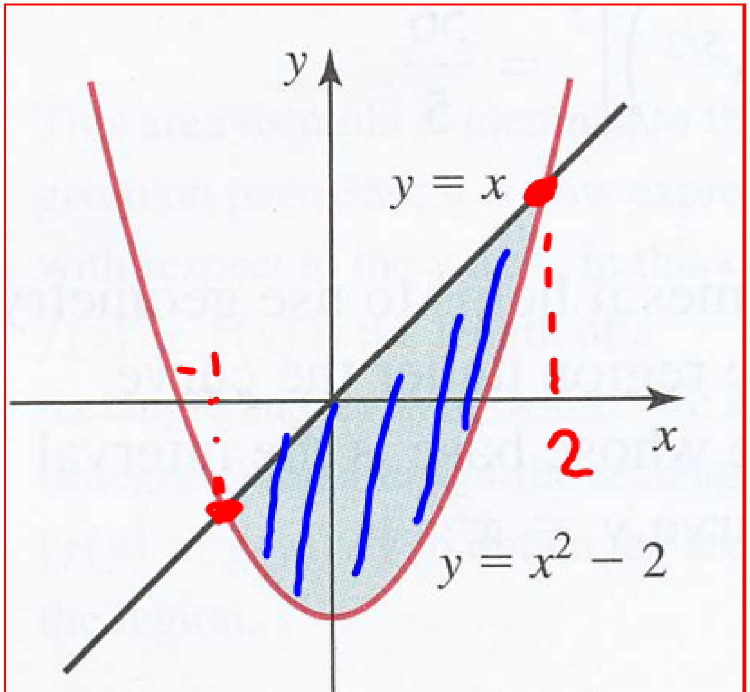
6.3 Area Between Two Curves



$$\int_a^b f(x)\,dx - \int_c^d g(x)$$

Find the total area of the shaded region.



Intersection Points

$x = x^2 - 2$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$\boxed{x = 2 \text{ or } x = -1}$

$$A = \int_{-1}^{2} \left(x - (x^2 - 2)\right) dx = \int_{-1}^{2} (x - x^2 + 2)\, dx$$

$$\text{Area} = \frac{x^2}{2} - \frac{x^3}{3} + 2x \Big|_{-1}^{2}$$

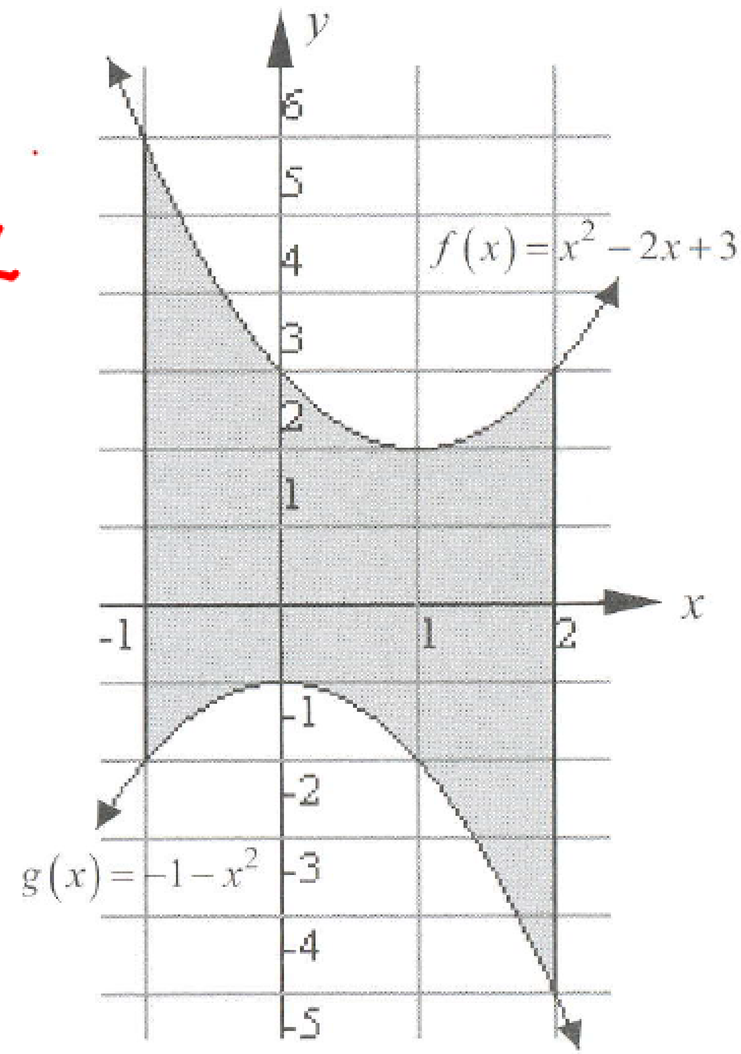
$$= \left[\frac{(2)^2}{2} - \frac{(2)^3}{3} + 2(2)\right] - \left[\frac{(-1)^2}{2} - \frac{(-1)^3}{3} + 2(-1)\right]$$

$$= \frac{9}{2}$$

In general if $f(x)$ and $g(x)$ are continuous functions with $f(x) \geq g(x)$ throughout $[a,b]$, then the area between the curves $f(x)$ and $g(x)$ from a to b is given by :

$$\mathrm{A} = \int_{\mathrm{a}}^{\mathrm{b}} \left(f(x) - g(x)\right) dx$$

Example 1 Find the area of the region bounded by $f(x)=x^2-2x+3$, $g(x)=-1-x^2$, and the lines $x=-1$ and $x=2$.



$$\text{Area}=\int_{-1}^{2}(x^2-2x+3)-(-1-x^2)\,dx$$

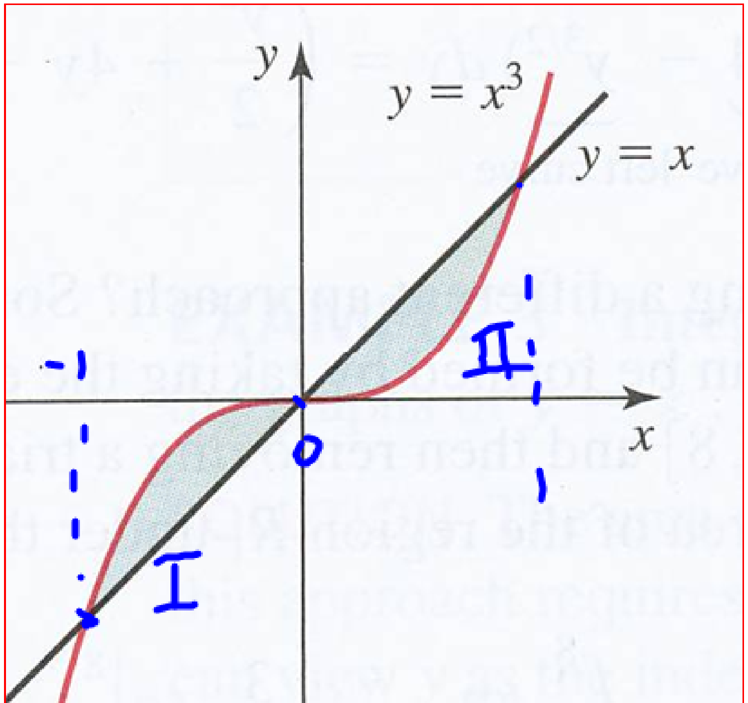
$$A=\int_{-1}^{2}(x^2-2x+3+1+x^2)\,dx$$

$$=\int_{-1}^{2}(2x^2-2x+4)\,dx$$

using Calc.

$=15$

Find the total area of the shaded region.



Intersection Points

$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0$

$x = 0 \quad x = 1 \quad x = -1$

$$\text{Area I} = \int_{-1}^{0} (x^3 - x)\,dx$$

calc

$$\text{Area II} = \int_{0}^{1} (x - x^3)\,dx$$

calc

Area I $= \frac{1}{4}$ Area II $= \frac{1}{4}$

Total Area $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Say we have the two functions $y=2x$ and $y=x^2$ graphed on the same graph, and we are asked to find the area between the two curves.

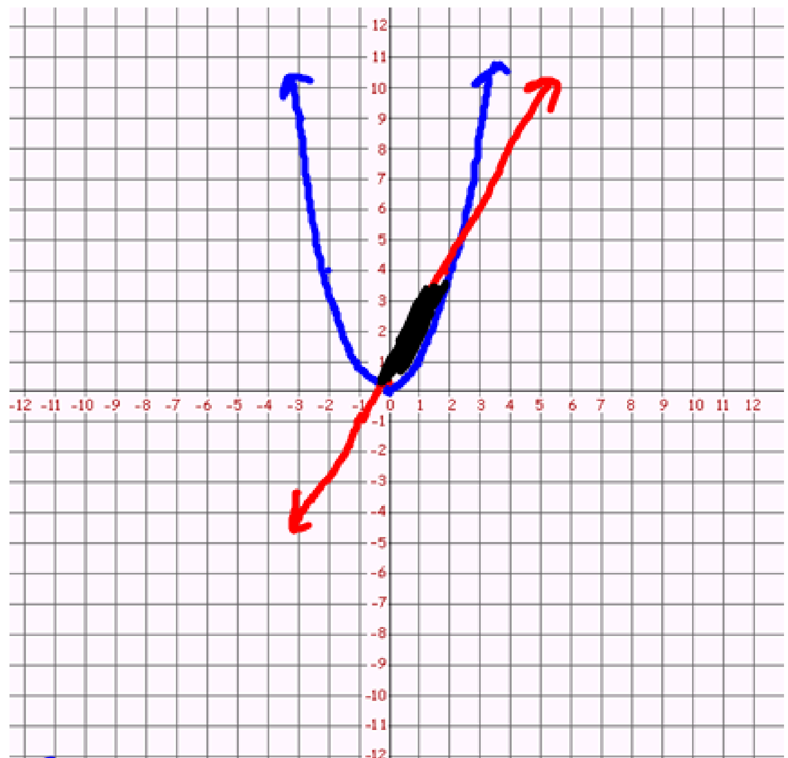
Intersection Points

$$x^2 = 2x$$

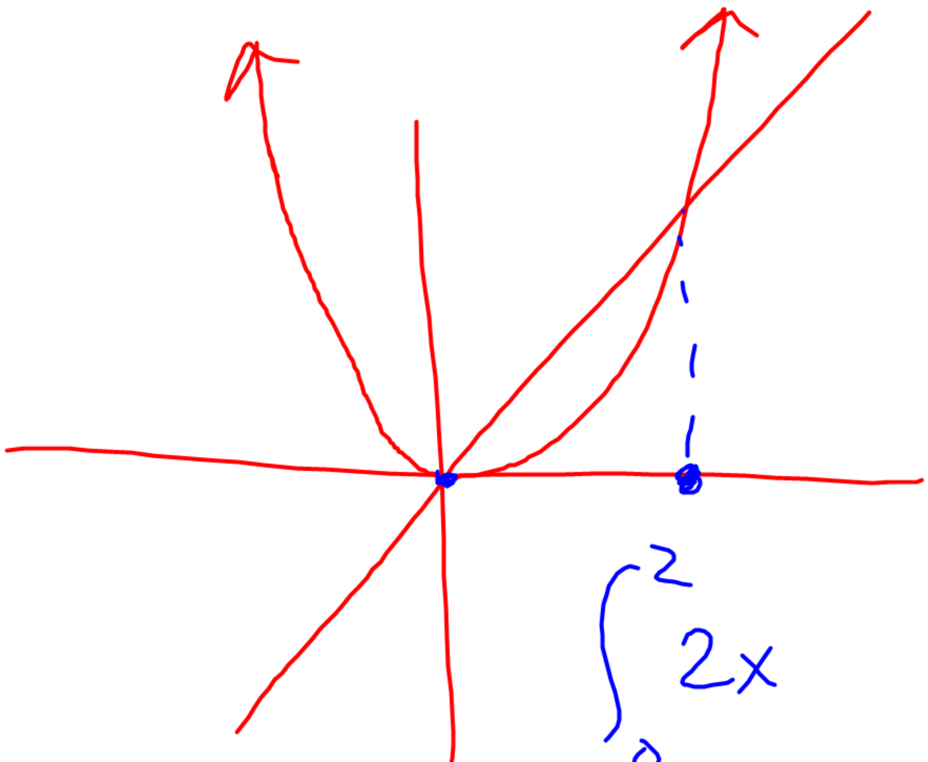
$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0 \qquad x = 2$$



$$A = \int_0^2 (2x - x^2)\,dx \qquad \text{calc} = 4/3$$



$$\int_0^2 2x \qquad \int_0^2 x^2$$

Example 1: Your Turn!

Find the area of the regon enclosed by the curves $y = 2\text{-}x^2$ and $y = \text{-}x$.

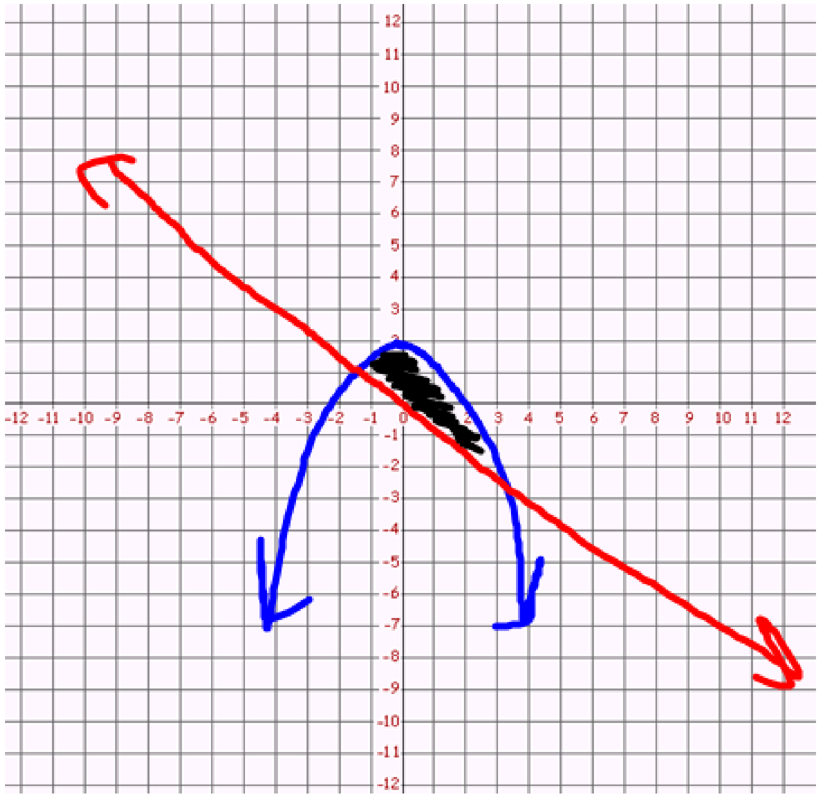
Intersection Points

$2 - x^2 = -x$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2$ or $x = -1$



$$\text{Area} = \int_{-1}^{2} 2 - x^2 - (-x)\, dx$$

$$= \int_{-1}^{2} (2 - x^2 + x)\, dx = 9/2$$

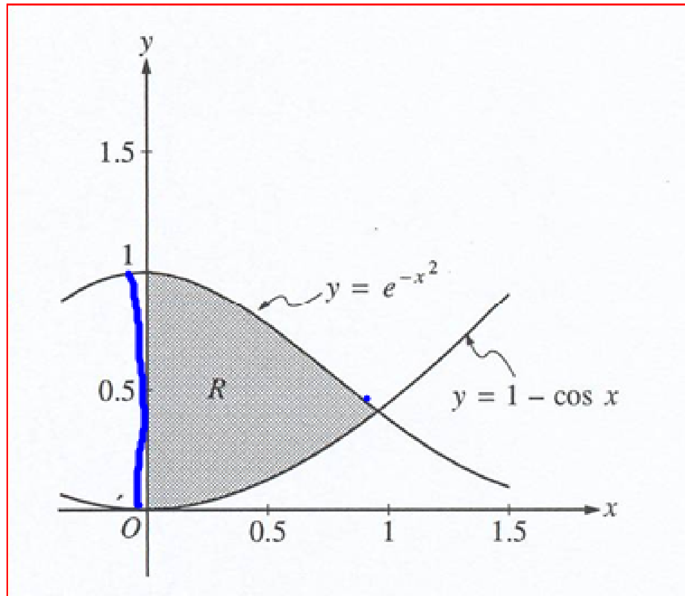
AP Calculus AB–1 / BC–1 2000 Free Response #1

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

$$A = \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$$

$$= .591$$



$A = .94194408$

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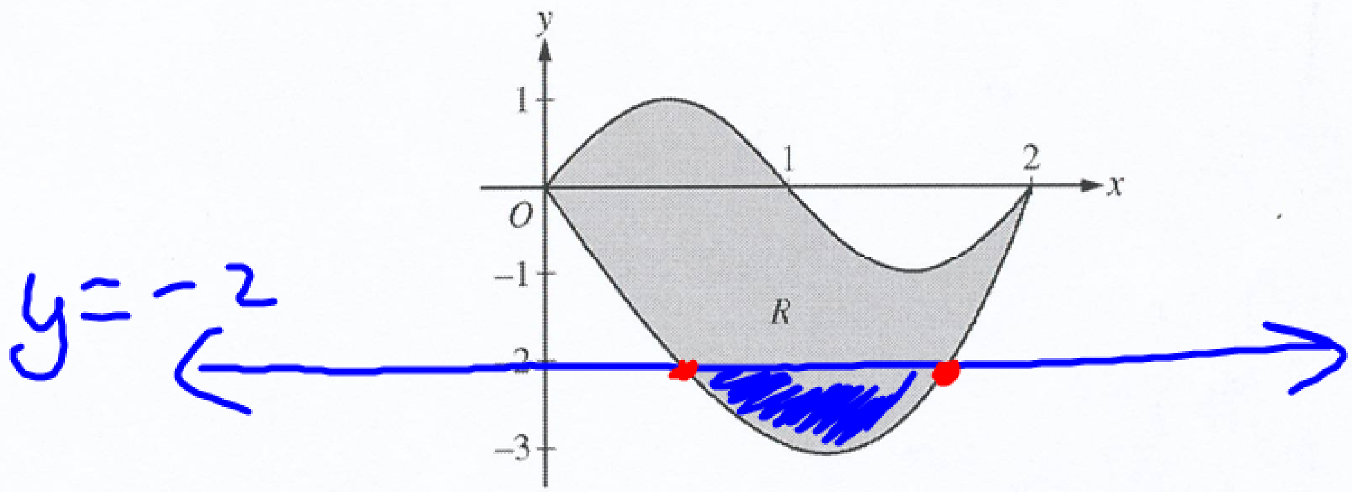
Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

$$A = \int_A^B \left(-2 - (x^3 - 4x)\right) dx$$

$A = .531887$

$B = 1.6751...$

a) Area $= \int_0^2 \left(\sin \pi x - (x^3 - 4x)\right) dx = 4$

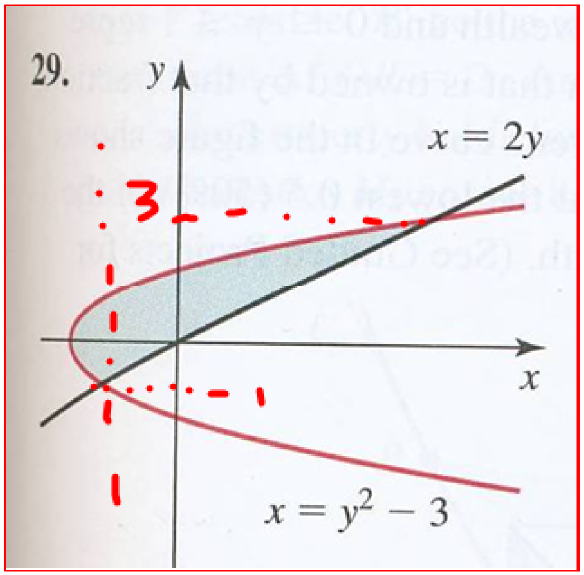
Integrating With Respect to y

Sometimes the boundaries of a region are more easily described by functions of *y* than by functions of *x*.

$x+3=y^2$

Find the total area of the shaded region.

Make equations be $x =$ ______



Intersection Points

$2y = y^2 - 3$

$0 = y^2 - 2y - 3$

$0 = (y-3)(y+1)$

$y = 3$ or $y = -1$

$$A = \int_{-1}^{3} 2y - (y^2 - 3)\,dy = \int_{-1}^{3} (2y - y^2 + 3)\,dy = 32/3$$

In general if $f(y)$ and $g(y)$ are continuous functions with $f(y) \geq g(y)$ throughout $[c,d]$, then the area between the curves $f(y)$ and $g(y)$ from c to d is given by :

$$\mathrm{A} = \int_{\mathrm{c}}^{\mathrm{d}} \left(f(y) - g(y)\right) dy$$

HUGE

sideways Parabola

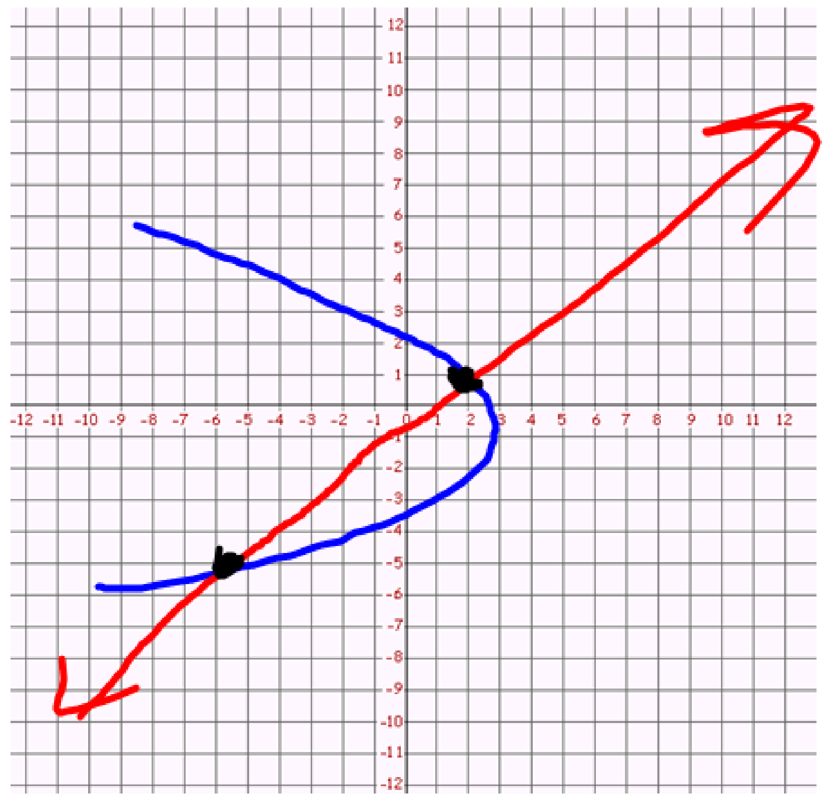
Find the area of the region bounded by the graphs

$x = 3 - y^2$ and $x = y + 1$.

$y = x - 1$

$y^2 = 3 - x$

$y = \pm\sqrt{3 - x}$



Intersection Points

$$3 - y^2 = y + 1$$

$$0 = y^2 + y - 2$$

$$0 = (y + 2)(y - 1)$$

$$y = -2 \qquad y = 1$$

$$\text{Area} = \int_{-2}^{1} 3-y^2 - (y+1)\, dy$$

$$= \int_{-2}^{1} (3-y^2-y-1)\, dy$$

$$= \int_{-2}^{1} (-y^2-y+2)\, dy = 9/2$$

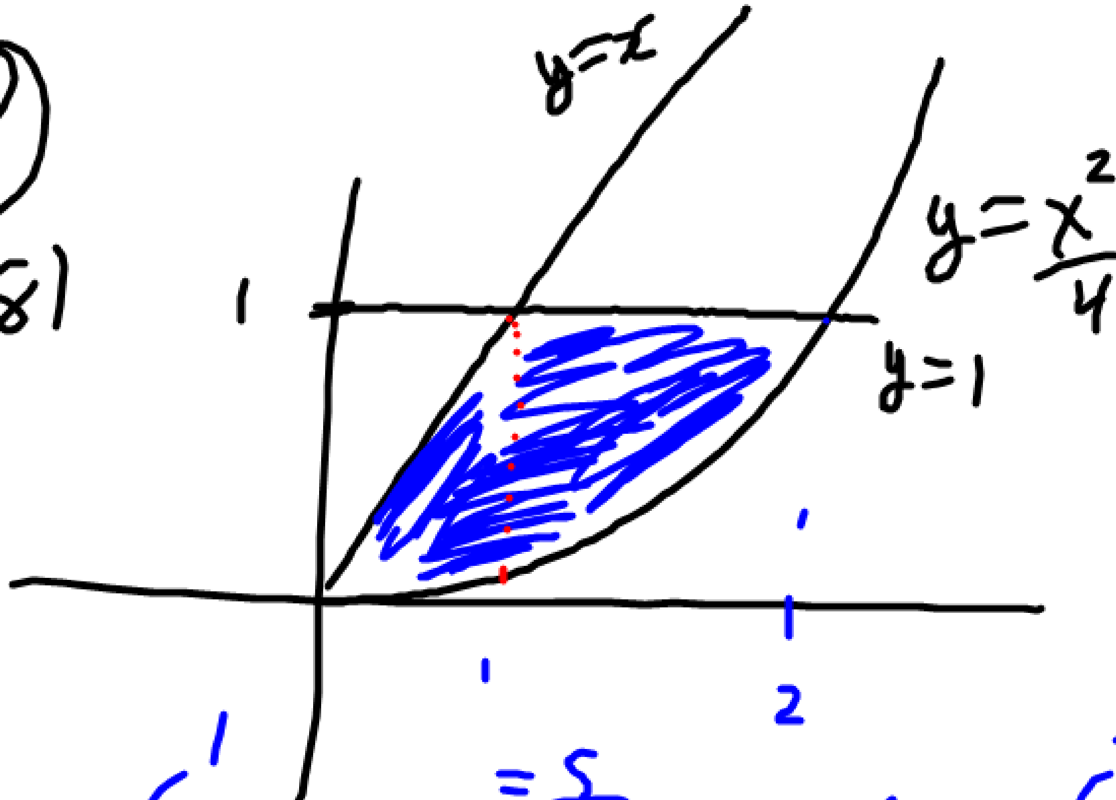
Assignment
Calc 30 Text Page 384
#'s 11, 12, 15, (use calculator to help sketch curve)

AP Calc Text Page 380
#'s 5, 6, 7, 8, 9
#'s 11, 13, 32 (use calculator to help sketch) with repsect to x axis

#'s 3, 4, 18, with respect to y axis

7

p.381



$$4y = x^2$$

$$\pm\sqrt{4y} = x$$

$$\sqrt{4y} = x$$

$$2\sqrt{y} = x$$

$$A_I = \int_0^1 \left(x - \frac{x^2}{4}\right) dx = \frac{5}{12}$$

$$A_{II} = \int_1^2 \left(1 - \frac{x^2}{4}\right) dx \quad \frac{5}{12}$$

$$= \frac{5}{6}$$

$$A = \int_0^1 \left(2\sqrt{y} - y\right) dy = \frac{5}{6}$$

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Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

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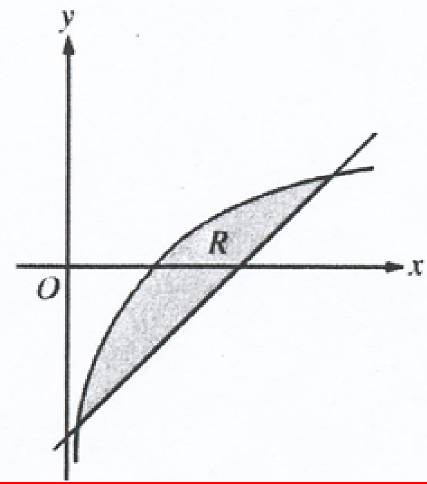
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



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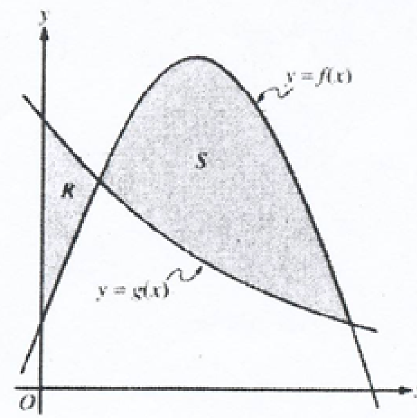
Question 1



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

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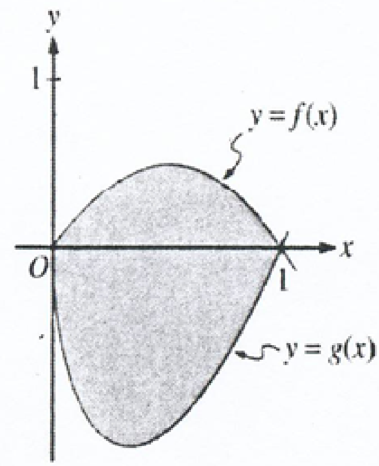
Question 2



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

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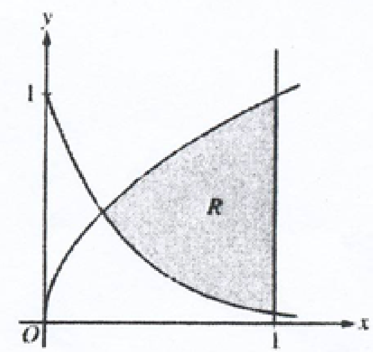
Question 1

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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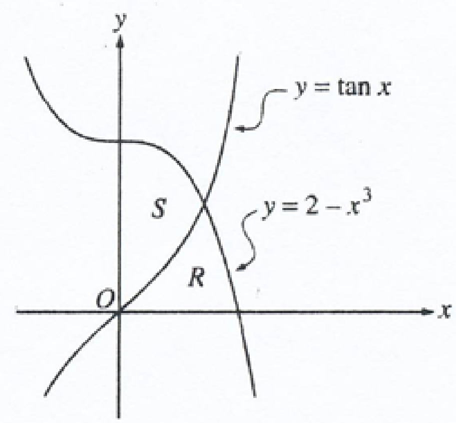
Question 1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the x-axis.