6.3 Answer Key

1b) $$\frac{\cos^2 x - \cos x - 2}{6\cos x - 12}$$

$$= \frac{(\cos x - 2)(\cos x - 1)}{6(\cos x - 2)}$$

$$= \frac{\cos x - 1}{6}$$

c) $$\frac{\sin x \cos x - \sin x}{\cos^2 x - 1}$$

$$= \frac{\sin x(\cos x - 1)}{(\cos x - 1)(\cos x + 1)}$$

$$= \frac{\sin x}{\cos x + 1}$$

d) $$\frac{\tan^2 x - 3\tan x - 4}{\sin x \tan x + \sin x}$$

$$= \frac{(\tan x - 4)(\tan x + 1)}{\sin x(\tan x + 1)}$$

$$= \frac{\tan x - 4}{\sin x}$$

$$= \frac{\tan x}{\sin x} - \frac{4}{\sin x}$$

$$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x} - 4\csc x$$

$$= \frac{1}{\cos x} - 4\csc x$$

$$= \sec x - 4\csc x$$

2. a) $\cos x + \cos x \tan^2 x = \sec x$

$= \cos x(1 + \tan^2 x)$ | $\frac{1}{\cos x}$

$= \cos x\ (\sec^2 x)$

$= \frac{\cos x}{\cos^2 x}$

$= \frac{1}{\cos x}$

b) $\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} = \sin x - \cos x$

$= \frac{(\sin x - \cos x)(\sin x + \cos x)}{(\sin x + \cos x)}$

$= \sin x - \cos x$.

c) $\frac{\sin x \cos x - \sin x}{\cos^2 x - 1} = \frac{1 - \cos x}{\sin x}$

$= \frac{\sin x(\cos x - 1)}{(\cos x - 1)(\cos x + 1)}$

$= \frac{\sin x}{\cos x + 1} \cdot \left(\frac{1 - \cos x}{1 - \cos x}\right)$

$= \frac{(\sin x)(1 - \cos x)}{1 - \cos^2 x}$

$= \frac{\sin x(1 - \cos x)}{\sin^2 x}$

$= \frac{1 - \cos x}{\sin x}$.

2d) $\dfrac{1-\sin^2 x}{1+2\sin x-3\sin^2 x} = \dfrac{1+\sin x}{1+3\sin x}$

$\dfrac{(1-\sin x)(1+\sin x)}{(1+3\sin x)(1-\sin x)}$

$\dfrac{1+\sin x}{1+3\sin x}$

3a) $\dfrac{\sin x}{\cos x} + \sec x$

$\dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}$

$\dfrac{\sin x+1}{\cos x}$

b) $\dfrac{1}{\sin x-1} + \dfrac{1}{\sin x+1}$

$= \dfrac{(\sin x+1)}{(\sin x+1)}\dfrac{1}{(\sin x-1)} + \dfrac{1}{(\sin x+1)}\left(\dfrac{\sin x-1}{\sin x-1}\right)$

$= \dfrac{\sin x+1+\sin x-1}{(\sin x+1)(\sin x-1)}$

$= \dfrac{2\sin x}{\sin^2 x-1}$

$= \dfrac{2\sin x}{-(\cos^2 x)}$

$= -\dfrac{2\sin x}{\cos x\cos x} = \dfrac{-2\tan x}{\cos x}$

c) $\frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x}$

$= \left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{1+\cos x} + \frac{\cos x}{\sin x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$= \frac{\sin^2 x + \cos x + \cos^2 x}{\sin x (1+\cos x)}$

$= \frac{1+\cos x}{\sin x (1+\cos x)}$

$= \frac{1}{\sin x}$

$= \csc x$

d) $\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1}$

$= \left(\frac{\sec x + 1}{\sec x + 1}\right)\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1}\left(\frac{\sec x - 1}{\sec x - 1}\right)$

$= \frac{1+\cos x + 1 - \cos x}{\sec^2 x - 1}$

$= \frac{2}{\tan^2 x}$

$= 2\cot^2 x$

7a) $\dfrac{\csc x}{2\cos x} = \csc 2x$

$\dfrac{\frac{1}{\sin x}}{2\cos x}$ | $\dfrac{1}{\sin 2x}$

$\dfrac{1}{2\sin x \cos x}$ | $\dfrac{1}{2\sin x \cos x}$

b) $\sin x + \cos x \cot x = \csc x$

$\sin x + \cos x \left(\dfrac{\cos x}{\sin x}\right)$ | $\dfrac{1}{\sin x}$

$\sin x + \dfrac{\cos^2 x}{\sin x}$

$\left(\dfrac{\sin x}{\sin x}\right) \sin x + \dfrac{\cos^2 x}{\sin x}$

$\dfrac{\sin^2 x + \cos^2 x}{\sin x}$

$\dfrac{1}{\sin x}$

10. a) $\dfrac{\csc x}{2\cos x} = \csc 2x$

* see 7a)

b) $\dfrac{\sin x \cos x}{1+\cos x} = \dfrac{1-\cos x}{\tan x}$

$\dfrac{\sin x \cos x}{1+\cos x}\left(\dfrac{1-\cos x}{1-\cos x}\right)$

$\dfrac{\sin x \cos x - \sin x \cos^2 x}{1-\cos^2 x}$

$\dfrac{\sin x(\cos x - \cos^2 x)}{\sin^2 x}$

$\dfrac{\cos x - \cos^2 x}{\sin x}$

$\dfrac{\cos x(1-\cos^2 x)}{\sin x}$

$\dfrac{\cos x}{\sin x}(1-\cos x)$

$\cot x(1-\cos x)$

$\dfrac{1-\cos x}{\tan x}$

10c) $\dfrac{\sin x + \tan x}{1+\cos x} = \dfrac{\sin 2x}{2\cos^2 x}$

$$\frac{\sin x + \frac{\sin x}{\cos x}}{1+\cos x}$$

$$\frac{\left(\frac{\cos x}{\cos x}\right)\sin x + \frac{\sin x}{\cos x}}{1+\cos x}$$

$$\frac{\left(\frac{\sin x \cos x + \sin x}{\cos x}\right)}{1+\cos x}$$

$$\left(\frac{\sin x \cos x + \sin x}{\cos x}\right)\left(\frac{1}{1+\cos x}\right)$$

$$\frac{\sin x(\cos x + 1)}{\cos x} \cdot \frac{1}{(1+\cos x)}$$

$$\frac{\sin x}{\cos x}$$

$$\frac{2\sin x \cos x}{2\cos^2 x}$$

$$\frac{\sin x}{\cos x}$$

11 a) $\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \csc x$

$\frac{\sin x}{\sin x}\left(\frac{2\sin x\cos x}{\cos x} + \frac{1-2\sin^2 x}{\sin x}\right)$ $\left(\frac{1}{\sin x}\right)$

$\frac{2\sin^2 x\cos x}{\sin x\cos x} + \frac{\cos x - 2\cos x\sin^2 x}{\sin x\cos x}$

$\frac{2\sin^2 x\cos x + \cos x - 2\cos x\sin^2 x}{\sin x\cos x}$

$\frac{\cos x\left(2\sin^2 x + 1 - 2\sin^2 x\right)}{\sin x\cos x}$

$\frac{\cos x}{\sin x\cos x}$

$\frac{1}{\sin x}$

b) $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$\left(\frac{\cos^2 x}{\cos^2 x}\right)\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{\sin^2 x}\right)$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x \sin^2 x}$$

$$\frac{1}{\cos^2 x \sin^2 x}$$

$$\sec^2 x \csc^2 x$$

c) $\dfrac{\cot x - 1}{1 - \tan x} = \dfrac{\csc x}{\sec x}$

$$= \frac{\frac{\cos x}{\sin x} - 1}{1 - \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\cos x}{\sin x} - 1\left(\frac{\sin x}{\sin x}\right)}{\left(\frac{\cos x}{\cos x}\right)1 - \frac{\sin x}{\cos x}}$$

$$= \frac{\left(\frac{\cos x - \sin x}{\sin x}\right)}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$= \frac{(\cos x - \sin x)}{\sin x} \cdot \frac{\cos x}{(\cos x - \sin x)}$$

$$= \frac{\cos x}{\sin x}$$

$$\frac{\frac{\cos x}{\sin x}}{\cot x}$$

(15) b) $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

$\dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)}$ | $\dfrac{\cos x}{\sin x}$

$\dfrac{2\sin x\cos x}{2\sin^2 x}$

$\dfrac{\cos x}{\sin x}$