6.2 Integration By U-Substitution

6.2 U-Substitution

Learning Targets:

1. SWBAT find the indefinite integral of a given function using the method of u-substitution.

2. SWBAT evaluate definite integrals using the method of u-substitution.

Say we were asked to find the antiderivative of the following:

$$\int \underbrace{(3x+5)}^3 dx$$

We could expand and integrate each term, but that would take a long time!

let $u = 3x + 5$

$$\frac{du}{dx} = 3$$

$$du = 3\,dx$$

$$\frac{1}{3}\,du = dx$$

$$\int u^3 \frac{1}{3}\, du$$

$$= \frac{1}{3}\int u^3\, du$$

$$= \frac{1}{3}\left[\frac{u^4}{4} + C\right]$$

$$= \frac{u^4}{12} + C$$

$$= \frac{(3x+5)^4}{12}\ |$$

Example 1: Find the following indefinite integrals.

a) $\int \underbrace{\sqrt{4x-1}}\,\underbrace{dx}$

let $u = 4x - 1$

$du = 4\,dx$

$\frac{1}{4}du = dx$

$$\frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\int u^{1/2}\,du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} + C\right]$$

$$\frac{1}{6}u^{3/2} + C$$

$$\frac{1}{6}(4x-1)^{3/2} + C$$

b) $\int \left(2x^3 - 1\right)^5 x^2 dx$

let $u = 2x^3 - 1$

$du = 6x^2 dx$

$\frac{1}{6} du = x^2 dx$

$\frac{1}{6} \int u^5 \, du$

$= \frac{1}{6}\left[\frac{u^6}{6} + C\right] = \frac{1}{36}\left(2x^3 - 1\right)^6 + C$

c) $\int \frac{4x^2}{(x^3-1)^3}\,dx$

let $u = x^3 - 1$

$du = 3x^2\,dx$

$\frac{4}{3}du = \frac{4}{\cancel{3}}\cancel{3}x^2\,dx$

$= \frac{4}{3}\int \frac{1}{u^3}\,du = \frac{4}{3}\int u^{-3}\,du$

$$= \frac{4}{3}\left[\frac{u^{-2}}{-2} + C\right]$$

$$= -\frac{2}{3}\left(x^3-1\right)^{-2} + C$$

Trigonometric Integrands

Example 2: Find the following indefinite integrals.

a) $\int \cos(7x+5)\underline{dx}$

Let: $u = 7x + 5$

$du = 7\,dx$

$\frac{1}{7}du = dx$

$\frac{1}{7}\int \cos u \, du = \frac{1}{7}\left[\sin u + C\right]$

$= \frac{1}{7}\sin(7x+5) + C$

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$$\text{b) } \int \frac{1}{\cos^2 2x} dx$$

$$\int \sec^2 2x \, dx$$

let $u = 2x$ $\quad du = 2\,dx$

$\frac{1}{2} du = dx$

$$\frac{1}{2} \int \sec^2 u \, du$$

$$\frac{1}{2} \left[\tan u + c \right]$$

$$\frac{1}{2} \tan 2x + c$$

c) $\int \sin^4 x \cos x dx$

$$\int (\sin x)^4 \cos x \, dx$$

let $u = \sin x$

$du = \cos x \, dx$

$$\int u^4 \, du$$

$$\frac{u^5}{5} + C = \frac{(\sin x)^5}{5} + C$$

2003 MC Question

8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

let $u = x^3$

$du = 3x^2\, dx$

$\frac{1}{3} du = x^2\, dx$

$\frac{1}{3}\int \cos u \, du$

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4\left(x^2-4\right)^2}+C$

(B) $\frac{1}{2\left(x^2-4\right)}+C$

(C) $\frac{1}{2}\ln\left|x^2-4\right|+C$

(D) $2\ln\left|x^2-4\right|+C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

let $u = x^2-4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + c$

U-Substitution To Evaluate Definite Integrals

Example 3: Evaluate the following definite integrals.

a) $\displaystyle\int_0^1 \underline{x}(x^2+1)^5 \,\underline{dx}$

let $u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int_1^2 u^5\,du$$

$$= \frac{1}{2}\left[\frac{u^6}{6}\Big|_1^2\right]$$

$$= \frac{1}{2}\left[\frac{2^6}{6} - \frac{1^6}{6}\right]$$

$$= \frac{1}{2}\left[\frac{63}{6}\right] = \frac{63}{12}$$

$$= 21/4$$

12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^{4} e^u\,du$ (C) $2\int_1^{2} e^u\,du$ (D) $\frac{1}{2}\int_1^{2} e^u\,du$ (E) $\int_1^{4} e^u\,du$

Let $u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int_1^2 e^u\,du$

Unique Example

$$\boxed{\int x(x+1)^{10}\,dx}$$

let $u = x+1 \longrightarrow u-1 = x$

$du = dx$

$$\int (u-1)(u^{10})\,du$$

$$\int (u^{11} - u^{10})\,du = \frac{u^{12}}{12} - \frac{u^{11}}{11} + C$$

$$= \frac{(x+1)^{12}}{12} - \frac{(x+1)^{11}}{11} + C$$

$$\int x\sqrt{x-2}\,dx$$

let $u = x-2 \longrightarrow u+2 = x$

$du = dx$

38. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

(A) $\frac{1}{x^3} + C$

(B) $\frac{8}{x^4} - \frac{2}{x^2} + C$

(C) $\ln(\ln x) + C$

(D) $\frac{\ln(x^2)}{2} + C$

(E) $\frac{(\ln x)^2}{2} + C$

$\int_1^x \frac{du}{u}$

$\ln|u| \Big|_1^x$

$\ln x - \ln 1$

$\ln x$

$\boxed{\int \frac{1}{x}(\ln x)\, d\cancel{x}}$

Let $u = \ln x$

$du = \frac{1}{x}\, dx$

$\int u\, du$

$\frac{u^2}{2} + C$

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