Unit #5 (Ch. 6)
Differential Equations and Mathematical Modelling

6.1 Antiderivatives

We evaluated **definite integrals** **in the last unit like the following:**

$$\text{b) } \int_1^4 (2x+3)\,dx = x^2 + 3x \Big|_1^4$$

$$\int (2x+3)\,dx = x^2 + 3x + C$$

Basic Integration Rules

$\frac{d}{dx}kx = k$	$\Rightarrow$	$\int k\,dx = kx + C$
$\frac{d}{dx}\left(2\sqrt{x}\right) = \frac{1}{\sqrt{x}}$	$\Rightarrow$	$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\Rightarrow$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
$\frac{d}{dx}\ln\lvert x\rvert = \frac{1}{x}$	$\Rightarrow$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\Rightarrow$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}\left(\frac{a^x}{\ln a}\right) = a^x$	$\Rightarrow$	$\int a^x\,dx = \frac{a^x}{\ln a} + C$
$\frac{d}{dx}\sin x = \cos x$	$\Rightarrow$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(-\cos x) = \sin x$	$\Rightarrow$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\Rightarrow$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(-\cot x) = \csc^2 x$	$\Rightarrow$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \cdot \tan x$	$\Rightarrow$	$\int \sec x \cdot \tan x\,dx = \sec x + C$
$\frac{d}{dx}(-\csc x) = \csc x \cdot \cot x$	$\Rightarrow$	$\int \csc x \cdot \cot x\,dx = -\csc x + C$
$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$	$\Rightarrow$	$\int \frac{1}{1+x^2}\,dx = \arctan x + C$
$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$	$\Rightarrow$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

In this section we will find indefinite integrals or antiderivatives.

Example 1: Find the most general antiderivative of the following functions:

a) $f(x) = \sqrt{x} + \sqrt[3]{x}$

$f(x) = x^{1/2} + x^{1/3}$

$\int (x^{1/2} + x^{1/3})\, dx$

$= \frac{2}{3} x^{3/2} + \frac{3}{4} x^{4/3} + C$

$$\int \frac{3}{\sqrt[3]{x^4}} \mathrm{dx}$$

$$\int 3x^{-4/3} dx$$

$$= \frac{3x^{-1/3}}{-1/3} + C$$

$$= -9x^{-1/3} + C$$

$$\frac{-9}{x^{1/3}} + C$$

$$\frac{-9}{\sqrt[3]{x}} + C$$

b) $f(t)=\sec^2 t+t^2$

$$\int \left(\sec^2 t + t^2\right) dt$$

$$= \tan t + \frac{t^3}{3} + C$$

Example 2: Find *f(x)*

$$f''(x) = x^2 + x^3$$

$$f' = \int (x^2 + x^3)\,dx$$

$$f' = \frac{x^3}{3} + \frac{x^4}{4} + C$$

$$f = \int \left(\frac{x^3}{3} + \frac{x^4}{4} + C\right) dx$$

$$f = \frac{x^4}{12} + \frac{x^5}{20} + Cx + D$$

* # Initial Value Problems

Example 1: Find *f(x)* if we are given the following:

a) $f'(x) = 4x + 3$ and $\underbrace{f(0) = -9}_{\text{initial condition}}$

$f(x) = \int (4x+3)\,dx$

$f(x) = 2x^2 + 3x + C$

$-9 = 2(0)^2 + 3(0) + C$

$-9 = C$

$f(x) = 2x^2 + 3x - 9$

b) $f''(x) = 20x^3 - 10$ and $f(1) = 1$ and $f'(1) = -5$

$$f' = \int (20x^3 - 10)\,dx$$

$$f' = 5x^4 - 10x + C$$

$$-5 = 5(1)^4 - 10(1) + C$$

$$-5 = 5 - 10 + C$$

$$0 = C$$

$$f' = 5x^4 - 10x$$

$$f = \int (5x^4 - 10x)\,dx$$

$$f = x^5 - 5x^2 + C$$

$$1 = (1)^5 - 5(1)^2 + C$$

$$5 = C$$

$$f(x) = x^5 - 5x^2 + 5$$

$$\int \frac{1}{3x+12}\,dx =$$

(A) $-3\ln|x+4|+C$

(B) $\frac{1}{3}\ln|x+4|+C$

(C) $\ln|x+4|+C$

(D) $3\ln|x+4|+C$

$$\int \frac{1}{3(x+4)}\,dx$$

$$\frac{1}{3}\int \frac{1}{x+4}\,dx$$

$$= \frac{1}{3}\ln|x+4| + C$$

$$\int \left(5e^{2x} + \frac{1}{x}\right) dx =$$

$$\frac{5e^{2x}}{2} + \ln|x| + C$$

(A) $\frac{5}{2}e^{2x} + \frac{2}{x^2} + C$

(B) $\frac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \frac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \frac{1}{x^2} + C$

$s(0) = 2$

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at time $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

$s(1)$

$s(t) = \int (3t^2 + 6t)\,dt$

$s(t) = t^3 + 3t^2 + C$

$2 = 0^3 + 3(0)^2 + C$

$2 = C$

$s(t) = t^3 + 3t^2 + 2$

$s(1) = (1)^3 + 3(1)^2 + 2 = 6$

$$\int_0^1 v(t) = s(1) - s(0)$$

$$\int_0^1 (3t^2+6t)\,dt = s(1) - 2$$

$$t^3+3t^2 \Big|_0^1 = s(1) - 2$$

$$1^3 + 3(1)^2 = s(1) - 2$$

$$6 = s(1)$$

A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 7$ for time $t \geq 0$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 0$?

$$s(t) = \int (3t^2 - 7)\, dt$$

$$s(t) = t^3 - 7t + c$$

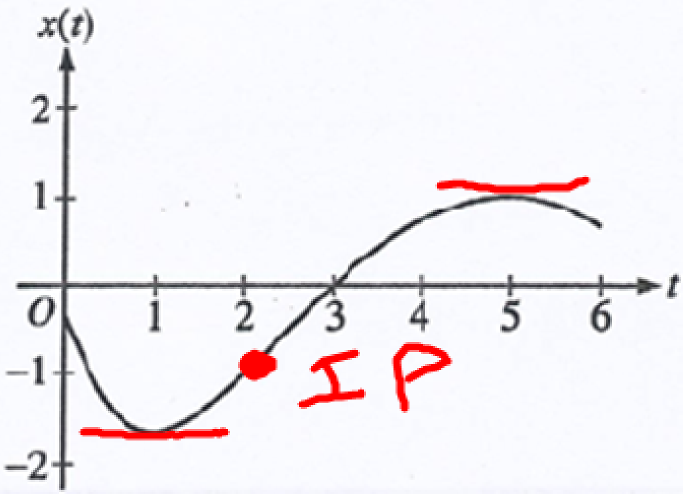
$$7 = (1)^3 - 7(1) + c$$

$$13 = c$$

$$s(t) = t^3 - 7t + 13$$

$$\underline{s(0) = 13}$$

s(t)



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

acceleration +

CU

Assignment
Handout
#'s Circled Problems