Chapter 6 Trigonometric Identities

6.1 Reciprocal, Quotient and Pythagorean Identities

An **equation** is a statement of equality that may be satisfied by none, one, two or more values of the variable

3

$$3x+7=16$$

3 3

$$x^2-x-6=0$$

(-2) (-2)

An **identity** is an equation that is satisfied by all values of the variable that are in the domain of each side of the equation.

$$2(x+1) = 2x + 2$$

A **trigonometric identity** is an identity that contains one or more trigonometric functions.

$(\cos\theta)^2$

$$(1-\sin\theta)^2+\cos^2\theta=2(1-\sin\theta)$$

$0 \qquad 0 \qquad 0$

$1 + 1 = 2$

Reciprocal Identities

1. $$\csc x = \frac{1}{\sin x}$$ $\sin x = \frac{1}{\csc x}$

2. $$\sec x = \frac{1}{\cos x}$$ $\cos x = \frac{1}{\sec x}$

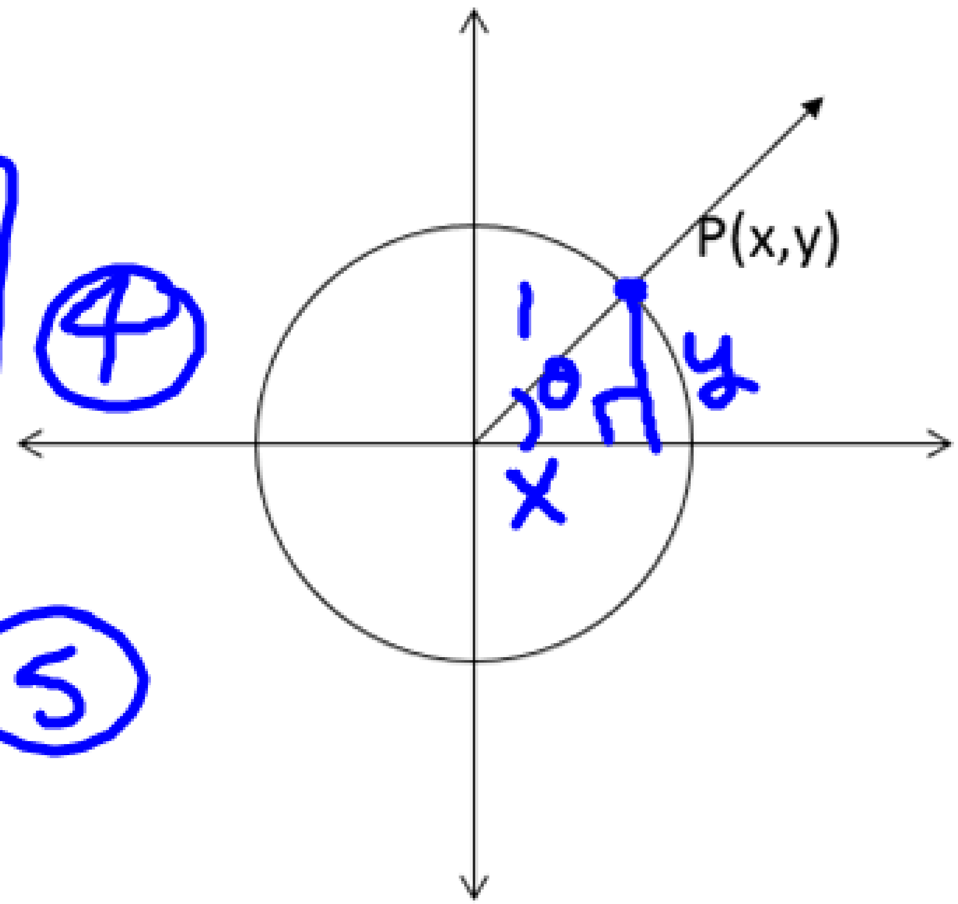
3. $$\cot x = \frac{1}{\tan x}$$ $\tan x = \frac{1}{\cot x}$

Let's develop the Quotient Identities

$$\tan\theta = \frac{y}{x}$$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$ 4

$$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}}$$ 5



Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

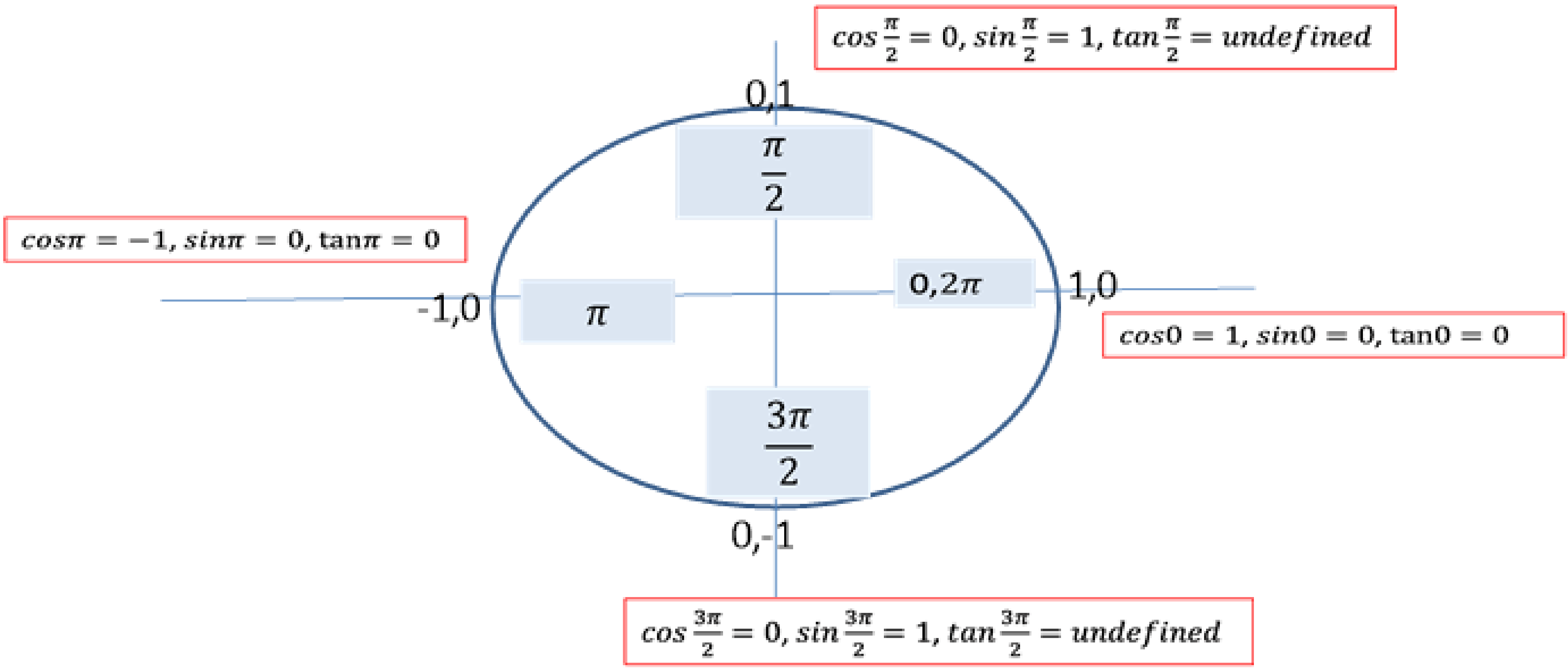
$$\frac{4}{(x-4)(x-3)}$$

$x \neq 4, 3$

Finding Non-Permissible Values

Non-permissible values are any values that make the expression or equation **undefined.**

To help us we are going to use the unit circle and quadrantal angles



Example 1: Find the non-permissible values for x in radians

a) $\frac{\sin x}{\cos x}$ b) $\frac{\sec x}{\sin x}$ c) $\frac{\tan x}{1-\cos x}$

a) $\cos x \neq 0$

$$\boxed{x \neq \frac{\pi}{2} \pm \pi n, n \in N}$$

b) $$\frac{\sec x}{\sin x} = \frac{\frac{1}{\boxed{\cos x}}}{\boxed{\sin x}}$$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} \pm \pi n, n \in N$

$\sin x \neq 0$

$x \neq \pi \pm \pi n, n \in N$

c) $\dfrac{\tan x}{1 - \cos x}$

$$\dfrac{\left(\dfrac{\sin x}{\boxed{\cos x}}\right)}{\boxed{1 - \cos x}}$$

$\to \cos x \neq 0$

$x = \frac{\pi}{2} \pm \pi n, n \in N$

$1 - \cos x \neq 0$

$1 \neq \cos x$

$x = 0 \pm 2\pi n, n \in N$

Example 2: Determine the non permissible values in degrees and verify that $x = 60°$ and $x = \cancel{\frac{\pi}{4}}$ are numerical solutions of the equation below

$$\sec x = \frac{\tan x}{\sin x}$$

$$\frac{1}{\boxed{\cos x}} = \frac{\left(\frac{\sin x}{\boxed{\cos x}}\right)}{\boxed{\sin x}}$$

$\cos x \neq 0$

$x \neq 90° \pm 180° n, n \in N$

$\sin x \neq 0$

$x \neq 0 \pm 180 n, n \in N$

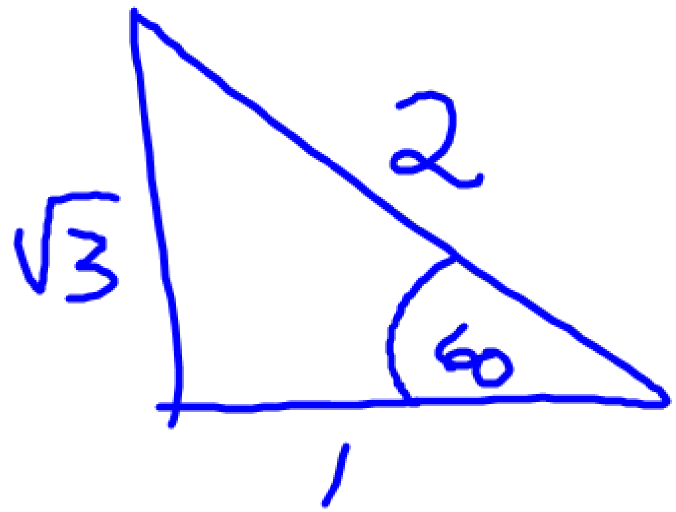
$$\sec x = \frac{\tan x}{\sin x}$$

$\sec 60^\circ$

$= \frac{2}{1}$

$= 2$

$$\frac{\tan 60^\circ}{\sin 60^\circ} = \frac{\left(\cancel{\frac{\sqrt{3}}{1}}\right)\frac{2}{\cancel{\sqrt{3}}}}{\cancel{\left(\frac{\sqrt{3}}{2}\right)}}$$

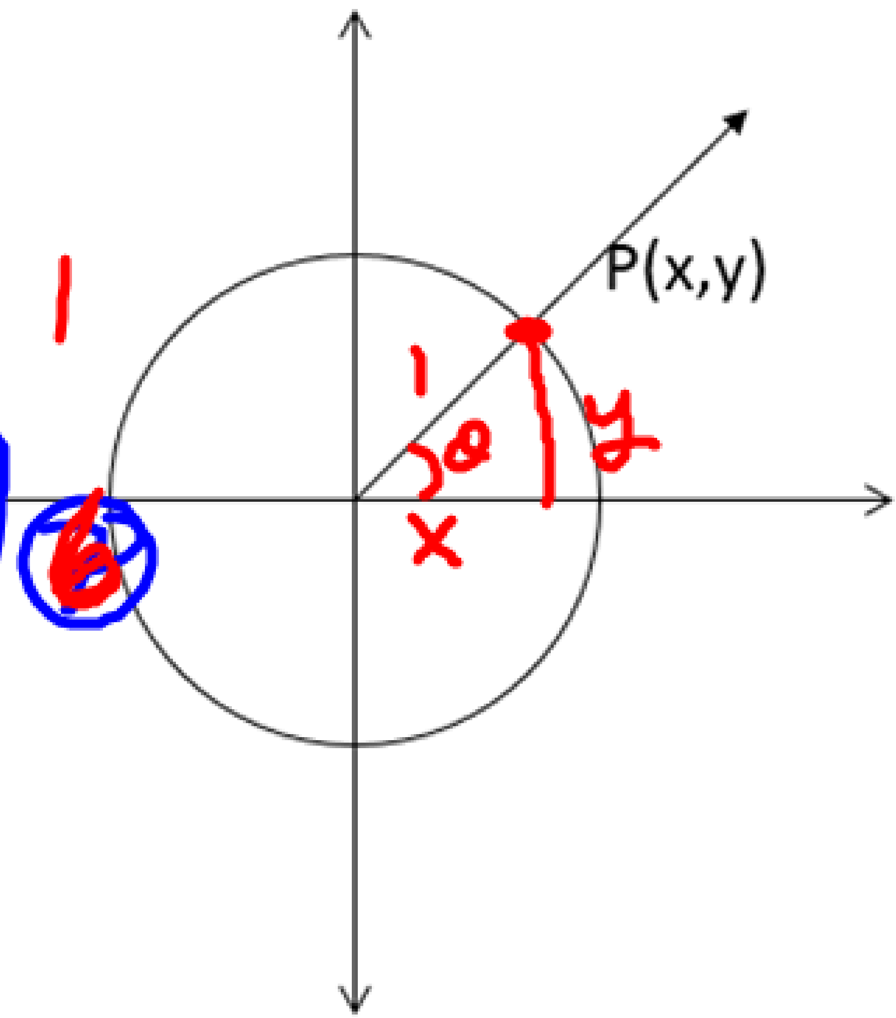


Let's develop the first Pythagorean Identity

$$x^2 + y^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$



$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta} \quad 8$$

Let's develop the remaining two **Pythagoran Identities**

The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

The Fundamental Eight Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

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1,5,6a,c

Example: Transform the expression on the left to the expression on the right using the eight fundamental identities.

$$a)\ \csc\theta \tan\theta \rightarrow \sec\theta$$

$$\left(\frac{1}{\cancel{\sin\theta}}\right)\left(\frac{\cancel{\sin\theta}}{\cos\theta}\right)$$

$$\frac{1}{\cos\theta}$$

$\blacksquare$

QED

$$\frac{1}{\cos\theta}$$

$b) \quad (1-\sin\theta)(1+\sin\theta) \rightarrow \cos^2\theta$

$1 + \cancel{\sin\theta} - \cancel{\sin\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta$

QED

$1 - \sin^2\theta$

$= (1-\sin\theta)(1+\sin\theta)$

c) $\dfrac{\cos^2\theta}{\sin\theta} + \underset{1}{\sin\theta}^{\sin\theta}_{\sin\theta} \rightarrow \csc\theta$

$$\frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$$

$$\frac{1}{\sin\theta}$$

QED

$$\frac{1}{\sin\theta}$$

$$\frac{1}{3} + \frac{1}{6}$$

Assignment

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10.

sinx
cosx
tanx

$$\frac{\csc x}{\tan x + \cot x}$$

$$\frac{\left(\frac{1}{\sin x}\right)}{\frac{\sin x}{\sin x}\left(\frac{\sin x}{\cos x}\right) + \left(\frac{\cos x}{\sin x}\right)\frac{\cos x}{\cos x}}$$

$$\frac{\left(\frac{1}{\sin x}\right)}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)}$$

$$\frac{\left(\dfrac{1}{\sin x}\right)}{\left(\dfrac{1}{\sin x \cos x}\right)}$$

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x} \cos x}{1}$$

$$\cos x$$

QED

$$\frac{\left(\dfrac{1}{\sin x}\right)}{\left(\dfrac{1}{\sin x \cos x}\right)}$$

$$\frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}\cos x}{1}$$

$$\cos x$$

QED

(12b)

$$\frac{\cot x}{\sec x} + \sin x = \csc x$$

$$\frac{\cos x}{1} \cdot \left(\frac{\cos x}{\sin x}\right) + \sin x \qquad \Bigg| \qquad \frac{1}{\sin x}$$

$$\frac{\cos^2 x}{\sin x} + \sin x \; \frac{\sin x}{\sin x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin x}$$

$$\frac{1}{\sin x}$$

QED

$$\csc^2\theta + \sin\theta = \frac{\cot^3\theta + \cot\theta + \cos\theta}{\cot\theta}$$

$$\frac{\cot^3\theta}{\cot\theta} + \frac{\cot\theta}{\cot\theta} + \frac{\cos\theta}{\cot\theta}$$

$$\underbrace{\cot^2\theta + 1} + \cancel{\cos\theta}\cdot\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)$$

$$\csc^2\theta + \sin\theta$$

QED