5.7 Chapter Review P 260 1-28

1. $x^2+4y^2=12$

$\frac{d}{dx}\left(x^2+4y^2\right)=\frac{d}{dx}(12)$

$2x+8y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-x}{4y}$

$\frac{d}{dx}\left(\frac{-x}{4y}\right)$

$=\frac{-1(4y)-(-x)\left(4\frac{dy}{dx}\right)}{(4y)^2}$

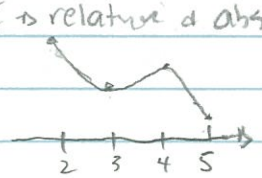
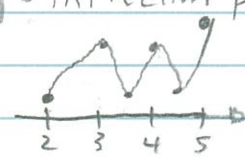
$=\frac{-4y+4x\left(\frac{-x}{4y}\right)}{16y^2}$

$=\frac{-4y^2-x^2}{y}\left(\frac{1}{16y^2}\right)$

$=\frac{-4y^2-x^2}{16y^3}=\frac{-1(x^2+4y^2)}{16y^3}=\frac{-1(12)}{16y^3}=\frac{-3}{4y^3}$

2. a) 0 b) 4 c) 4 and 3 (why?) d) 2

3. a → none b → rel. min c → inflection pt d → rel. max e → none
f → relative & absolute min g → inflection pt h → absolute max

4. 
5. 
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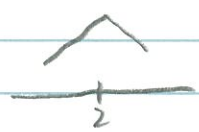
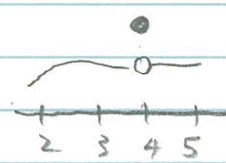
7. 
8. 

9. a) $(0,5)\cup(9,10)$ b) $(5,9)$ c) $(0,3)\cup(7,10)$
d) $(3,7)$ e) $(5,5)$ - rel. max $(9,1)$ rel. min
f) $(3,3)$ $(7,3)$

10. a) $(0,7)\cup(9,10)$ b) $(7,9)$ c) $(1,3)\cup(5,8)$
d) $(0,1)\cup(3,5)\cup(8,10)$ e) $x\in\{1,3,5,8\}$
f) rel max $x=7$ rel. min $x=9$ g) $x\in\{7,9\}$

11. y-int → $\frac{-100}{-25}=4$

5.7 - continued

12. $f(x) = \frac{x^4 - 9x^2}{x^3+1}$

$0 = \frac{x^4-9x^2}{x^3+1}$

$0 = x^2(x^2-9)$

$x = 0, +3, -3$

13. V.A → $x^4 + 8x = 0$

$x(x^3+8) = 0$

$x(x+2)(x^2-2x+4)$

$x = 0 \quad x = -2$

14. $f(x) = \frac{3x^2}{x^2+2x+1}$ $\lim_{x\to 0} \frac{\frac{3x^2}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2}} = \lim_{x\to 0} \frac{3}{1+\frac{2}{x}+\frac{1}{x^2}} = \frac{3}{1+0+0} = 3$

15. $f(x) = \frac{\sqrt{9x^2+3x+2}}{x-2}$ $\lim_{x\to 0} \frac{\sqrt{x^2(9+\frac{3x}{x^2}+\frac{2}{x^2})}}{x(1-\frac{2}{x})} = \lim_{x\to 0} \frac{|x|\sqrt{9+\frac{3}{x}+\frac{2}{x^2}}}{x(1-\frac{2}{x})} = 1(3)$ or $-1(3)$

3 or -3

16. $f(x) = 2x^3 - 3x^2 - 12x \quad [-2, 4]$

absolute max $(4, 32)$

absolute min $(-2, -20)$

17. $x^2 + xy = 6$

a) $\frac{d}{dx}(x^2 + xy) = \frac{d}{dx}(6)$

$2x + 1y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x-y}{x}$

b) at $(-3, 1)$ $\frac{dy}{dx} = \frac{-2(-3)-1}{-3} = \frac{5}{-3}$

decreasing because $\frac{dy}{dx}$ is negative

c) $\frac{d^2y}{dx^2} = \frac{\left(-2 - \frac{dy}{dx}\right)(x) - (-2x-y)(1)}{x^2}$

$= \frac{-2x - 2\cancel{x}\left(\frac{-2x-y}{\cancel{x}}\right) + 2x + y}{x^2}$

$= \frac{-2x + 2x + y + 2x + y}{x^2}$

$= \frac{2x+2y}{x^2}$

at $(-3, 1) = \frac{2(-3)+2(1)}{(-3)^2} = \frac{-4}{9}$

concave down because $\frac{d^2y}{dx^2}$ is ne

5.7 - Continued

18 a) $F'(3)=0$, $f''(3)=-4$ - relative maximum

b) $f'(3)=0$ $f''(3)=4$ - relative minimum

c) $F'(2.9)=1$ $f'(3)=0$ $f'(3.1)=-1$ - relative maximum

d) $F''(2.9)=-4$, $F'(3)=0$ $F''(3.1)=5$ - point of inflection

19. $f''(x) = \dfrac{(x-1)^3}{\sqrt{x^2+4}}$

$- - - \ 0 + + + \quad (x-1)^3$

$+ + + + + + \quad \sqrt{x^2+4}$

$- - - \ 1 + + + +$

Concave up $(1, \infty)$

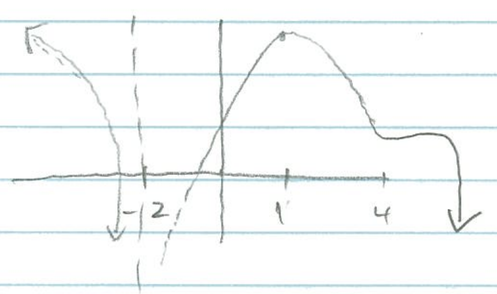
Concave down $(-\infty, 1)$

20. $f(x) = \dfrac{x^3-3x^2+5x-7}{x^2-x+1}$

$$
\begin{array}{r|l}
 & x-2 \leftarrow \text{oblique} \\
\hline
x^2-x+1 & x^3-3x^2+5x-7 \\
 & x^3-x^2+x \\
\hline
 & -2x^2-4x-7 \\
 & -2x^2+2x-2 \\
\hline
 & 0
\end{array}
$$

21. First derivative of a cube is a square ∴ only 2 roots

22.



23.

5.7 - Continued

23. $f(x) = \dfrac{2x^2}{x^2+12}$

$$f'(x) = \frac{(4x)(x^2+12) - (2x^2)(2x)}{(x^2+12)^2}$$

$$= \frac{4x^3 + 48x - 4x^3}{(x^2+12)^2}$$

$$= \frac{48x}{(x^2+12)^2}$$

$$F''(x) = \frac{48(x^2+12)^2 - 48x(2)(x^2+12)'(2x)}{(x^2+12)^4}$$

$$= \frac{48(x^2+12)[x^2+12-4x^2]}{(x^2+12)^4}$$

$$= \frac{48(-3x^2+12)}{(x^2+12)^3}$$

$$= \frac{-144(x^2-4)}{(x^2+12)^3}$$

$$= \frac{-144(x-2)(x+2)}{(x^2+12)^3}$$

$f'(x)$ a)

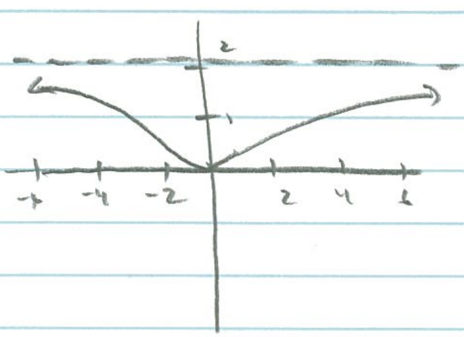
- - - - 0 + + + + + + $48x$
+ + + + + + + + $(x^2+12)^2$
- - - - 0 + + + + +

b) increasing $(0, \infty)$
decreasing $(-\infty, 0)$
c) relative mini $(0,0)$

$f''(x)$ d)

+ + + + + + + 0 - - $-144(x-2)$
- - - - 0 + + + + + + $(x+2)$
+ + + + + + + + + + + $(x^2+12)^3$
- - - - -2 + + + 2 - - -

e) Concave up $(-2, 2)$
Concave down $(-\infty, -2) \cup (2, \infty)$
f) inflection points
$(2, \frac{1}{2})$ $(-2, \frac{1}{2})$
g) x-int $\rightarrow$ $x = 0$
y-int $\rightarrow$ $y = 0$
h) H.A $\rightarrow$ $y = 2$



5.7 - Continued

24. $f(x) = 3x^5 - 10x^3$

$f'(x) = 15x^4 - 30x^2$

$= 15x^2(x^2 - 2)$

$= 15x^2(x - \sqrt{2})(x + \sqrt{2})$

$f''(x) = 60x^3 - 60x$

$= 60x(x^2 - 1)$

$= 60x(x+1)(x-1)$

$f'(x)$ a)

+ + + + 0 + + + + + $15x^2$

- - - - - - - 0 + + $(x - \sqrt{2})$

- - - 0 + + + + + + + $(x + \sqrt{2})$

+ + + $-\sqrt{2}$ - - - 0 - - - $\sqrt{2}$ + + +

b) increasing $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

decreasing $(-\sqrt{2}, \sqrt{2})$

c) relative min $(\sqrt{2}, -11.31)$

relative max $(-\sqrt{2}, 11.31)$

$f''(x)$ d)

- - - - - - 0 + + + + $60x$

- - - - 0 + + + + + + + $(x+1)$

- - - - - - - - 0 + + + $(x-1)$

- - - - | + + 0 - - | + + +

e) Concave up $(-1, 0) \cup (1, \infty)$

concave down $(-\infty, -1) \cup (0, 1)$

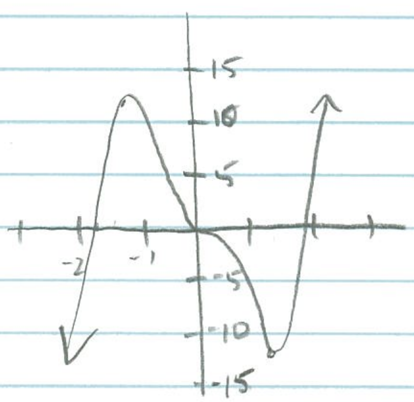
F) inflection points

$(-1, 7)$ $(0, 0)$ $(1, -7)$

g) x-int → $x = 0$, $x = \pm\frac{\sqrt{30}}{3}$

y-int → 0

h) no asymptotes



5.7 · Continued

25 $f(x) = \dfrac{3x}{(x+1)^2}$

$f'(x) = \dfrac{3(x+1)^2 - 3x(2)(x+1)}{(x+1)^4}$

$f'(x) = \dfrac{3(x+1)((x+1) - 2x)}{(x+1)^4}$

$f'(x) = \dfrac{3(-x+1)}{(x+1)^3}$

$f'(x) = \dfrac{-3(x-1)}{(x+1)^3}$

$f''(x) = \dfrac{(-3)(x+1)^3 - (-3(x-1))(3)(x+1)^2)}{(x+1)^6}$

$f''(x) = \dfrac{-3(x+1)^3 + 9(x-1)(x+1)^2}{(x+1)^6}$

$f''(x) = \dfrac{-3((x+1) - 3(x-1))}{(x+1)^4}$

$f''(x) = \dfrac{-3(-2x+4)}{(x+1)^4}$

$f''(x) = \dfrac{6(x-2)}{(x+1)^4}$

f'(x) a)
+ + + + + 0 - - - - $-3(x-1)$
- - - 0 + + + + + + + $(x+1)^3$
- - - -1 + + 1 - - -

b) increasing $(-1, 1)$
decreasing $(-\infty, -1) \cup (1, \infty)$

c) relative max $(1, \frac{3}{4})$

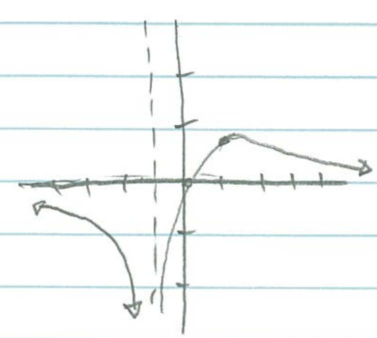
f''(x) d)
- - - - - 0 + + $6(x-2)$
+ + 0 + + + + + $(x+1)^4$
- - - -1 - - - 2 + + +

e) concave up $(2, \infty)$
concave down $(-\infty, -1) \cup (-1, 2)$

f) inflection point $(2, \frac{2}{3})$

g) x-int → $x = 0$
y-int → $y = 0$

h) V.A → $x = -1$
H A → $y = 0$



5.7 Continued

26 $f(x) = \dfrac{x^2-7x+10}{x-1}$

$f'(x) = \dfrac{(2x-7)(x-1)-(x^2-7x+10)(1)}{(x-1)^2}$

$f'(x) = \dfrac{2x^2-9x+7-x^2+7x-10}{(x-1)^2}$

$f'(x) = \dfrac{x^2-2x-3}{(x-1)^2}$

$f'(x) = \dfrac{(x-3)(x+1)}{(x-1)^2}$

$f''(x) = \dfrac{(2x-2)(x-1)^2-(x^2-2x-3)(2)(x-1)}{(x-1)^4}$

$f''(x) = \dfrac{2[(x-1)^2-x^2+2x+3]}{(x-1)^3}$

$f''(x) = \dfrac{2(x^2-2x+1-x^2+2x+3)}{(x-1)^3}$

$f''(x) = \dfrac{2(4)}{(x-1)^3}$

$f''(x) = \dfrac{8}{(x-1)^3}$

$f'(x)$ a)
- – – – – – – – 0 + + + $(x-3)$
- – – – 0 + + + + + $(x+1)$
- + + + + 0 + + + + $(x-1)^2$
- $-1 \quad 1 \quad 3$
- + + + | – – – | – – – | + + +

b) increasing $(-\infty,-1) \cup (3,\infty)$
decreasing $(-1,1) \cup (1,3)$

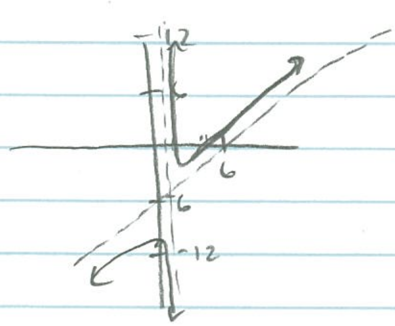
c) rel. max pt $(-1,-9)$
rel. min pt. $(3,-1)$

$f''(x)$ d)
- + + + + + + + 8
- – – – 0 + + + + $(x-1)^3$
- – – – 1 + + + +

e) concave up $(1,\infty)$
concave down $(-\infty,1)$

f) no inflection pt (und at $x=1$)

g) x-int → $x^2-7x+10=0$ y-int → $y=-10$
$(x-2)(x-5)=0$
$x=2 \quad x=5$



h) V.A → $x=1$

$x-6$ ← slant

$x-1 \overline{)\,x^2-7x+10}$
x^2-x
$-6x+10$

5.7 Continued

27 $f(x) = 3x^{2/3} - x^2$

$f'(x) = 2x^{-1/3} - 2x$

$= 2x^{-1/3}(1 - x^{4/3})$

$= \dfrac{2(1 - x^{4/3})}{x^{1/3}}$

$f''(x) = -\frac{2}{3}x^{-4/3} - 2$

$= -\frac{2}{3}x^{-4/3}(1 + 3x^{4/3})$

$= \dfrac{-2(3x^{4/3}+1)}{3x^{4/3}}$

f'(x) a) -- 0 + + + + 0 - - - $2(1-x^{4/3})$

- - - - - 0 + + + + + + + $x^{1/3}$

+ + + -1 - - - 0 + + + 1 - - - -

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b) increasing $(-\infty, -1) \cup (0, 1)$

decreasing $(-1, 0) \cup (1, \infty)$

c) relative maxima $(-1, 2)$ $(1, 2)$

relative min $(0, 0)$

f''(x) d) - - - - - - - $-2(3x^{4/3}+1)$

+ + + 0 + + + + + $3x^{4/3}$

0

- - - ∞ - - - - -

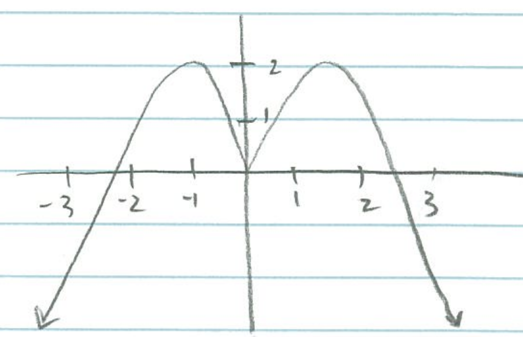
e) concave down $(-\infty, 0) \cup (0, \infty)$

f) no inflection pts

g) x-int → $x = 0$ $x = \pm\sqrt[4]{27}$

y-int → $y = 0$

h) no asymptotes



5.7 - Continued

28 $f(x) = \dfrac{x+2}{\sqrt{x^2+2}}$

$$f'(x) = \frac{(1)(x^2+2)^{1/2} - (x+2)(\frac{1}{2})(x^2+2)^{-1/2}(2x)}{x^2+2}$$

$$f'(x) = \frac{(x^2+2)^{-1/2}\left[x^2+2-x^2-2x\right]}{(x^2+2)^1}$$

$$f'(x) = \frac{2-2x}{(x^2+2)^{3/2}}$$

$$f'(x) = \frac{-2(x-1)}{(x^2+2)^{3/2}}$$

$$f''(x) = \frac{-2(x^2+2)^{3/2} - (-2(x-1))\frac{3}{2}(x^2+2)^{1/2}(2x)}{(x^2+2)^3}$$

$$f''(x) = \frac{-2(x^2+2)^{1/2}\left[x^2+2-(x-1)(3x)\right]}{(x^2+2)^3}$$

$$f''(x) = \frac{-2\left[x^2+2-3x^2+3x\right]}{(x^2+2)^{5/2}}$$

$$f''(x) = \frac{-2\left[-2x^2+3x+2\right]}{(x^2+2)^{5/2}}$$

$$f''(x) = \frac{2(2x+1)(x-2)}{(x^2+2)^{5/2}}$$

$2x^2-3x-2$
$2x^2-4x+1x-2$
$2x(x-2)+1(x-2)$
$(2x+1)(x-2)$

$f'(x)$ a)
+ + + + + 0 - - - - $-2(x-1)$
+ + + + + + + + + + $(x^2+2)^{3/2}$
1

b) increasing $(-\infty, 1)$
decreasing $(1, \infty)$

c) relative max point $(1, \sqrt{3})$

d)
- - - 0 + + + + + + $2(2x+1)$
- - - - - - 0 + + $(x-2)$
+ + + + + + + + + + $(x^2+2)^{5/2}$
+ + $-\frac{1}{2}$ - - 2 + + +

e) concave up $(-\infty, -\frac{1}{2}) \cup (2, \infty)$

f) concave down $(-\frac{1}{2}, 2)$

g) x-int → $x = -2$
y-int → $y = \sqrt{2}$

h) H.A → $y = 1$ if $x > 0$ $y = -1$ if $x < 0$
No V.A.