5.6 Curve Sketching   P. 259 1-10

1. $f(x) = 2x^2 + 5x - 12$

$f'(x) = 4x + 5$

$f''(x) = 4$

a) $f'(x) = 4x + 5$

$- - - 0 + + +$ $(4x+5)$

$- - - -5/4 + + +$

b) increasing $\left(\frac{-5}{4}, \infty\right)$

decreasing $\left(-\infty, \frac{-5}{4}\right)$

c) critical numbers $x = \frac{-5}{4}$

d) relative minimum $\left(\frac{-5}{4}, \frac{-121}{8}\right)$

e) $f''(x) = 4$

$+ + + + + +$ 4

$+ + + + + +$

f) concave up $(-\infty, \infty)$

g) no inflection points

h) x-intercepts → $0 = 2x^2 + 5x - 12$

$0 = 2x^2 + 8x - 3x - 12$

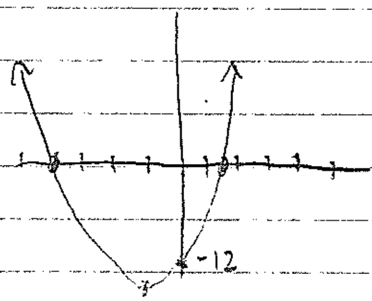
$0 = 2x(x+4) - 3(x+4)$

$0 = (x+4)(2x-3)$

$x = -4$ $x = \frac{3}{2}$

y-int → $y = -12$

i) No asymptotes



2. $f(x) = x^3 + 6x^2 - 15x - 90$

$f'(x) = 3x^2 + 12x - 15$

$f''(x) = 6x + 12$

a) $f'(x) = 3x^2 + 12x - 15$

$= 3(x^2 + 4x - 5)$

$= 3(x+5)(x-1)$

$- - 0 + + + +$ $(x+5)$

$- - - - - 0 + +$ $(x-1)$

$+ + -5 - - 1 + +$

b) increasing $(-\infty, -5) \cup (1, \infty)$

decreasing $(-5, 1)$

c) critical numbers $x = -5$, $x = 1$

d) relative max → $(-5, 10)$

relative min → $(1, -98)$

e) $f''(x) = 6x + 12$

$f''(x) = 6(x+2)$

$- - - 0 + + + +$ $(x+2)$

$- - -2 + + +$

f) concave down $(-\infty, -2)$

concave up $(-2, \infty)$

g) inflection point $(-2, -44)$

h) x-intercepts $0 = x^3 + 6x^2 - 15x - 90$

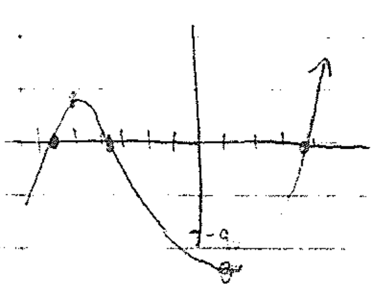
$0 = (x+6)(x^2 - 15)$

y-int → $y = -90$

$x = -6$ $x = \pm\sqrt{15}$

i) No asymptotes

-6	1	6	-15	-90
	↓	-6	0	90
	1	0	-15	0



5.6 - continued

3. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

a) $f'(x) = 4x^3 - 6x^2$

$f'(x) = 2x^2(2x - 3)$

+ + + 0 + + + + + $2x^2$

- - - - - 0 + + + $2x - 3$

- - - 0 - - $\frac{3}{2}$ + + +

b) increasing $(\frac{3}{2}, \infty)$

decreasing $(-\infty, \frac{3}{2})$

c) critical numbers $x = 0$ $x = \frac{3}{2}$

d) relative minimum $(\frac{3}{2}, \frac{-27}{16})$

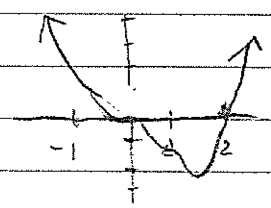
e) $f''(x) = 12x^2 - 12x$

$f''(x) = 12x(x-1)$

- - - - 0 + + + + + x

- - - - - - 0 + + + $x - 1$

+ + + 0 - - 1 + + +



F) concave up $(-\infty, 0) \cup (1, \infty)$

concave dn $(0, 1)$

g) inflection pts $(0, 0)$ and $(1, -1)$

h) x-int $\Rightarrow$ $0 = x^4 - 2x^3$

$0 = x^3(x-2)$ $x = 0$, $x = 2$

y-int $\Rightarrow$ $y = 0$

i) No asymptotes

4. $f(x) = 6x^2 - x^4$

$f'(x) = 12x - 4x^3$

$f''(x) = 12 - 12x^2$

a) $f'(x) = 12x - 4x^3$

$f'(x) = 4x(3 - x^2)$

- - - - 0 + + + + $4x$

+ + + + + + 0 - - $\sqrt{3} - x$

- - 0 + + + + + + $\sqrt{3} + x$

+ + + $-\sqrt{3}$ - - 0 + + $\sqrt{3}$ -

b) increasing $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

decreasing $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

c) critical numbers $x = 0$, $x = \pm\sqrt{3}$

d) relative max $(\sqrt{3}, 9)$ $(-\sqrt{3}, 9)$

relative min $(0, 0)$

e) $f''(x) = 12 - 12x^2$

$= 12(1 - x^2)$

$= 12(1 + x)(1 - x)$

- - 0 + + + + + $1 + x$

+ + + + + 0 - - - $1 - x$

- - - -1 + + 1 - - -

f) Concave up $(-1, 1)$

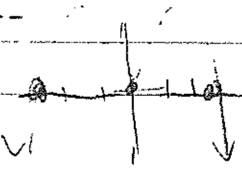
Concave down $(-\infty, -1) \cup (1, \infty)$

g) inflection pt. $(-1, 5)$ $(1, 5)$

h) x-int $\Rightarrow$ $0 = 6x^2 - x^4$

$0 = x^2(6 - x^2)$ $x = 0$, $x = \pm\sqrt{6}$

i) No asymptotes



5.6 - Continued

5. $f(x) = \frac{1}{5}x^5 - x^4$

$f'(x) = x^4 - 4x^3$

$f''(x) = 4x^3 - 12x^2$

a) $f'(x) = x^4 - 4x^3$

$f'(x) = x^3(x-4)$

$- - - \; 0 \; + + + + + \quad x^3$

$- - - - - \; 0 \; + + \quad x-4$

$+ + \; 0 \; - - - \; 4 \; + + +$

b) increasing $(-\infty, 0) \cup (4, \infty)$

decreasing $(0, 4)$

c) critical numbers $x=0$ and $x=4$

d) relative minimum $(4, -51.2)$

relatative maximum $(0, 0)$

e) $f''(x) = 4x^3 - 12x^2$

$f''(x) = 4x^2(x-3)$

$+ + + + + + + + \quad 4x^2$

$- - - \; 0 \; + + \quad x-3$

$- - - \; 0 \; - - \; 3 \; + + + +$

f) concave up $(3, \infty)$

concave down $(-\infty, 3)$

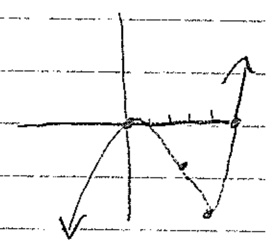
g) inflection points $(3, -32.4)$

h) x-int $\Rightarrow$ $0 = \frac{1}{5}x^5 - x^4$

$0 = x^4(\frac{1}{5}x - 1)$

$x=0 \quad x=5$

i) No asymptotes



6. $f(x) = x - 3x^{1/3}$

$f'(x) = 1 - 1x^{-2/3}$

$f''(x) = \frac{2}{3}x^{-5/3}$

a) $f'(x) = 1 - x^{-2/3}$

$f'(x) = x^{-2/3}(x^{2/3} - 1)$

$+ + + + + \; 0 \; + + + + \quad x^{-2/3}$

$- - \; 0 \; + + + + + \quad (x^{1/3}+1)$

$- - - - - - \; 0 \; + + \quad (x^{1/3}-1)$

$+ + + \; {-1} \; - - - \; 0 \; - - - \; 1 \; + + +$

DNE (undefined at 0)

b) increasing $(-\infty, -1) \cup (1, \infty)$

decreasing $(-1, 1)$

c) critical numbers $x = \pm 1 \quad x=0$

he says f'(x) Does not exist

d) relative maximum $(-1, 2)$

relative minimum $(1, -2)$

e) $f''(x) = \frac{2}{3}x^{-5/3}$

$+ + + + + + \quad 2$

$- - - - \; + + + \quad 3x^{5/3}$

$- - - \; 0 \; + + +$

f) concave up $(0, \infty)$

concave down $(-\infty, 0)$

g) inflection pt $(0, 0)$

h) x-int $\Rightarrow$ $0 = x - 3x^{1/3}$ $\quad 0 = x^{1/3}(x^{2/3} - 3)$

$x=0 \quad x = \pm 3\sqrt{3}$

y-int $\Rightarrow$ $y=0$

i) no asymptotes

$x^{2/3} = 3$

$x^{\frac{2}{3} \cdot \frac{3}{2}} = 3^{\frac{3}{2}}$

$\sqrt{3^3}$

$= 3\sqrt{3}$

56 - continued

7. $f(x) = x^{4/3} + 8x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{8}{3}x^{-2/3}$

$f''(x) = \frac{4}{9}x^{-2/3} - \frac{16}{9}x^{-5/3}$

a) $f'(x) = \frac{4}{3}x^{-2/3}(x+2) = \frac{4(x+2)}{3x^{2/3}}$

--- 0 + + + + + + $4(x+2)$

+ + + + + 0 + + + + $3x^{2/3}$

--- -2 + + + 0 + + + +

DNE

b) increasing $(-2, \infty)$ tangent line is vertical

decreasing $(-\infty, -2)$ except $x = 0$

c) critical numbers $x = -2$ $x = 0$

d) relative minimum $(-2, -7.56)$

e) $f''(x) = \frac{4}{9}x^{-2/3} - \frac{16}{9}x^{-5/3}$

$= \frac{4}{9}x^{-5/3}(x-4)$

$= \frac{4(x-4)}{9(x)^{5/3}}$

------- 0 + + + $4(x-4)$

---- 0 + + + + + $9(x^{5/3})$

+ + + 0 --- 4 + + +

∞

f) Concave up $(-\infty, 0) \cup (4, \infty)$

Concave down $(0, 4)$

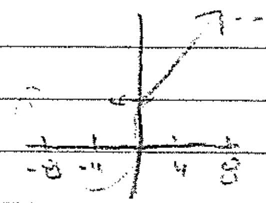
g) inflection points $(0,0)$ $(4, 19.05)$

h) x-int → $0 = x^{1/3}(x+8)$

$x = 0$ $x = -8$

y-int → $y = 0$

i) No asymptotes



8 $f(x) = \frac{4x}{x^2-4}$

$f'(x) = \frac{4(x^2-4) - 4x(2x)}{(x^2-4)^2} = \frac{-4x^2-16}{(x^2-4)^2} = \frac{-4(x^2+4)}{(x^2-4)^2}$

$f''(x) = \frac{-8x(x^2-4)^2 - (-4)(x^2+4)(2)(x^2-4)(2x)}{(x^2-4)^4} = \frac{-8x(x^2-4)^2 + 16x(x^2+4)(x^2-4)}{(x^2-4)^4}$

$\frac{-8x(x^2-4)[x^2-4-2(x^2+4)]}{(x^2-4)^3} = \frac{-8x(-x^2-12)}{(x^2-4)^3}$

a) $f'(x) = \frac{-4(x^2+4)}{(x^2-4)^2}$

- - - - - - - - - - - - - $-4(x^2+4)$

+ + + 0 + + + 0 + + + $(x^2-4)^2$

- - - - -2 - - - 2 - - -

b) decreasing $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

c) no critical numbers

d) no relative extrema

e) $f''(x) = \frac{8x(x^2+12)}{(x^2-4)^3}$

- - - 0 + + + + $8x$

+ + + + + + + x^2+12

+ + + 0 - - 0 + + + $(x^2-4)^3$

-2 0 2

- - - ∞ + + - - - ∞ + +

f) Concave up $(-2, 0) \cup (2, \infty)$

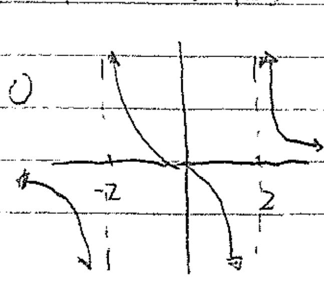
Concave down $(-\infty, -2) \cup (0, 2)$

g) inflection pt $(0, 0)$

h) x-int is 0 y-int = 0

i) VA → $x = \pm 2$

h.A → $y = 0$



5.6- continued

9. $f(x) = \dfrac{x^2-2x+1}{x^2+1}$

$f'(x) = \dfrac{(2x-2)(x^2+1) - (x^2-2x+1)(2x)}{(x^2+1)^2} = \dfrac{2x^3-2x^2+2x-2-2x^3+4x^2-2x}{(x^2+1)^2}$ $\quad 2x^2-2$

$f''(x) = \dfrac{(4x)(x^2+1)^2-(2x^2-2)(2)(x^2+1)(2x)}{(x^2+1)^4} = \dfrac{4x(x^2+1)[x^2+1-2x^2+2]}{(x^2+1)^4}$

a) $f'(x) = \dfrac{2(x^2-1)}{(x^2+1)^2}$

+ + + 0 - - - 0 + + + $\quad 2(x^2-1)$

+ + + + + + + + + $\quad (x^2+1)^2$

+ + -1 - - 1 + + +

b) increasing $(-\infty,-1) \cup (1,\infty)$

decreasing $(-1,1)$

c) critical numbers $x = \pm 1$

d) relative max $(-1,2)$ rel. min $(1,0)$

e) $f''(x) = \dfrac{-4x(x^2-3)}{(x^2+1)^3}$

+ + + + + 0 - - - - $-4x$

+ + + 0 - - 0 + + x^2-3

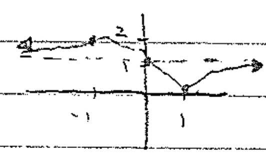
+ + + + + + + $(x^2+1)^3$

+ + $-\sqrt{3}$ - 0 + $\sqrt{3}$ - -

f) concave up $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$

concave down $(-\sqrt{3},0) \cup (\sqrt{3},\infty)$

g) inflection at $(-\sqrt{3}, 1.87)$ $(0,1)$ $(\sqrt{3}, 0.13)$



h) x-int $x=1$ y-int $y=1$

i) HA $y=1$

10. $f(x) = \dfrac{x^2+3}{x-1}$

$f'(x) = \dfrac{2x(x-1)-(x^2+3)(1)}{(x-1)^2} = \dfrac{2x^2-2x-x^2-3}{(x-1)^2} = \dfrac{x^2-2x-3}{(x-1)^2} = \dfrac{(x-3)(x+1)}{(x-1)^2}$

$f''(x) = \dfrac{(2x-2)(x-1)^2-(x^2-2x-3)(2)(x-1)(1)}{(x-1)^4} = \dfrac{2(x-1)(x^2-2x+1-x^2+2x+3)}{(x-1)^4} = \dfrac{8}{(x-1)^3}$

a) $f'(x) = \dfrac{(x-3)(x+1)}{(x-1)^2}$

- - - - - - - 0 + + $(x-3)$

- - - 0 + + + + + + $(x+1)$

+ + + + + 0 + + + + $(x-1)^2$

+ + + -1 - - 1 - - 3 + + +

b) increasing $(-\infty,-1) \cup (3,\infty)$

decreasing $(-1,1) \cup (1,3)$

c) critical numbers $x=-1$ and $x=3$

d) relative max $(-1,-2)$

relative min $(3,6)$

e) $f''(x) = \dfrac{8}{(x-1)^3}$

+ + + + + + $\quad 8$

- - - 0 + + + $(x-1)^3$

- - - 1 + + + +

f) concave up $(1,\infty)$ concave down $(-\infty,1)$

g) no inflection pts

h) no x-int $\quad$ y-int $\Rightarrow y=-3$

i) V.A.: $x=1$

$x+1$

$x-1 \,\overline{)\, x^2+0x+3}$

x^2-x