5.3 Increasing and Decreasing Intervals P. 235 1-20
The First Derivative Test

1. $f(x)=4$
$f'(x)=0$
never increasing or decreasing
no relative extrema

2. $f(x)=6-3x$
$f'(x)=-3$
never increasing, decreasing $(-\infty,\infty)$
no relative extrema

3. $f(x)=x^2+6x-8$
$f'(x)=2x+6$
$f'(x)=2(x+3)$
+ + + + + + + + 2
- - - - + + + + $(x+3)$
- - - - -3 + + +
decreasing → $(-\infty,-3)$
increasing → $(-3,\infty)$
relative min pt → $(-3,-17)$

4. $f(x)=8x-2x^2$
$f'(x)=8-4x$
$f'(x)=4(2-x)$
+ + + + + + + + 4
+ + + + - - - - - $(2-x)$
+ + + 2 - - -
decreasing → $(2,\infty)$
increasing → $(-\infty,2)$
relative max pt → $(2,8)$

5. $f(x)=x^3-27x$
$f'(x)=3x^2-27$
$f'(x)=3(x^2-9)$
$f'(x)=3(x+3)(x-3)$
- - - - 0 + + + + + + $(x+3)$
- - - - - - 0 + + + $(x-3)$
+ + + -3 - - - 3 + + +
decreasing $(-3,3)$
increasing $(-\infty,-3)\cup(3,\infty)$
relative max pt. $(-3,54)$
relative min pt $(3,-54)$

6. $f(x)=x^3-3x^2$
$f'(x)=3x^2-6x$
$f'(x)=3x(x-2)$
- - - 0 + + + + + x
- - - - - - 0 + + + $x-2$
0 2
+ + + - - - + + +
decreasing $(0,2)$
increasing $(-\infty,0)\cup(2,\infty)$
relative max pt $(0,0)$
relative min pt $(2,-4)$

7. $f(x)=-x^3-3x^2+24x+20$
$f'(x)=-3x^2-6x+24$
$f'(x)=3(-x^2-2x+8)$
$f'(x)\quad 3(8-2x-x^2)$
$f'(x)=3(4+x)(2-x)$
- - - - 0 + + + + + $4+x$
+ + + + + + 0 - - $2-x$
- - - -4 + + + 2 - - -

decreasing $(-\infty,-4)\cup(2,\infty)$
increasing $(-4,2)$
relative min pt → $(-4,-60)$
relative max pt → $(2,48)$

5.3 - Continued

8. $f(x) = x^4 - 8x^2$

$f'(x) = 4x^3 - 16x$

$f'(x) = 4x(x^2 - 4)$

$f'(x) = 4x(x-2)(x+2)$

- - - - - 0 + + + + + $4x$
- - - - - - - 0 + + + $x-2$
- - 0 + + + + + + + $x+2$

- - - -2 + + + 0 - - - 2 + + +

decreasing $(-\infty, -2) \cup (0, 2)$
increasing $(-2, 0) \cup (2, \infty)$
relative max at $(0, 0)$
relative min at $(-2, -16)$ + $(2, -16)$

9. $f(x) = 3x^4 + 4x^3 - 36x^2 + 11$

$f'(x) = 12x^3 + 12x^2 - 72x$

$f'(x) = 12x(x^2 + x - 6)$

$f'(x) = 12x(x+3)(x-2)$

- - - - - 0 + + + + + + $12x$
- - - 0 + + + + + + + + $x+3$
- - - - - - - - - 0 + + $x-2$

- - - -3 + + 0 - - 2 + + +

decreasing $(-\infty, -3) \cup (0, 2)$
increasing $(-3, 0) \cup (2, \infty)$
relative max at $(0, 11)$
relative min at $(-3, -178)(2, -53)$

10. $f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$f'(x) = 15x^2(x^2 - 1)$

$f'(x) = 15x^2(x+1)(x-1)$

+ + + + + + 0 + + + + + $15x^2$
- - 0 + + + + + + $x+1$
- - - - - - - - - 0 + + $x-1$

+ + + -1 - - - 0 - - - 1 + + +

decreasing $(-1, 1)$
increasing $(-\infty, -1) \cup (1, \infty)$
relative max $(-1, 2)$
relative min $(1, -2)$

11. $f(x) = 2\sqrt{x}$

$f'(x) = 1x^{-1/2}$

$f'(x) = \dfrac{1}{\sqrt{x}}$

+ + + + $\sqrt{x}$

0 + + + +

never decreasing
increasing $(0, \infty)$
no relative extrema

5.3 - Continued

12. $f(x) = 5\sqrt[5]{x}$

$f'(x) = x^{-4/5}$

$+ + + + + + + + + +$ $x^{-4/5}$

$+ + + 0 + + + +$

never decreasing

increasing $(-\infty, \infty)$

no relative extrema

13. $f(x) = x - 12x^{1/3}$

$f'(x) = 1 - 4x^{-2/3}$

$0 = 1 - 4x^{-2/3}$

$1 = 4x^{-2/3}$

$\left(\frac{1}{4}\right)^{-3/2} = \left(x^{-2/3}\right)^{-3/2}$

$\pm 8 = x$

$+ + \quad - - - - \quad + + + + +$ $(1 - 4x^{-2/3})$

$+ + + \; -8 \; - - - \; 8 \; + + +$

decreasing $(-8, 8)$

increasing $(-\infty, -8) \cup (8, \infty)$

relative max $(-8, 16)$

relative min $(8, -16)$

14. $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^{2/3}$

$f'(x) = x^2 + x^{-1/3}$

$f'(x) = x^{-1/3}(x^{7/3} + 1)$

$- - - - - \; 0 \; + + +$ $x^{-1/3}$

$- - - \; 0 \; + + + + + + +$ $x^{7/3} + 1$

$+ + \; -1 \; - - - - \; 0 \; + + +$

$x^{7/3} = -1$

$x^{7/3 \cdot 3/7} = (-1)^{3/7}$

$x = -1$

decreasing $(-1, 0)$

increasing $(-\infty, -1) \cup (0, \infty)$

relative max $(-1, \frac{7}{6})$

relative min $(0, 0)$

15. $f(x) = \dfrac{2x}{x+2}$

$f'(x) = \dfrac{2(x+2) - (2x)(1)}{(x+2)^2}$

$f'(x) = \dfrac{2x + 4 - 2x}{(x+2)^2}$

$f'(x) = \dfrac{4}{(x+2)^2}$

$+ + + + \quad + + + +$ $4(x+2)^2$

$+ + + \; -2 \; + + +$

undefined

never decreasing

increasing

$x \in (-\infty, -2) \cup (-2, \infty)$

no relative extrema

5.3 - continued

16. $f(x) = \dfrac{x^2}{x+1}$

$f'(x) = \dfrac{2x(x+1) - x^2(1)}{(x+1)^2}$

$f'(x) = \dfrac{2x^2 + 2x - x^2}{(x+1)^2}$

$f'(x) = \dfrac{x^2 + 2x}{(x+1)^2}$

$f'(x) = \dfrac{x(x+2)}{(x+1)^2}$

$- - - - - 0 + + +$ x

$- - - 0 + + + + + +$ $x+2$

$+ + + + 0 + + + +$ $(x+1)^2$

$+ + -2 - - -1 - - 0 + + +$

decreasing:
$x \in (-2,-1) \cup (-1,0)$

increasing:
$x \in (-\infty,-2) \cup (0,\infty)$

relative max: $(-2,-4)$

relative min $(0,0)$

17. $f(x) = \sqrt{x}\,(6-x)$

$f'(x) = \frac{1}{2}x^{-1/2}(6-x) + x^{1/2}(-1)$

$f'(x) = 3x^{-1/2} - \frac{1}{2}x^{1/2} - x^{1/2}$

$f'(x) = 3x^{-1/2} - \frac{3}{2}x^{1/2}$

$f'(x) = \frac{3}{2}x^{-1/2}(2-x)$

$+ + + + 2 - - -$ $(2-x)$

und $0 + + + + + +$ $(\frac{3}{2}x^{-1/2})$ $\dfrac{3}{2\sqrt{x}}$

und 0 $+ + +$ 2 $- - -$

Increasing for $x \in (0,2)$

decreasing for $x \in (2,\infty)$

relative maximum at $(2, 4\sqrt{2})$

18. $f(x) = \dfrac{x}{\sqrt{x-3}}$

$f'(x) = \dfrac{(1)(x-3)^{1/2} - 1x(\frac{1}{2})(x-3)^{-1/2}}{(x-3)^{1/2(2)}}$

$f'(x) = \dfrac{\frac{1}{2}(x-3)^{-1/2}\left[2(x-3) - x\right]}{x-3}$

$f'(x) = \frac{1}{2}(x-3)^{-3/2}(x-6)$

$\Rightarrow$ $- - - - 0 + + +$ $(x-6)$

und $0 + + + + + +$ $(x-3)^{-3/2}$

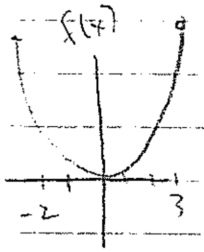
und 3 $- - -$ 6 $+ + +$

decreasing $(3,6)$

increasing $(6,\infty)$

relative min
$(6, 2\sqrt{3})$

5.3 - continued.



19. a) $f(x)$ is increasing for $x \in (0,3)$ because $f'(x) > 0$ (+ve)

b) $f(x)$ is decreasing for $x \in (-2,0)$ since $f'(x) < 0$ (-ve)

c) relative minimum at $x = 0$ because $f'(x)$ switches from negative to 0 to positive

there is no relative maximum

d) Since $f(x)$ decreases on the interval $(-2,0)$ because $f'(x) < 0$, $f(x)$ must reach max value at $x = -2$

20. a) v b) iii c) i d) ii e) iv.