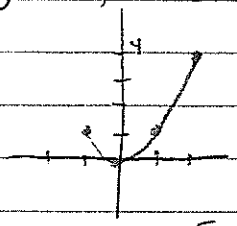
5.2 Relative and Absolute Extrema P 226 3, 4a-h, 5ab

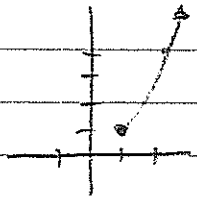
3. a) $y = x^2$; $x \in [-1, 2]$



relative and absolute min: $f(0) = 0$ $(0,0)$

absolute max: $f(2) = 4$ $(2,4)$

b) $y = x^2$; $x \in [1, \infty)$

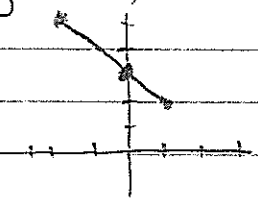


absolute min $f(1) = 1$ $(1,1)$

no absolute max

no relative extrema

c) $y = 3 - x$; $x \in [-2, 1]$

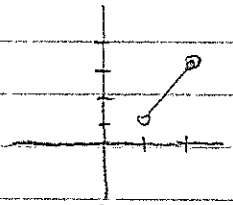


no relative extrema

absolute max: $f(-2) = 5$ $(-2, 5)$

absolute min: $f(1) = 2$ $(1, 2)$

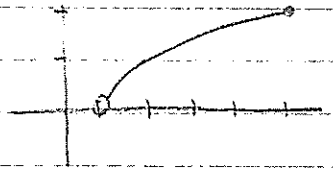
d) $y = 2x - 1$; $x \in (1, 2)$



no relative extrema

no absolute extrema

e) $y = \sqrt{x-1}$; $x \in (1, 5]$

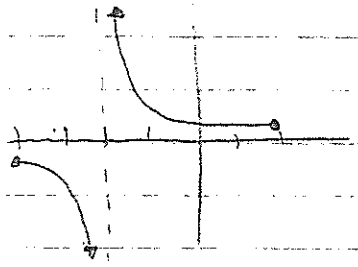


no relative extrema

no absolute min

absolute max $f(5) = 2$ $(5, 2)$

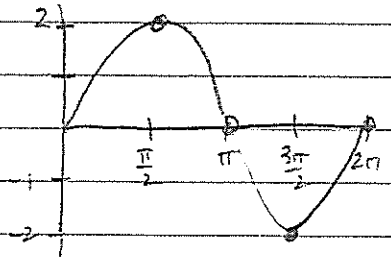
f) $y = \dfrac{1}{x+2}$; $x \in [-4, 2]$



no absolute or relative extrema

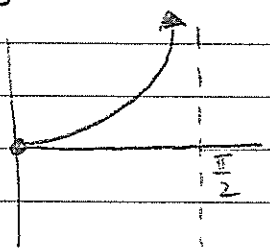
5.2 - continued

g) $y = 2\sin x$; $x \in [0, 2\pi]$



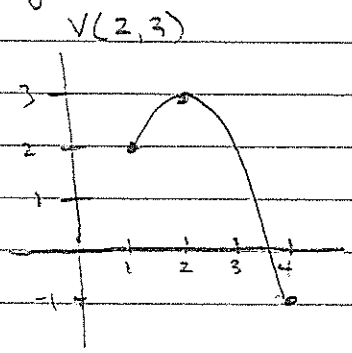
absolute and relative max
$f(\frac{\pi}{2}) = 2$ $(\frac{\pi}{2}, 2)$
absolute and relative min
$f(\frac{3\pi}{2}) = -2$ $(\frac{3\pi}{2}, -2)$

h) $y = \tan x$; $x \in [0, \frac{\pi}{2})$



no absolute or relative max
no relative min
absolute min $f(0) = 0$ $(0,0)$

i) $y = -(x-2)^2 + 3$; $x \in [1, 4]$



absolute and relative max $f(2) = 3$ $(2,3)$
no relative min
absolute min $f(4) = -1$ $(4,-1)$

4. a) $f(x) = 2x + 5$
$f'(x) = 2$ ← no critical #

b) $f(x) = 4x^2 - 8x + 11$
$f'(x) = 8x - 8$
$f'(x) = 8(x-1)$
$0 = 8(x-1)$
$x = 1$

c) $y = 2x^3 - 24x + 21$
$y' = 6x^2 - 24$
$0 = 6x^2 - 24$
$0 = 6(x^2 - 4)$
$0 = (x+2)(x-2)$
$x = -2$ $x = 2$

d) $y = x^4 - \frac{4}{3}x^3$
$y' = 4x^3 - 4x^2$
$0 = 4x^2(x-1)$
$x = 0$ $x = 1$

5.2 - continued

4 e) $f(x) = 6x^{2/3}$

$f'(x) = 4x^{-1/3}$

$0 = 4x^{-1/3}$

$0 = x$

f) $f(x) = \frac{x}{x-3}$

$f'(x) = \frac{(1)(x-3) - (x)(1)}{(x-3)^2}$

$f'(x) = \frac{x-3-x}{(x-3)^2}$

$f'(x) = \frac{-3}{(x-3)^2}$

$\frac{0}{1} = \frac{-3}{(x-3)^2}$

$0 = -3$ ∴ NONE

Note: not $x=3$ because 3 not in domain of function

g) $y = \frac{x^2}{x-2}$ ← $x \neq 2$

$y' = \frac{2x(x-2) - x^2(1)}{(x-2)^2}$

$y' = \frac{2x^2 - 4x - x^2}{(x-2)^2}$

$y' = \frac{x^2 - 4x}{(x-2)^2}$

$0 = x(x-4)$

$x = 0 \quad x = 4$

Note: not $x=2$ because 2 is not in domain of function (undefined at $x=2$)

h) $y = x - 3x^{1/3}$

$y' = 1 - x^{-2/3}$

$0 = 1 - x^{-2/3}$

$0 = 1 - \frac{1}{x^{2/3}}$

$0 = \frac{x^{2/3} - 1}{x^{2/3}}$

$0 = \frac{(x^{1/3} - 1)(x^{1/3} + 1)}{x^{2/3}}$

$x = 1, \quad x = -1, \quad x = 0$

5 a) $f(x) = 3x - 12, \quad [-1, 3] \quad f'(x) = 3$

$f(-1) = 3(-1) - 12$ $\quad f(3) = 3(3) - 12$

$f(-1) = -15$ $\quad f(3) = -3$

abs min → $(-1, -15)$ $\quad$ abs max → $(3, -3)$

b) $f(x) = 6x - x^2 \quad [1, 4] \quad f'(x) = 6 - 2x = 2(3 - x)$ ↓ $x = 3$

$f(1) = 6(1) - (1)^2$ $\quad f(3) = 6(3) - (3)^2$ $\quad f(4) = 6(4) - (4)^2$

$f(1) = 5$ $\quad f(3) = 9$ $\quad = 8$ $(4, 8)$

abs min → $(1, 5)$ $\quad$ abs max $(3, 9)$ $\quad (3, 9)$

rel: $(1, 5)$