Unit 5 - Graphical Applications of the Derivative P. 220 1-11, 16, 17, 18

5.1 Higher Order Derivatives P. 220 1-17

1. $f(x) = 5x^{10}$
$f'(x) = 50x^9$
$f''(x) = 450x^8$

2. $F(x) = 8\sqrt{x}$
$F'(x) = 4x^{-1/2}$
$F''(x) = -2x^{-3/2}$

3. $y = \frac{12}{x}$
$y' = -12x^{-2}$
$y'' = 24x^{-3}$

4. $y = \frac{18}{\sqrt[3]{x}}$
$y = 18x^{-1/3}$
$y' = -6x^{-4/3}$
$y'' = 8x^{-7/3}$

5. $f(x) = \pi^3$
$f'(x) = 0$
$f''(x) = 0$

6. $f(x) = 2x^4 - 5x^3$
$f'(x) = 8x^3 - 15x^2$
$f''(x) = 24x^2 - 30x$

7. $y = -4x^3 - 5x^2 + 11x - 9$
$y' = -12x^2 - 10x + 11$
$y'' = -24x - 10$

8. $y = \frac{2}{x} + \frac{4}{x^2} - \frac{5}{x^3}$
$y' = -2x^{-2} - 8x^{-3} + 9x^{-4}$
$y'' = 4x^{-3} + 24x^{-4} - 36x^{-5}$

9. $f(x) = (3x+5)^6$
$f'(x) = 6(3x+5)^5 \cdot (3)$
$F'(x) = 18(3x+5)^5$
$f'(x) = 90(3x+5)^4 \cdot 3$
$f''(x) = 270(3x+5)^4$

10. $F(x) = \sqrt{3x-1}$
$F'(x) = \frac{1}{2}(3x-1)^{-1/2} \cdot 3$
$F'(x) = \frac{3}{2}(3x-1)^{-1/2}$
$F''(x) = -\frac{3}{4}(3x-1)^{-3/2} \cdot 3$
$F''(x) = -\frac{9}{4}(3x-1)^{-3/2}$

11. $y = x^3(x^2+1)^5$
$y' = 3x^2(x^2+1)^5 + x^3(6)(x^2+1)^5(2x)$
$y' = 3x^2(x^2+1)^5 + 12x^4(x^2+1)^5$
$y' = 3x^2(x^2+1)^5\left[(x^2+1) + 4x^2\right]$
$y' = 3x^2(x^2+1)^5(5x^2+1)$
$y' = (15x^4+3x^2)(x^2+1)^5$
$y'' = (60x^3+6x)(x^2+1)^5 + (15x^4+3x^2)5(x^2+1)^4(2x)$

$y'' = 6x(x^2+1)^4\left[(10x^2+1)(x^2+1) + 5x^2(5)\right]$
$y'' = 6x(x^2+1)^4(10x^4+11x^2+1+25x^4+5x^2)$
$y'' = 6x(x^2+1)^4(35x^4+16x^2+1)$

5.1 - continued.

16. $x^2 + y - 3 = 0$

$2x + \frac{dy}{dx} - 0 = 0$

$\frac{dy}{dx} = -2x$

$\frac{d^2y}{dx^2} = \boxed{-2}$

17. $x^2 - 2y^2 = 4$

$2x - 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{-4y}$

$\frac{dy}{dx} = \frac{x}{2y}$ ✓ replace with $\frac{x}{2y}$

$\frac{d^2y}{dx^2} = \frac{(1)(2y) - (x)(2\frac{dy}{dx})}{(2y)^2}$

$= \frac{2y - x\left(2\left(\frac{x}{2y}\right)\right)}{4y^2}$

$= \frac{2y - \frac{x^2}{y}}{4y^2}$

$= \left(\frac{2y^2 - x^2}{y}\right)\left(\frac{1}{4y^2}\right)$

stop here → $= \frac{2y^2 - x^2}{4y^3}$ ← sub in original eq → $x^2 - 2y^2 = 4$, $-4 = 2y^2 - x^2$

$= \frac{-4}{4y^3}$

$= \boxed{\frac{-1}{y^3}}$

18. $xy = 12$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-y}{x}$

$\frac{dy^2}{dx^2} = \frac{(-1)\frac{dy}{dx}(x) - (-y)(1)}{x^2}$

$= \frac{(-1)\left(\frac{-y}{x}\right)(x) + y}{x^2}$

$= \frac{y + y}{x^2}$

$= \boxed{\frac{2y}{x^2}}$