5.6 Properties of Linear Relations

Lesson Focus

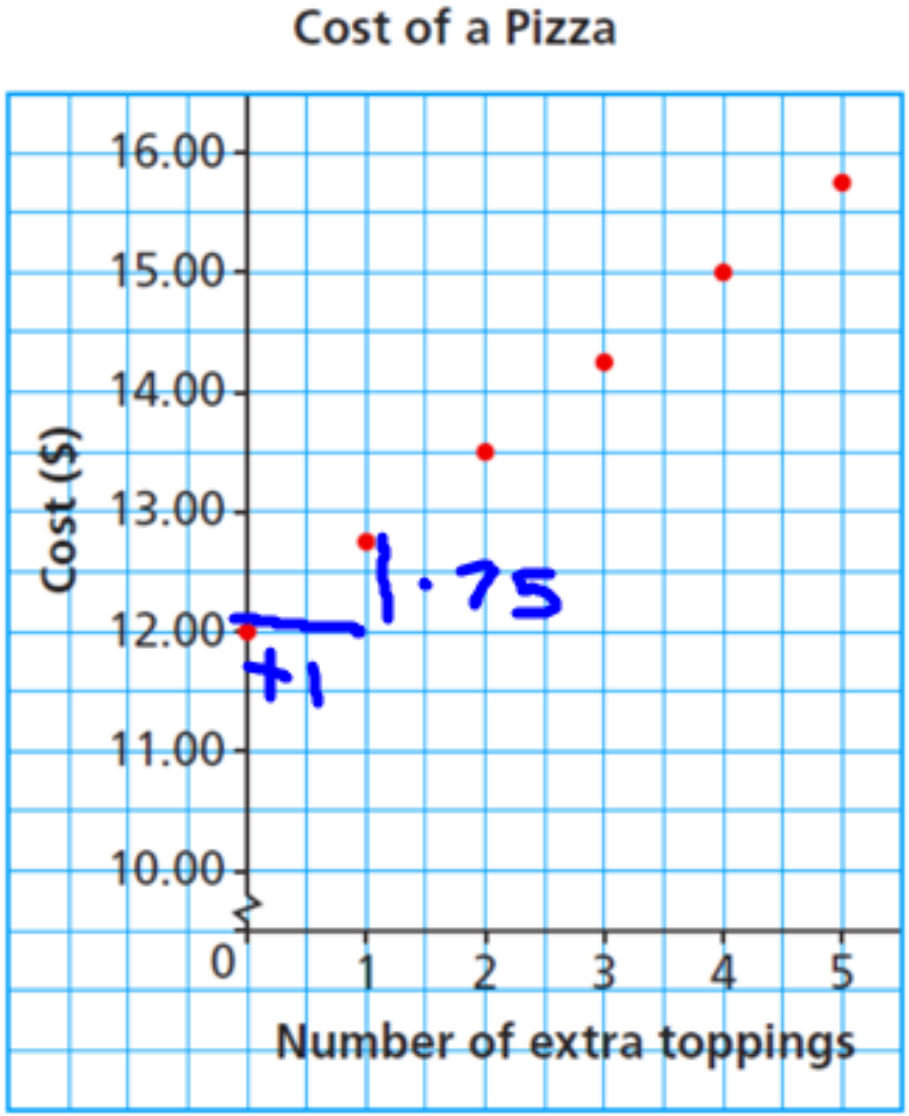
Identify and represent linear relations in different ways

Explore

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

+1

+.75



Linear Relation

For a linear relation:

a **constant change** in the ***independent variable***
results in a **constant change** in the ***dependent variable***

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

Example

Which table of values represents a linear relation? Justify the answer.

a) The relation between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F

C	F
0	32
5	41
10	50
15	59

b) The relation between the current, I amps, and power, P watts, in an electrical circuit

I	P
0	0
5	75
10	300
15	675

Example – Your Turn

Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

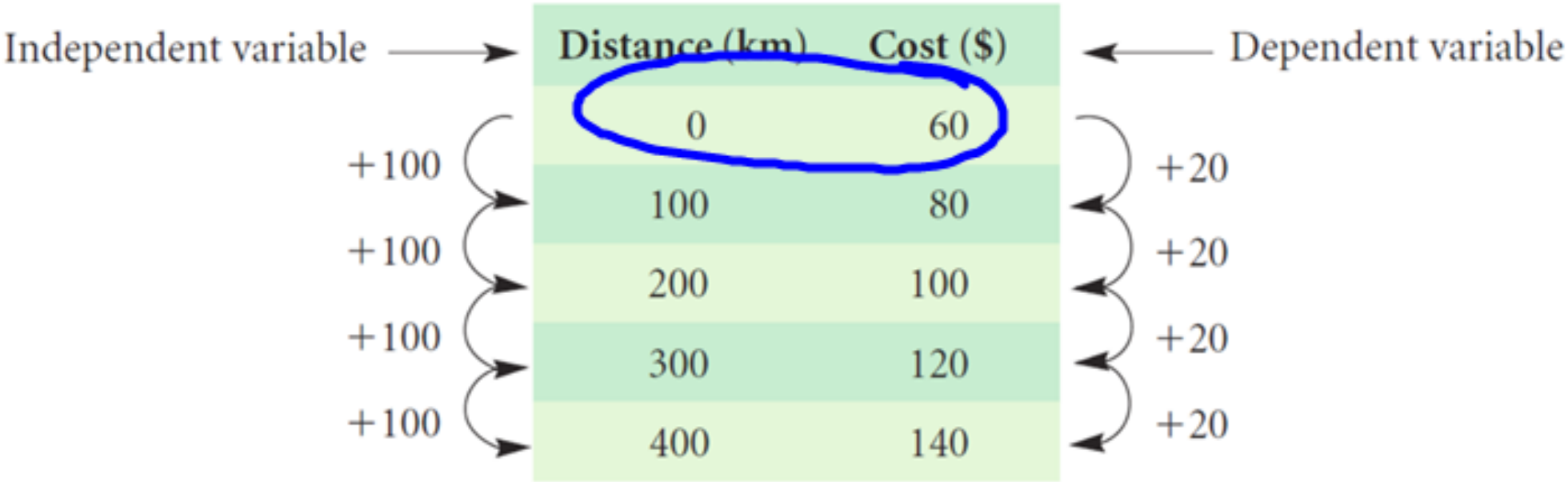
A	T
60	3
120	6
180	9
240	12
300	15

Linear Relation

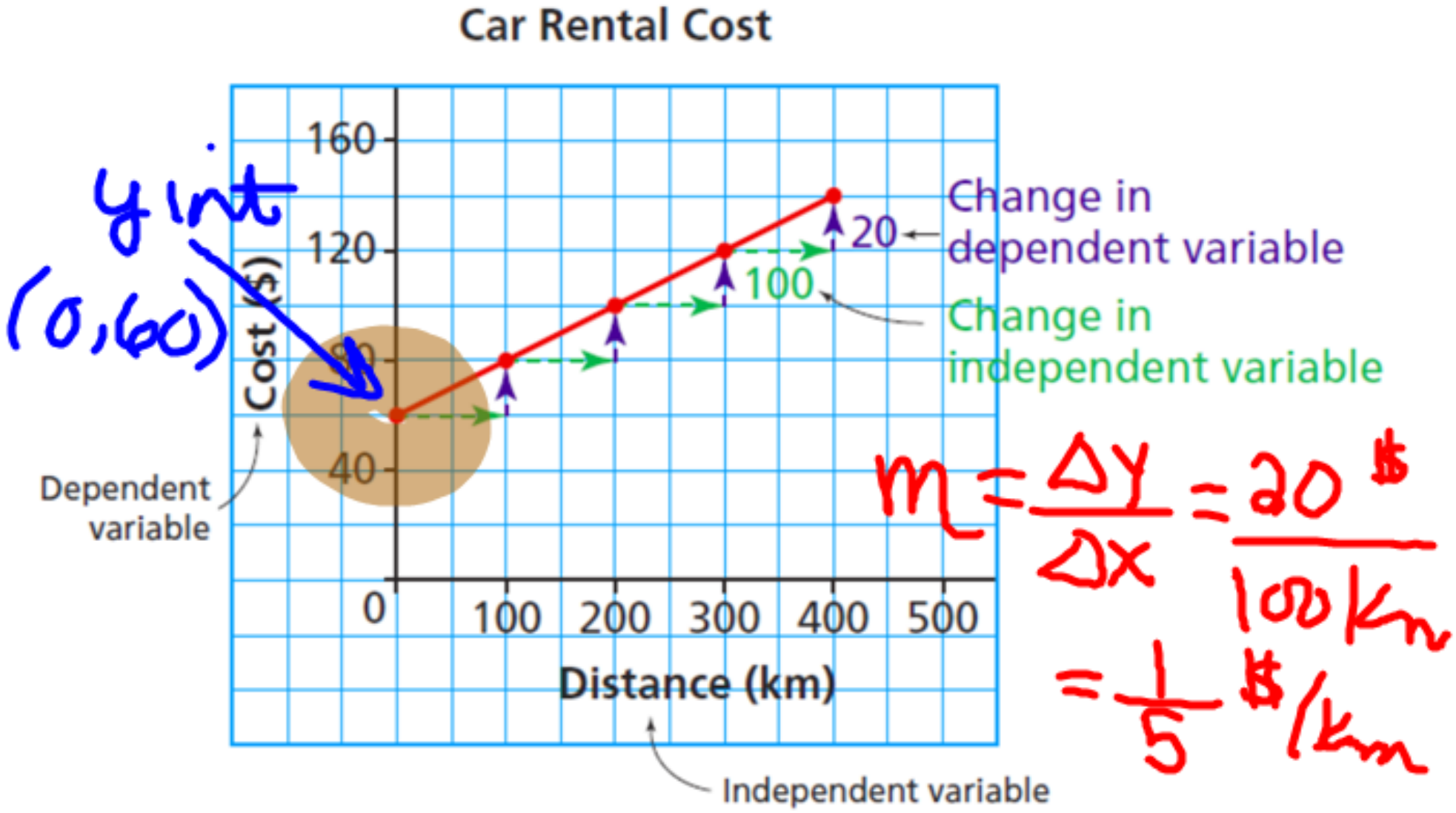
For a linear relation:

*a **constant change** in the **independent variable***

*results in a **constant change** in the **dependent variable***



Graph of a Linear Relation



Rate of Change

- For a linear relation we can calculate the **rate of change**

$$Rate\ of\ Change = \frac{change\ in\ dependent\ variable}{change\ in\ independent\ variable}$$

- *This is commonly referred to as the **slope***

$$slope = \frac{rise}{run} \quad or \quad slope = \frac{y_2 - y_1}{x_2 - x_1}$$

coordinates

Example

Determine the rate of change for each of the linear relations

$ROC = \frac{68-32}{20-0} = \frac{36}{20} = \frac{9}{5}$

C	F
0	32
5	41
10	50
15	59
20	68

2
1

$ROC = \frac{32-41}{0-5} = \frac{9}{5}$

$ROC = \frac{6-3}{120-60}$

t	T
60	3
120	6
180	9
240	12
300	15

$= \frac{3}{60}$

$= \frac{1}{20}$

Asssignment Page 307
#'s 3,4,7,9,15

3. Which tables of values represent linear relations? Explain your answers.

a)

Time (min)	Distance (m)
0	10
2	50
4	90
6	130

b)

Time (s)	Speed (m/s)
0	10
1	20
2	40
3	80

c)

Speed (m/s)	Time (s)
15	7.5
10	5
5	2.5
0	0

d)

Distance (m)	Speed (m/s)
4	2
16	4
1	1
9	3

4. Which sets of ordered pairs represent linear relations? Explain your answers.

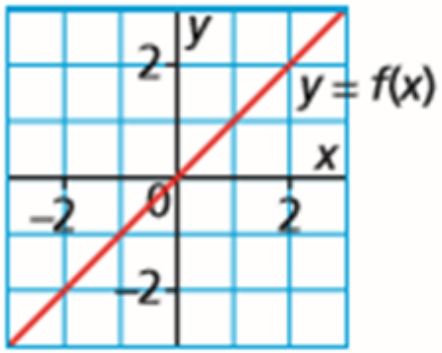
a) $\{(3, 11), (5, 9), (7, 7), (9, 5)\}$

b) $\{(-2, 3), (0, 1), (2, -3), (4, -7)\}$

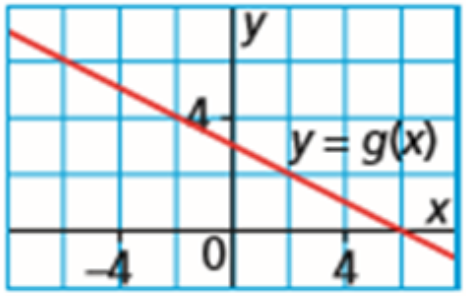
c) $\{(1, 1), (1, 3), (2, 1), (2, 3)\}$

5. Which graphs represent linear relations? How do you know?

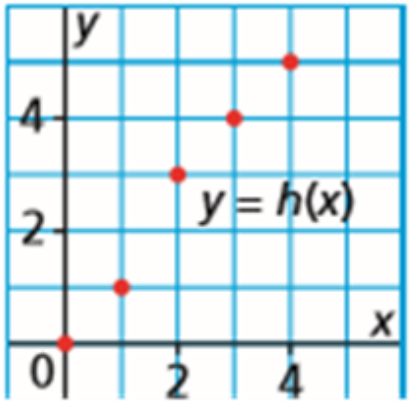
a)



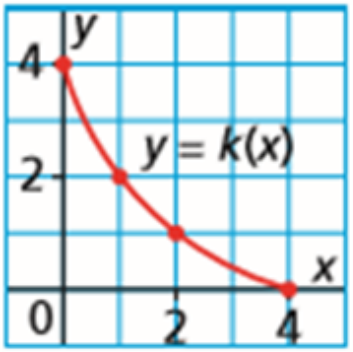
b)



c)



d)

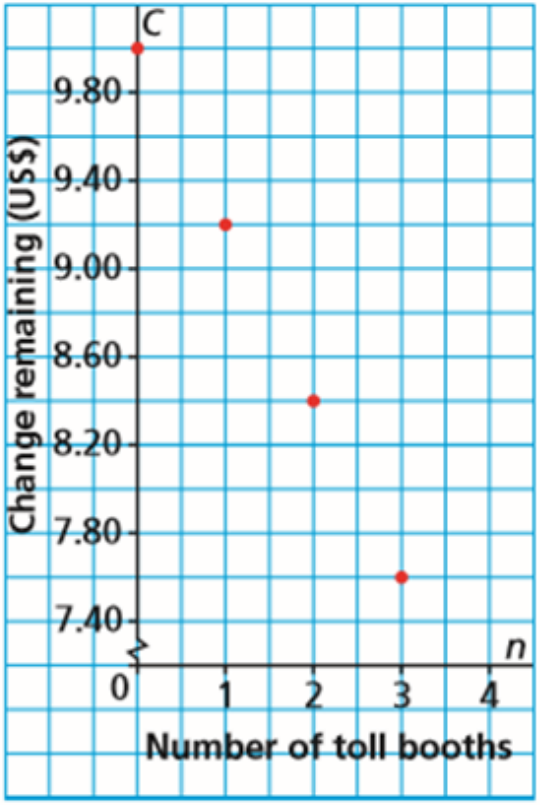


9. Earth rotates through approximately 360° every 24 h. The set of ordered pairs below describes the rotation. The first coordinate is the time in hours, and the second coordinate is the approximate angle of rotation in degrees. Describe two strategies you could use to determine if this relation is linear.

{(0, 0), (6, 90), (12, 180), (18, 270), (24, 360)}

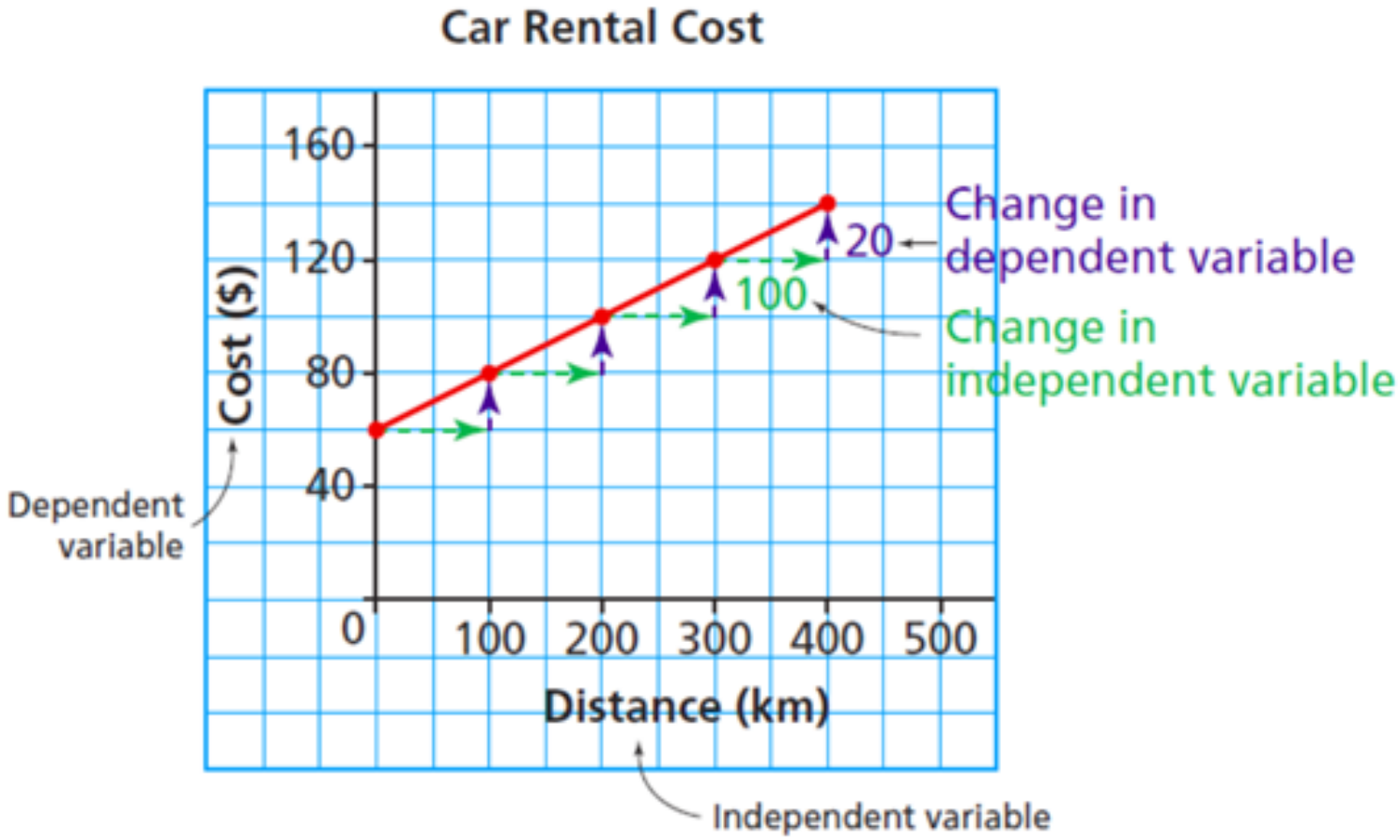
15. Kashala takes a cross-country trip from her home in Lethbridge through the United States. In Illinois, she drives on a toll highway. This graph represents the cost of Kashala's drive on the toll highway. She is charged a constant amount at each toll booth and she starts with US$10 in change. Determine the rate of change, then describe what it represents.

Kashala's Drive on the Toll Highway



Linear Equation

$$C = 0.20d + 60$$



Creating a Graph from an Equation

Set up a table of values

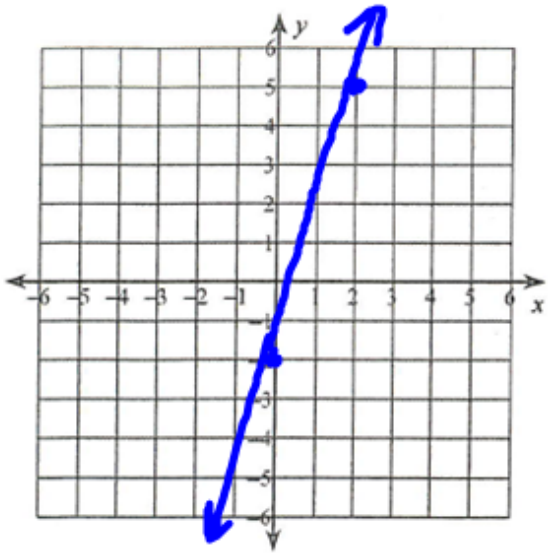
Independent variable (left), dependent variable (right)

Choose a few independent values and solve for the associated dependent values

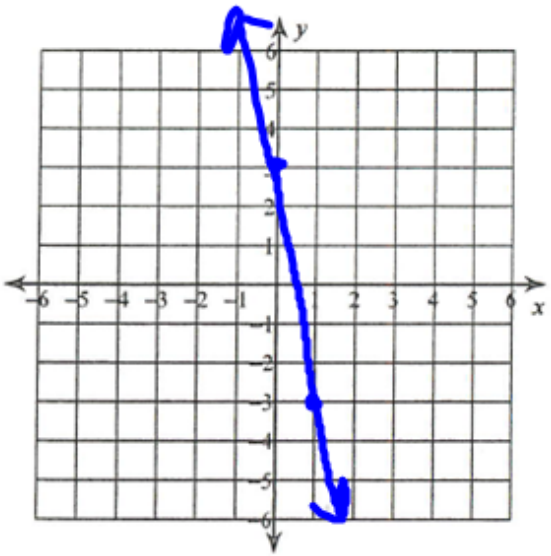
Then plot the ordered pairs on a coordinate plane

Sketch the graph of each line.

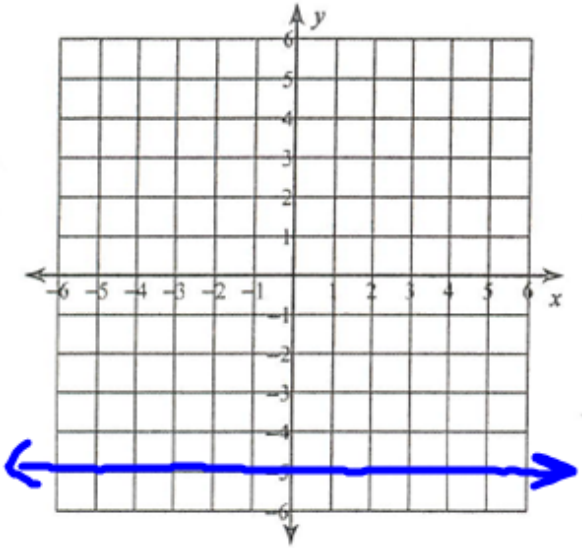
1) $y = \frac{7}{2}x - 2$



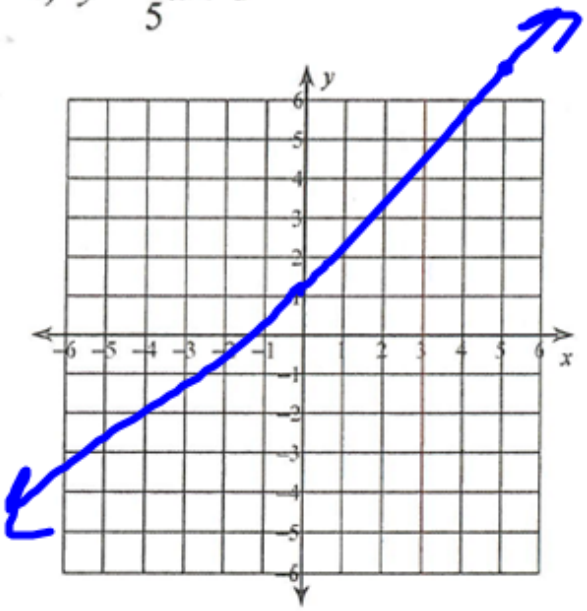
2) $y = -6x + 3$



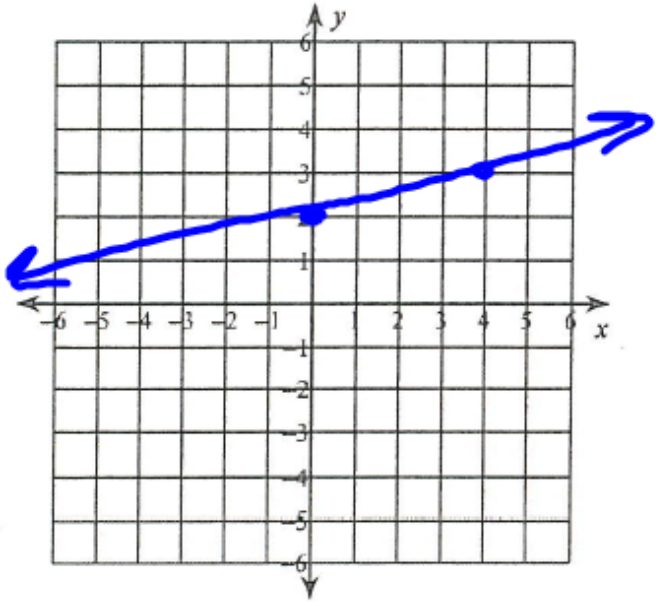
3) $y = -5$



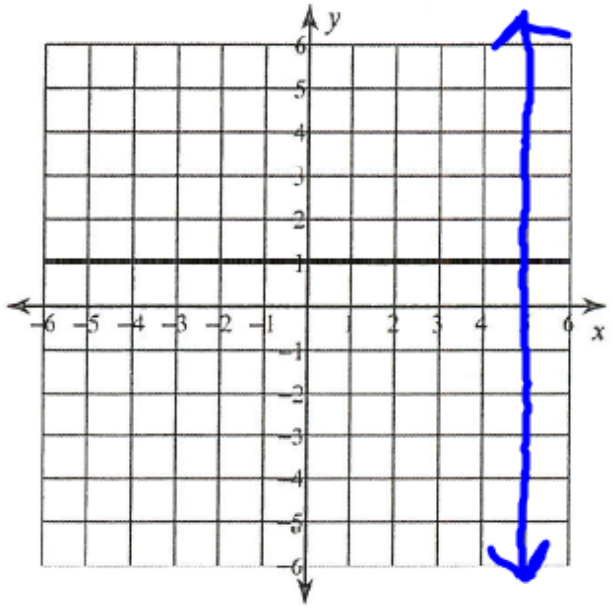
4) $y = \frac{6}{5}x + 1$



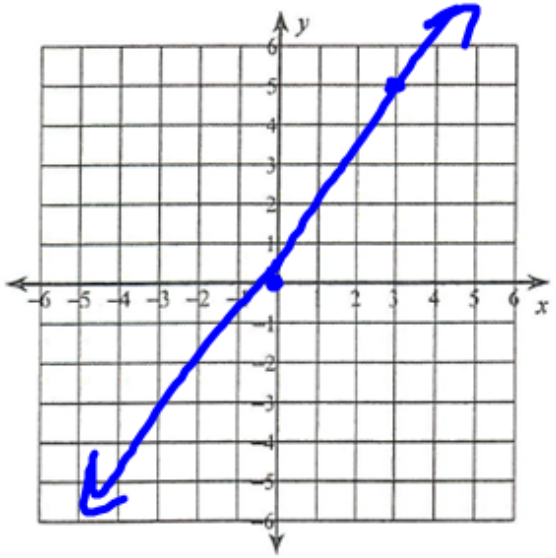
5) $y = \frac{1}{4}x + 2$



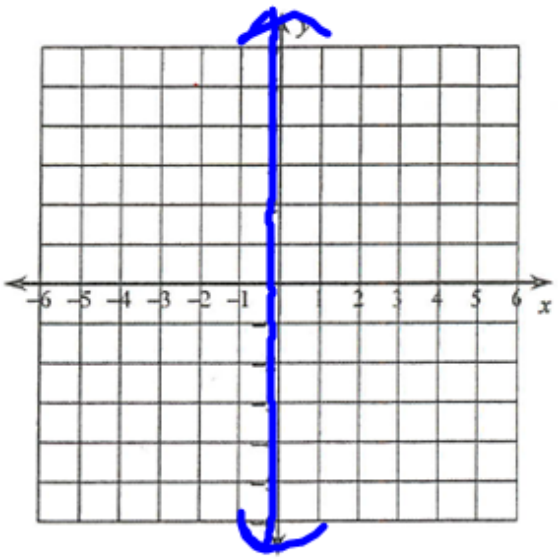
6) $x = 5$



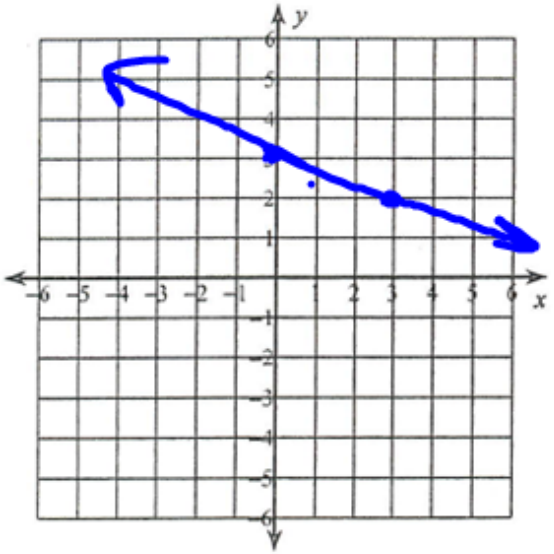
7) $y = \frac{5}{3}x$



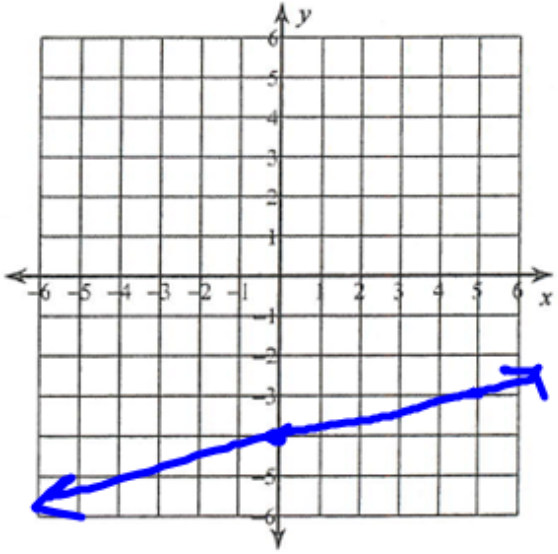
8) $x = 0$



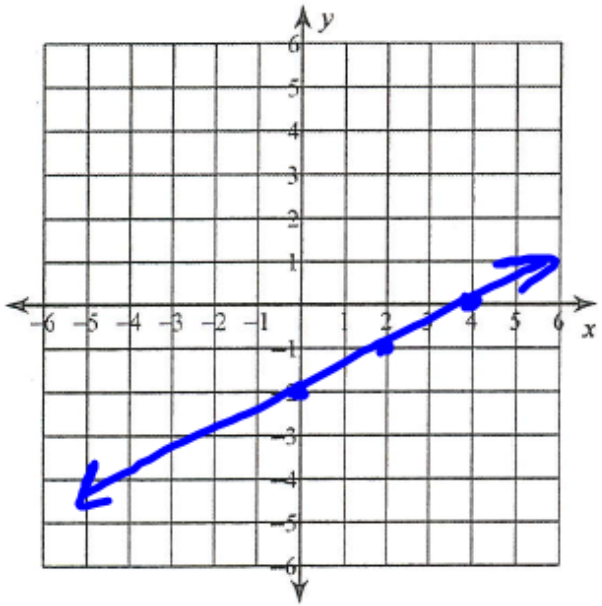
9) $y = -\frac{1}{3}x + 3$



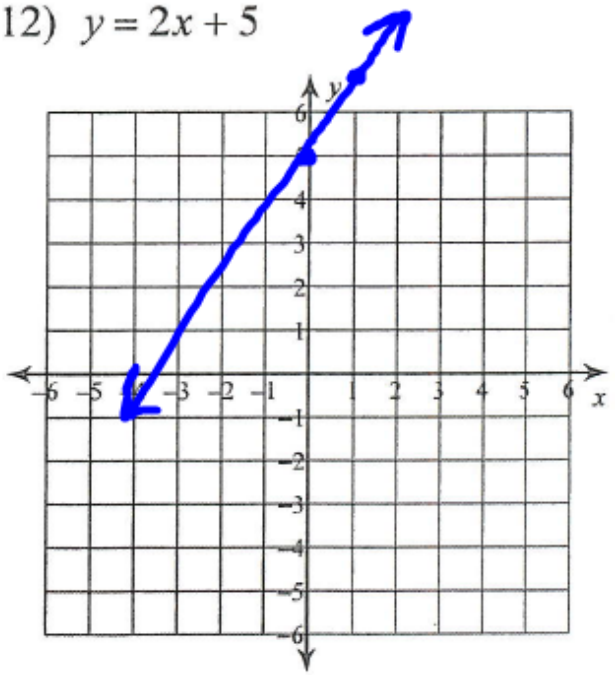
10) $y = \frac{1}{5}x - 4$



11) $y = \frac{1}{2}x - 2$



12) $y = 2x + 5$



Example

Which relation is linear? Justify the answer.

a) A new car is purchased for $24 000. Every year, the value of the car decreases by 15%. The value is related to time.

b) For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

$C(t) = 50t + 75$

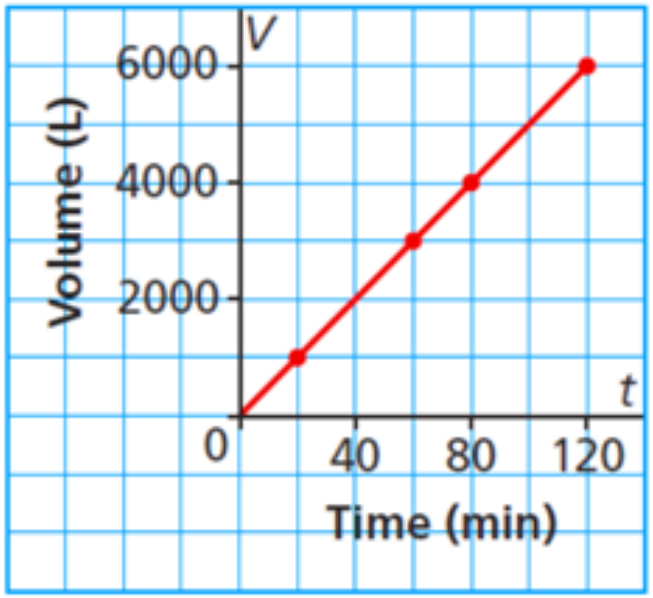
time	Price
0	24000
1	20400
2	17340

$\Delta = -3600$

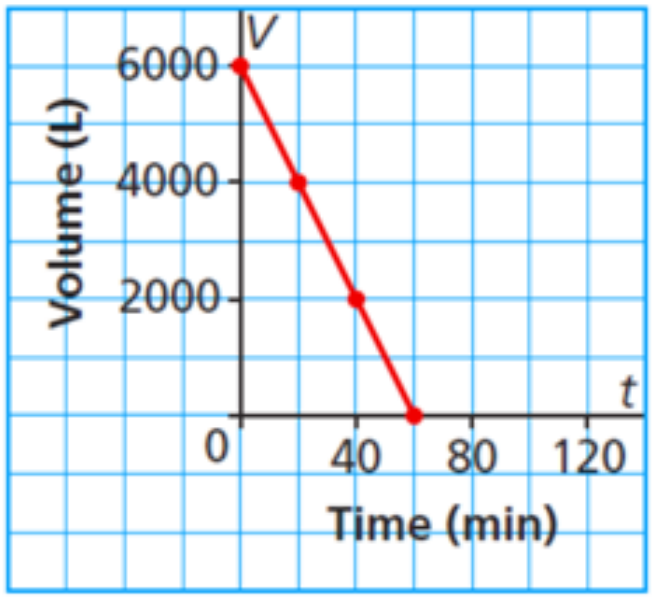
$\Delta = -3060$

Example

Graph A
Filling a Water Tank



Graph B
Emptying a Water Tank



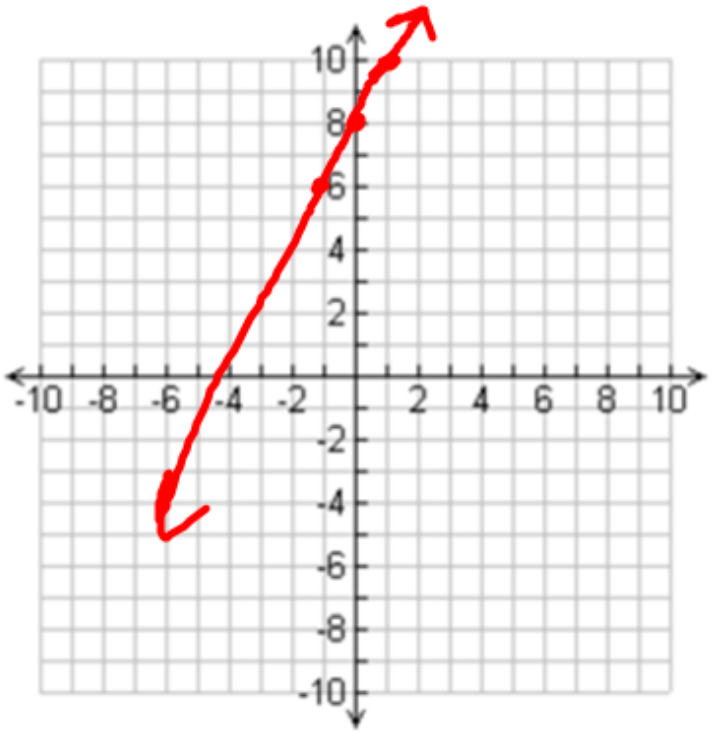
a) Identify the independent and dependent variables.

b) Determine the rate of change of each relation, then describe what it represents.

Homework

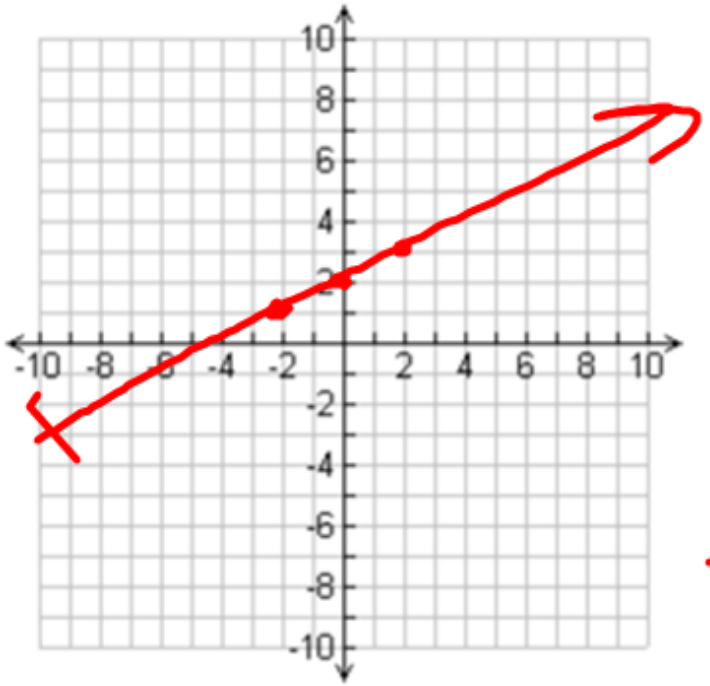
P. 307-310

 # 5, 6, 12, 14, 16, 17, 20



6. $i)$ $y = 2x + 8$

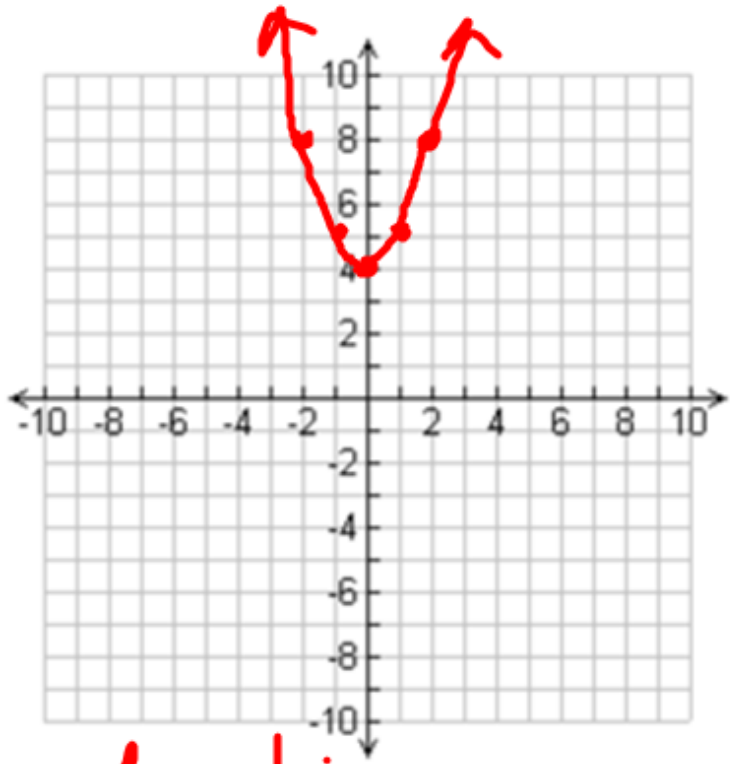
x	y
0	8
1	10
-1	6



6. *ii*) $y = 0.5x + 2$

$y = \frac{1}{2}x + 2$

x	y
0	2
2	3
-2	1



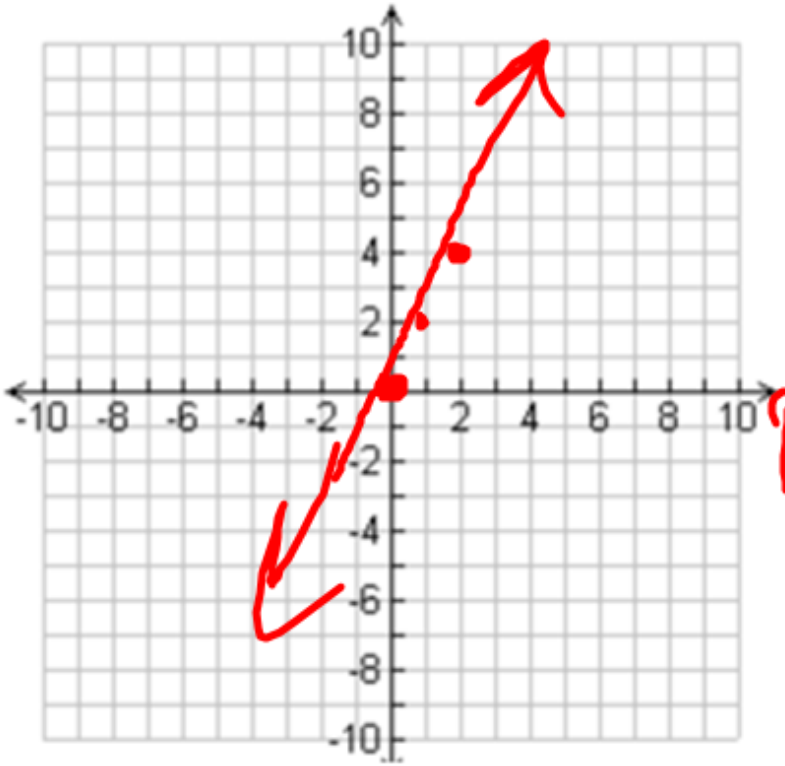
quadratic

Parabola

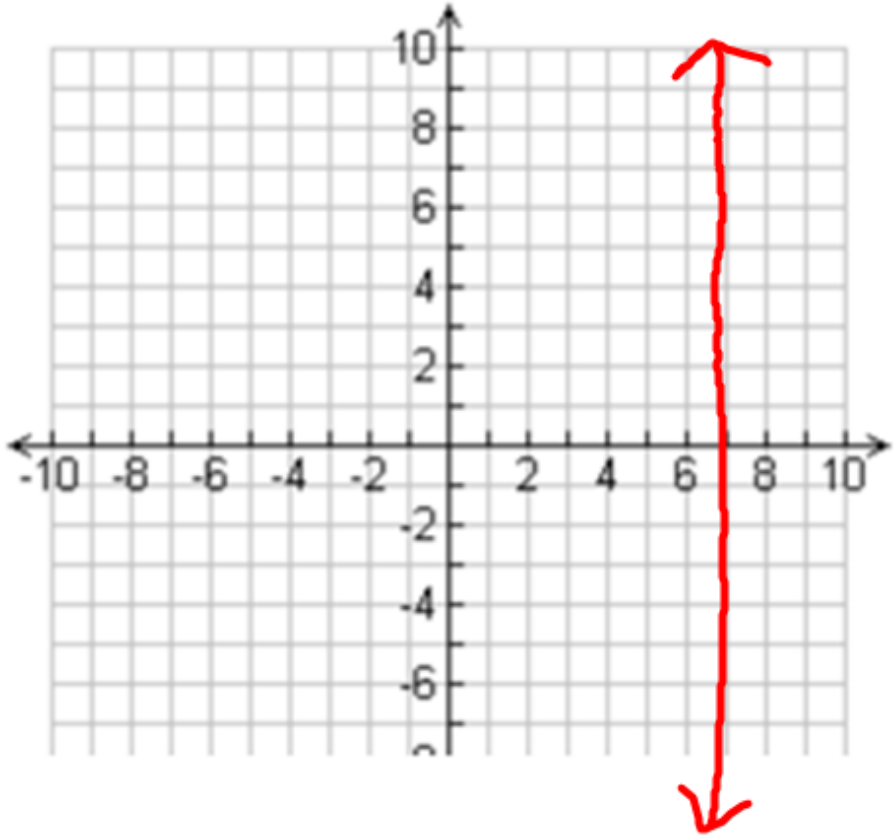
6. *iii*) $y = x^2 + 4$

x	y
0	4
1	5
-1	5
2	8
-2	8

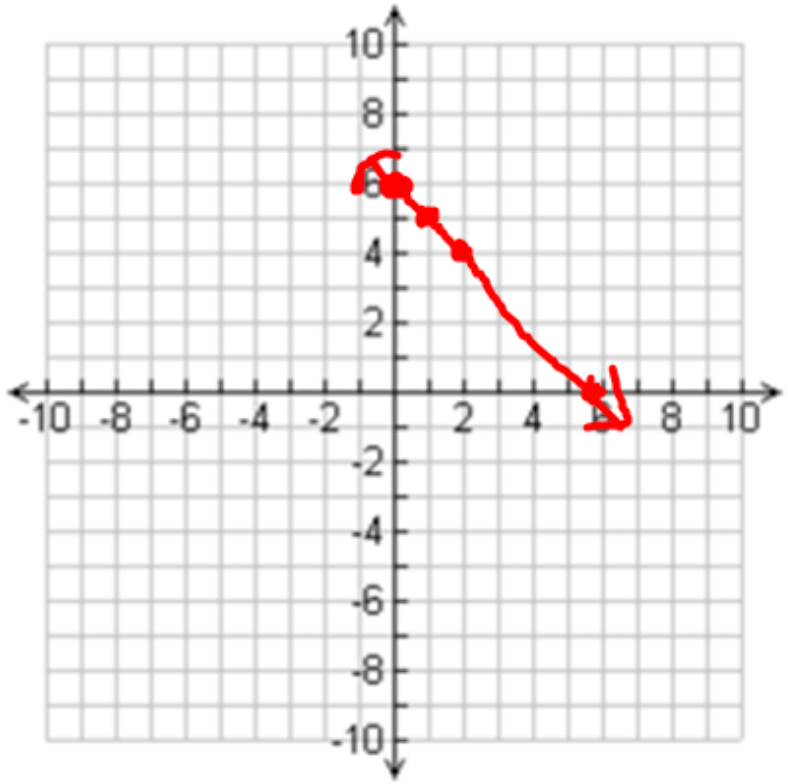
6. $iv)$ $y = 2x$



$ROC = \frac{2}{1}$



6. $v)$ $x = 7$



6. *vi*) $x + y = 6$

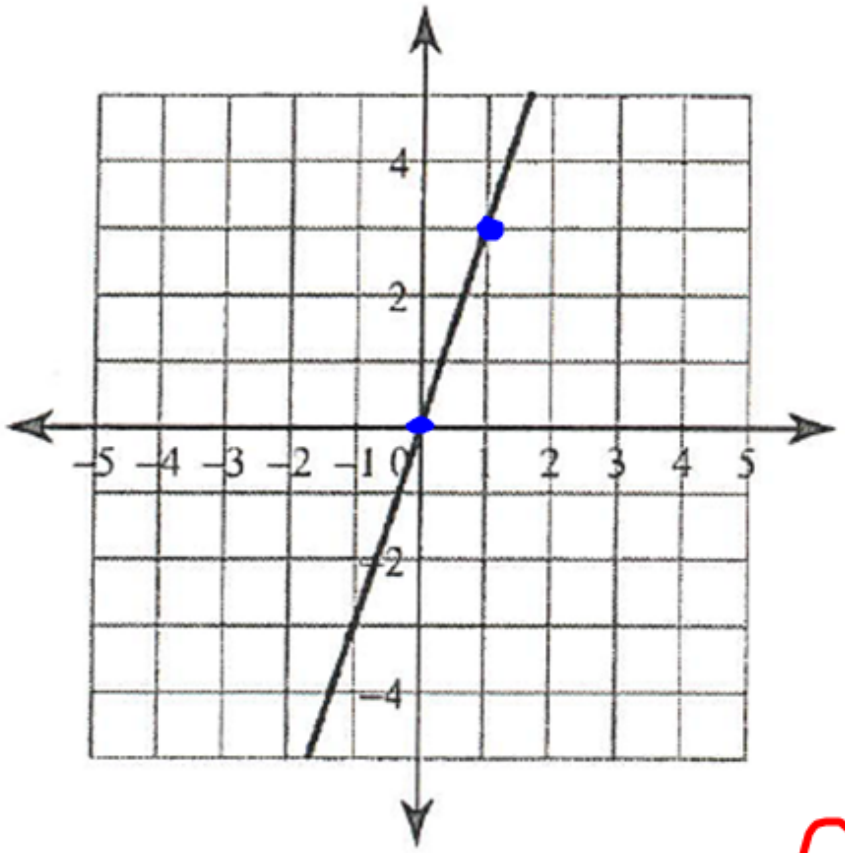
$y = -x + 6$

$\text{RoC} = \frac{-1}{1}$

Writing Equations of Lines Given the Graph

Write the slope-intercept form of the equation of each line.

1)

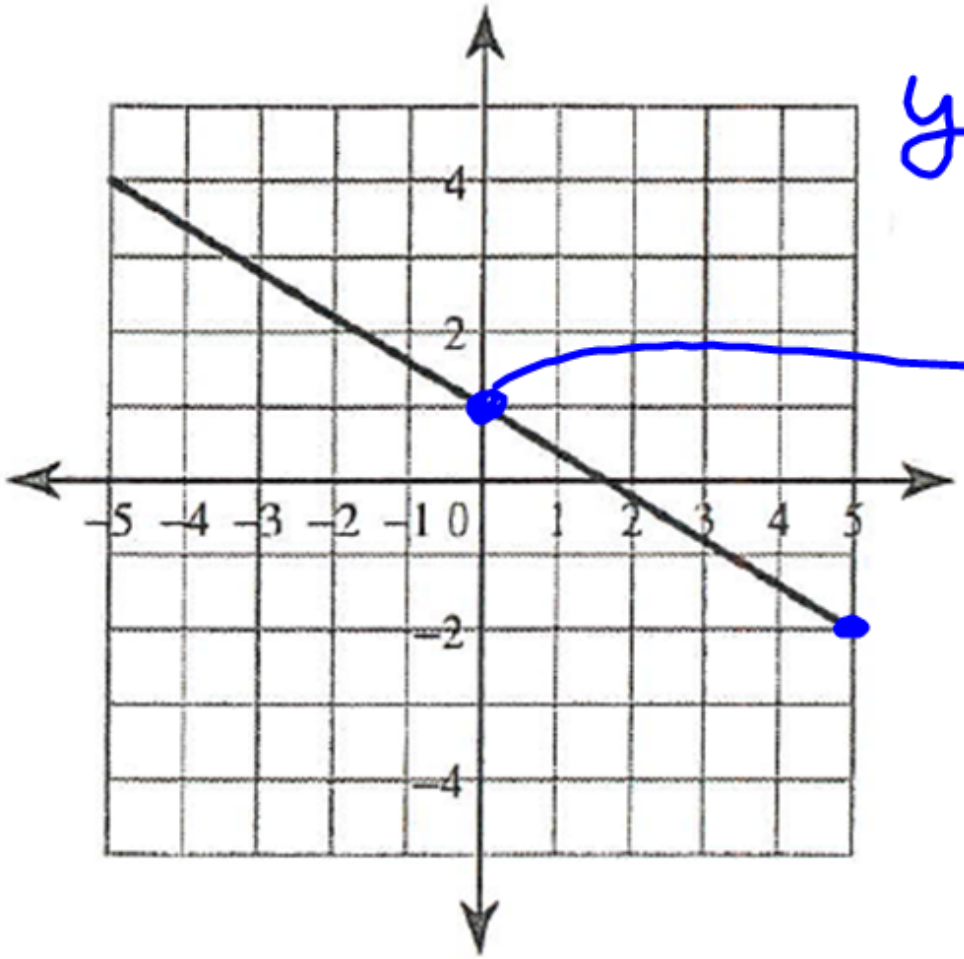


$\frac{3}{1}$

$y = Roc\,x + \#$

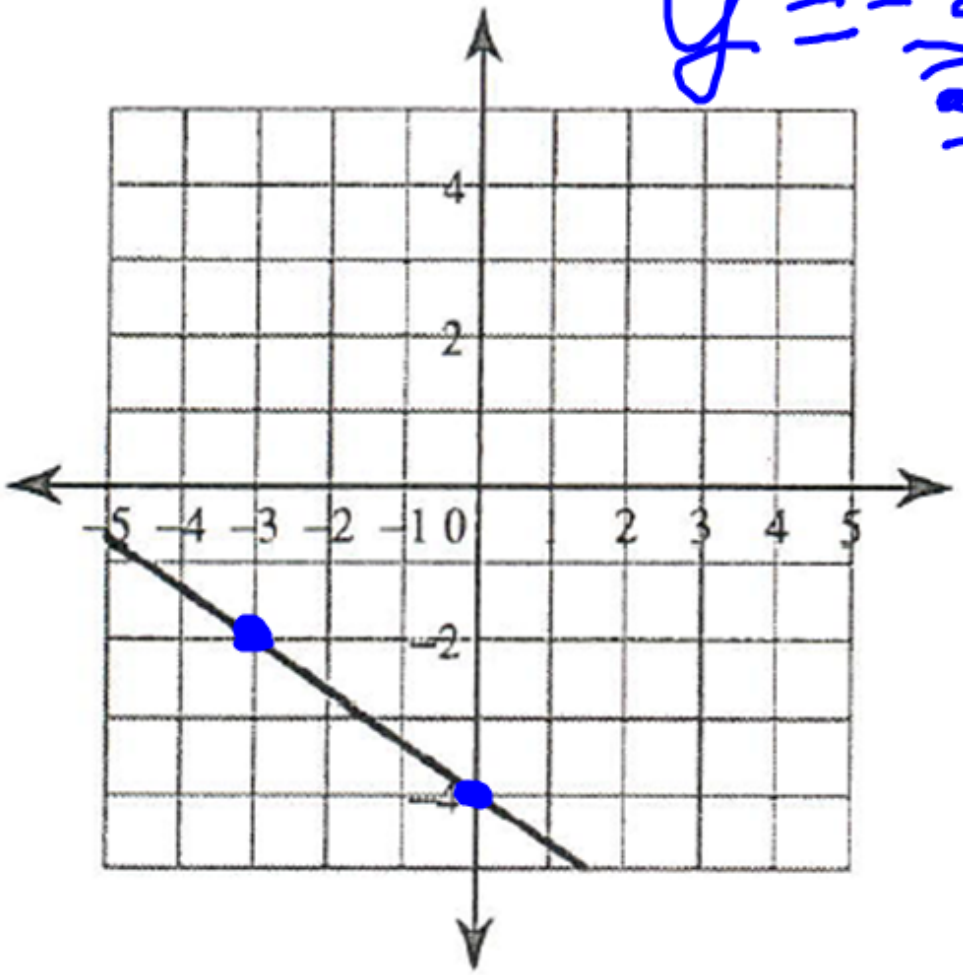
$y = 3x + 0$

2)



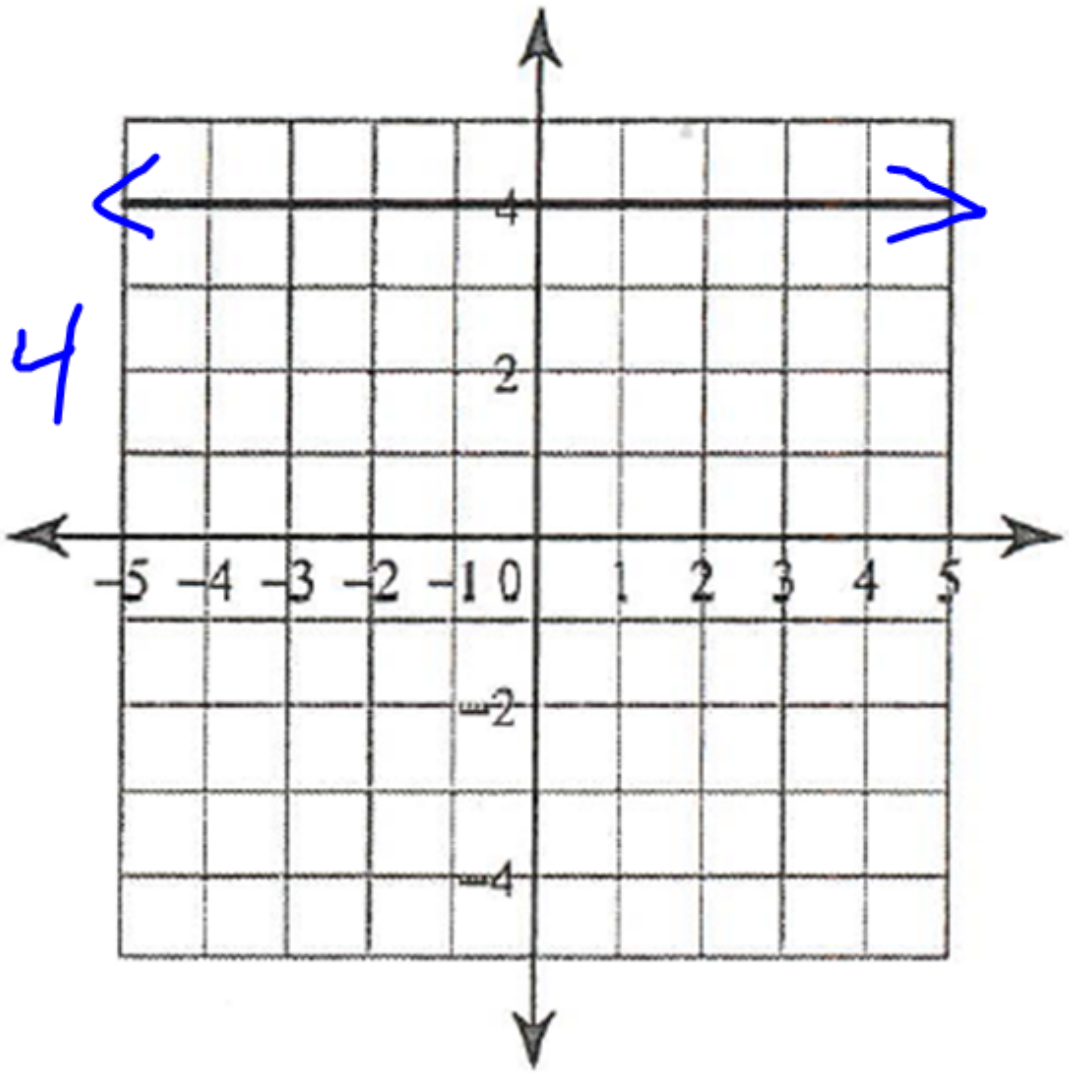
$y = -\frac{3}{5}x + 1$

3)



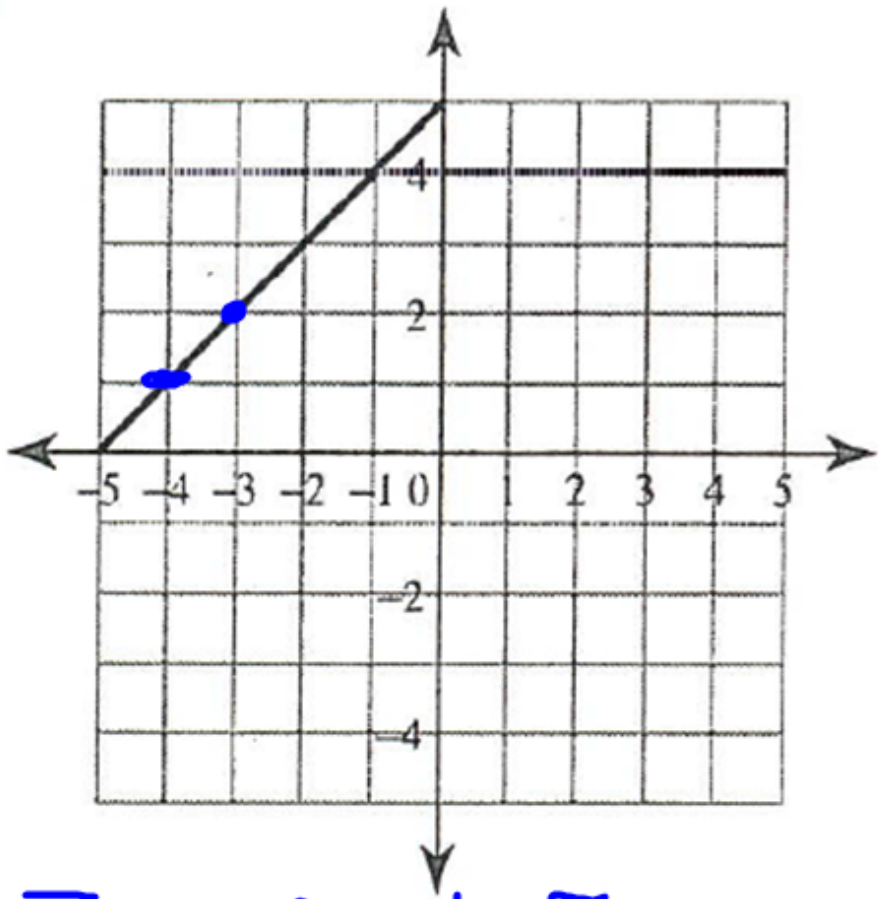
$y = -\frac{2}{3}x - 4$

4)



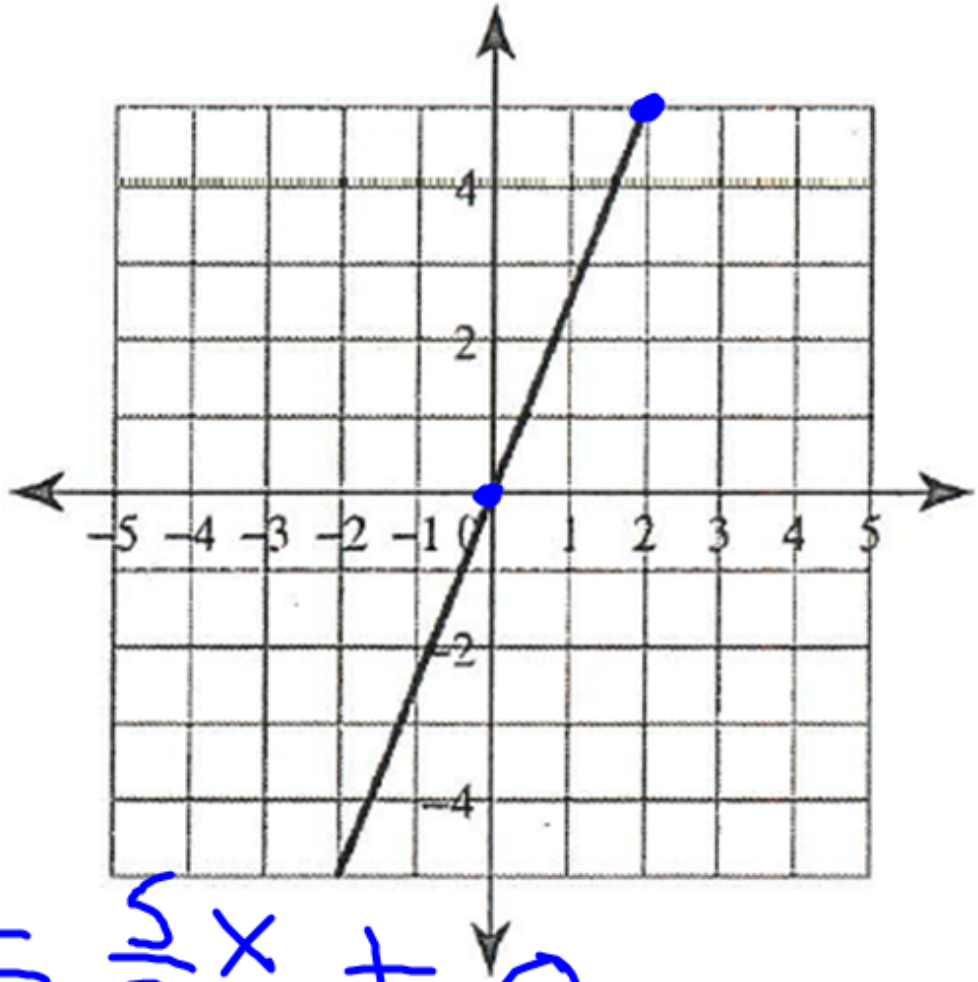
$y = 4$

5)



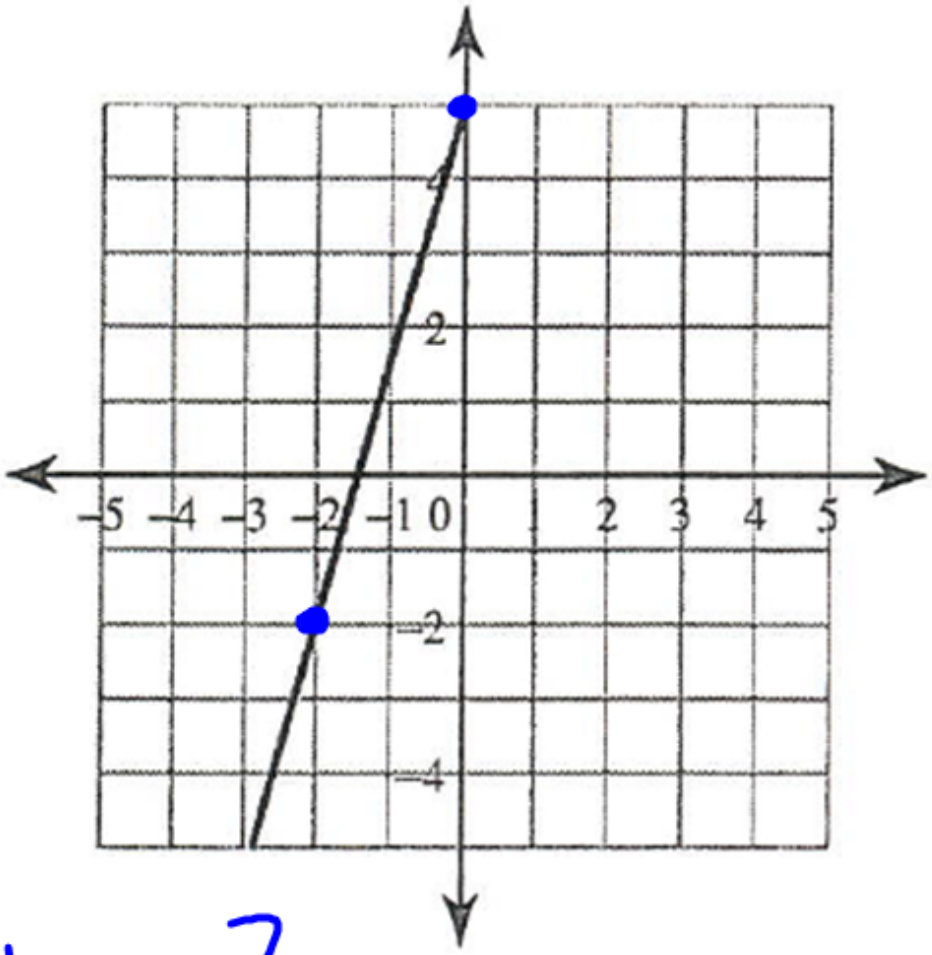
$y = x + 5$

6)



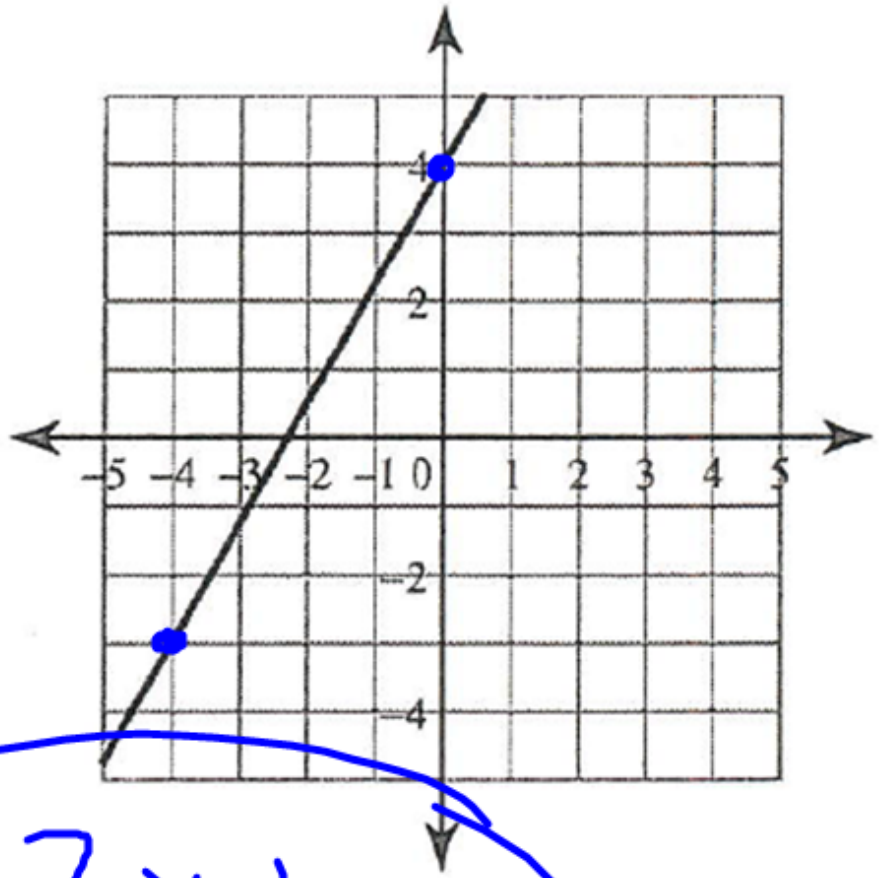
$y = \frac{5}{2}x + 0$

7)



$y = \frac{7}{2}x + 5$

8)



$y = \frac{7}{4}x + 4$