5.6 Curve Sketching

Polynomials

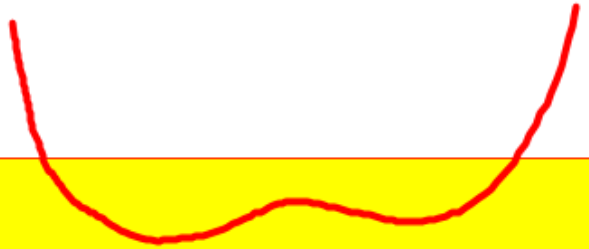
Rationals

5.6 Curve Sketching

Learning Targets:

1. SWBAT sketch graphs of polynomial functions.
2. SWBAT sketch graphs of rational functions.

Today we are going to put all things learned in this unit together and sketch accurate pictures of functions.



Example 1:

For the function $f(x) = x^4 + 4x^3$ find:

a) Intervals of increase and decrease

b) Relative max and mins

} f'

c) Intervals of concavity

d) Inflection Points

e) Horizontal and Vertical Asymptotes

f) Intercepts

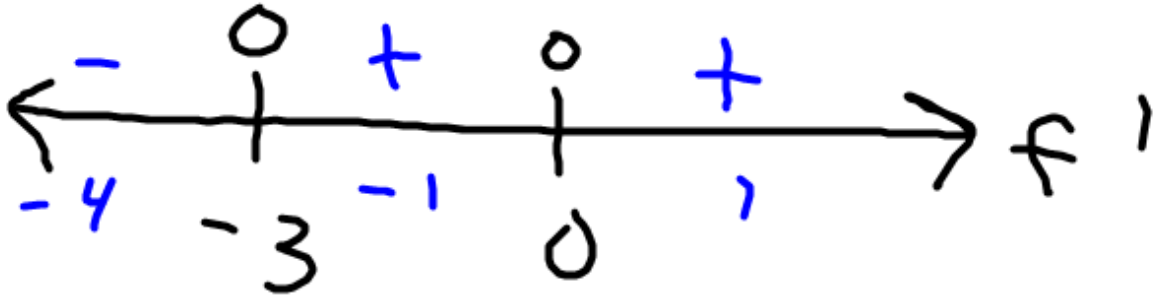
g) Sketch

a) $f' = 4x^3 + 12x^2$

$4x^3 + 12x^2 = 0$

$4x^2(x+3) = 0$

$x = 0$ OR $x = -3$



inc

$(-3, 0) \cup (0, \infty)$

dec

$(-\infty, -3)$

b) Rel min

$f(-3)$

$= (-3)^4 + 4(-3)^3$

$= 81 - 108$

$= -27$

$(-3, -27)$

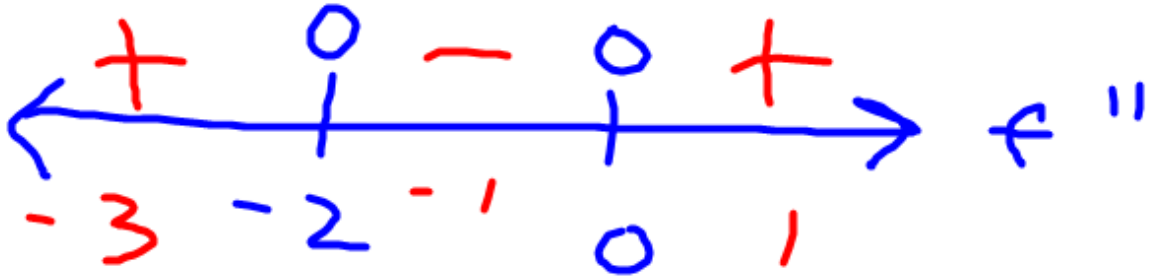
Rel Min

c) $f'' = 12x^2 + 24x$

$12x^2 + 24x = 0$

$12x(x+2) = 0$

$x = 0 \quad x = -2$



CU

$(-\infty, -2) \cup (0, \infty)$

CD

$(-2, 0)$

d) $f(-2)$

$= (-2)^4 + 4(-2)^3$

$= 16 - 32$

$= -16$

$f(0) = 0$

$(-2, -16)$ IP

$(0, 0)$

e) No asymptotes

f) Intercepts

y int

Let $x=0$

$y=0$

$(0,0)$

x int

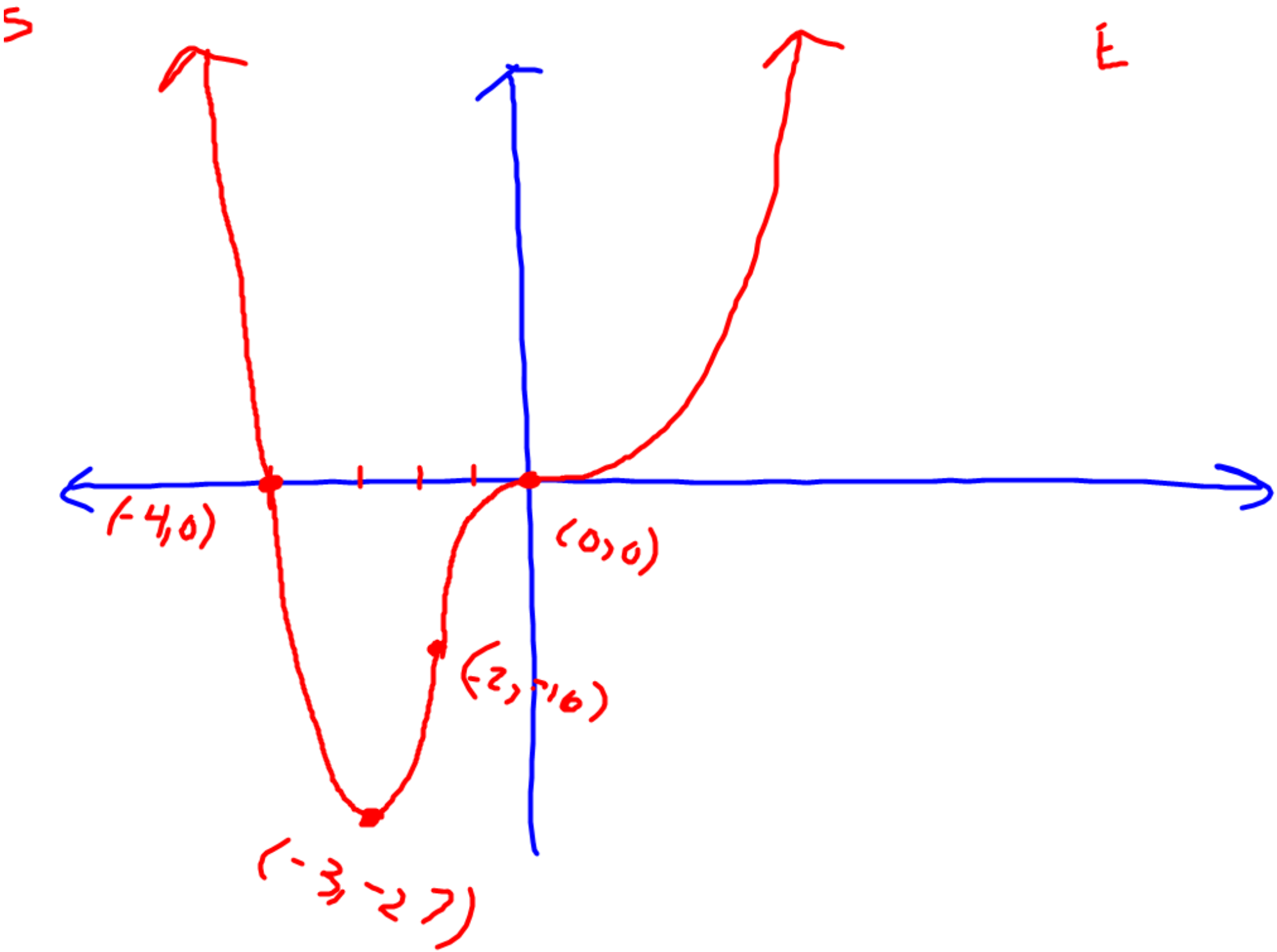
Let $y=0$

$0=x^4+4x^3$

$0=x^3(x+4)$

$x=0, -4$

$(0,0)$ $(-4,0)$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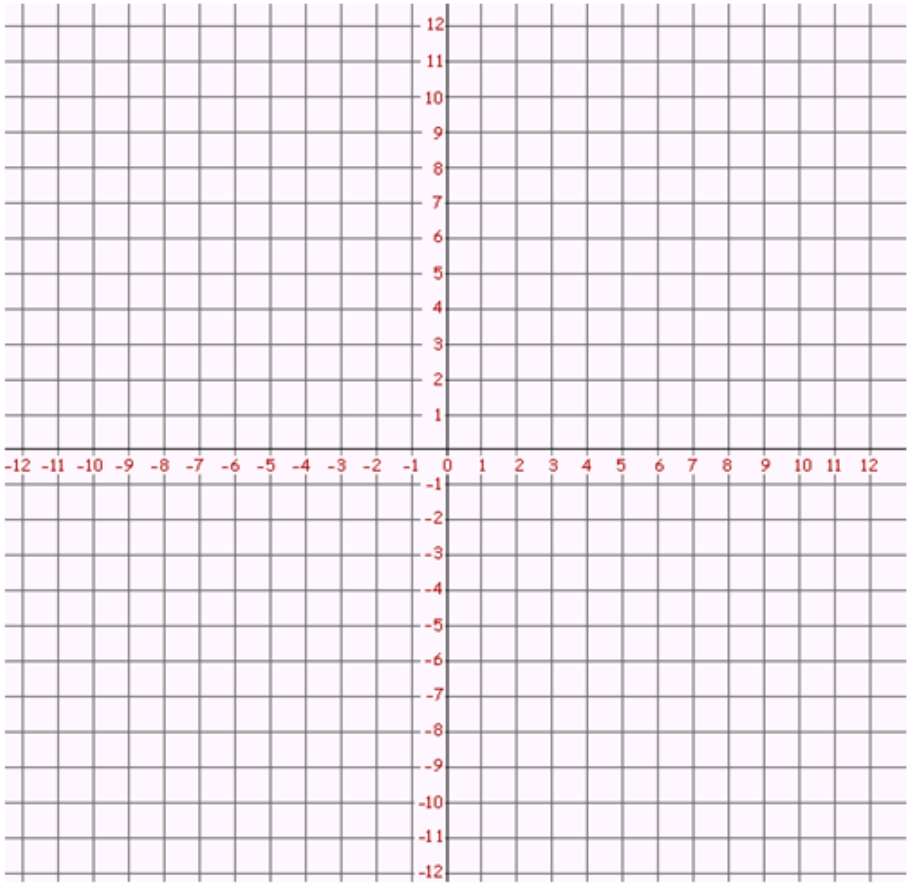
(0,0)
(-2,-16)
(-3,-27)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Example 2 :

For the function $f(x) = 3x^4 - 4x^3 + 2$

find :

a) Intervals of increase and decrease

Inc $(1, \infty)$

Dec $(-\infty, 1)$

b) Relative max and mins

$(1, 0)$ min

c) Intervals of concavity

CU $(-\infty, 0) \cup (2/3, \infty)$

CD $(0, 2/3)$

d) Inflection Points

$(0, 2)$ $(2/3, 1.41)$

e) Horizontal and Vertical Asymptotes

f) Intercepts

$(0, 2)$

g) Sketch

Example 2 :

For the function $f(x) = 3x^4 - 4x^3 + 2$

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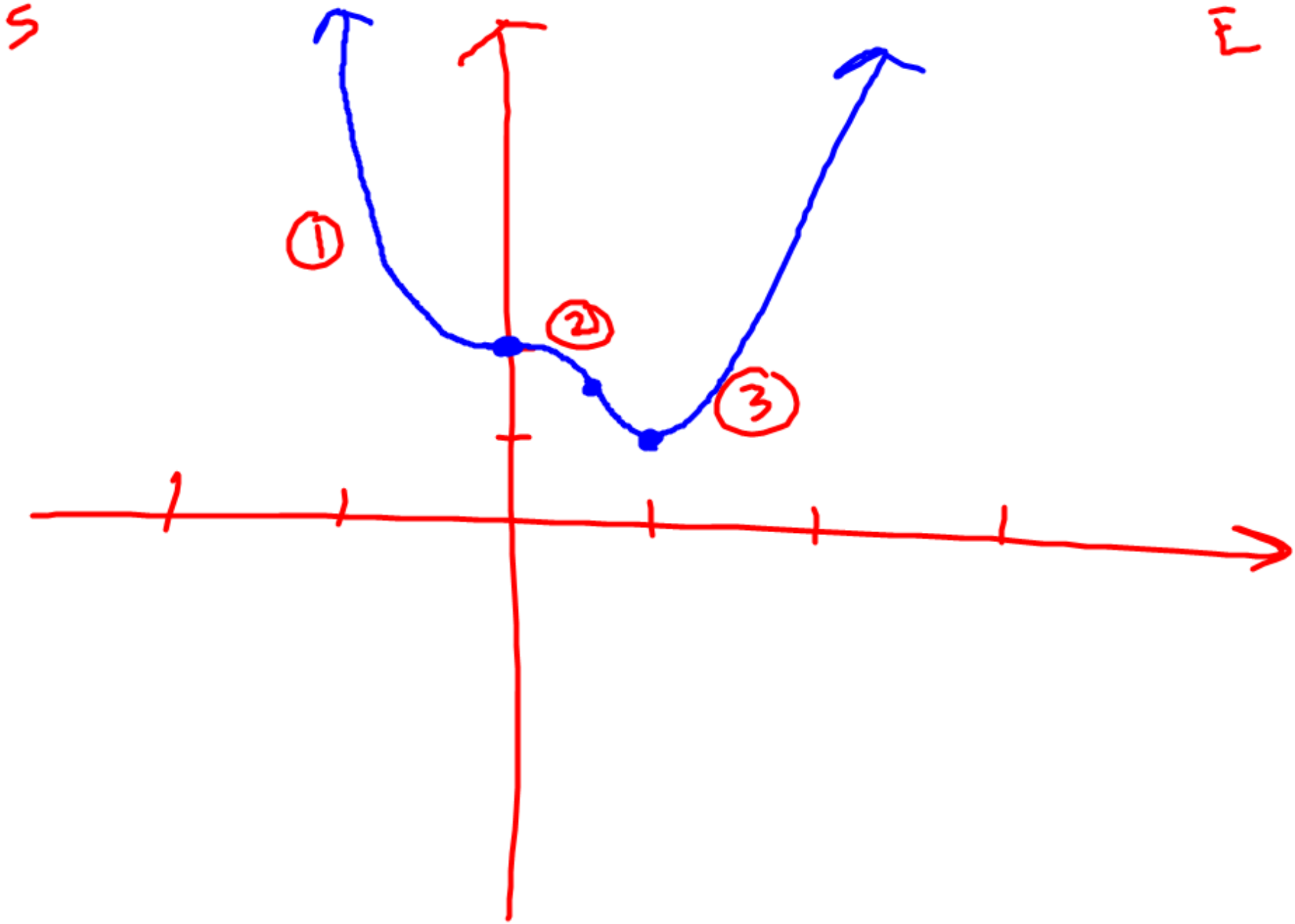
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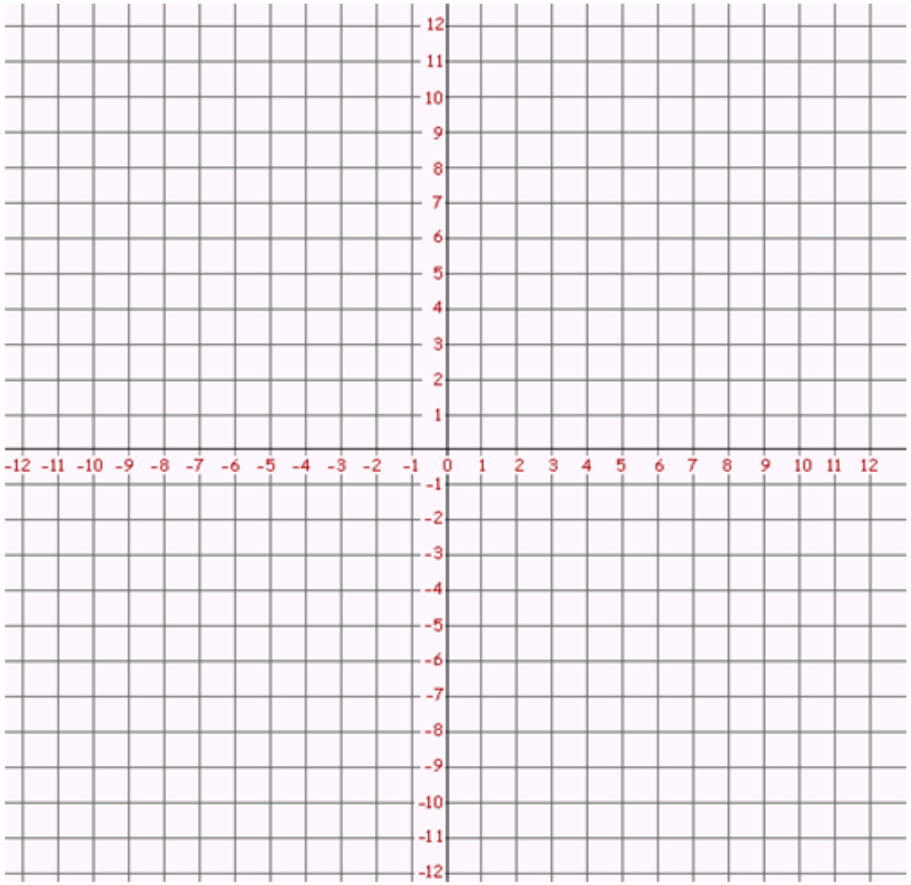
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Assignment
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Also try: $f(x) = \frac{1}{3}x^3 - 3x$

Graphing Rational Functions

Recall from Pre-Calc 30 and from our previous work with limits, how to find horizontal and vertical asymptotes

Find the horizontal and vertical asymptotes for the function $f(x)=\frac{2x}{x^2-1}$

HA

$\lim_{x\to\infty} \frac{2x^1}{x^2-1} = 0$

$y = 0$

VA

$x^2-1=0$

$(x-1)(x+1)=0$

$x=1 \quad x=-1$

Find the horizontal and vertical asymptotes

for the function $f(x)=\dfrac{2x^2}{x^2-x-6}$

Example 3 :

For the function $f(x)=\dfrac{x}{x-1}$ find :

$x \neq 1$

a) Intervals of increase and decrease

b) Relative max and mins

c) Intervals of concavity

d) Inflection Points

e) Horizontal and Vertical Asymptotes

f) Intercepts

g) Sketch

a) $f' = \dfrac{(x-1)(1) - x(1)}{(x-1)^2}$

$f' = \dfrac{-1}{(x-1)^2}$

b) None

c) $f'(x) = -(x-1)^{-2}$

$f'' = \dfrac{2}{(x-1)^3}$

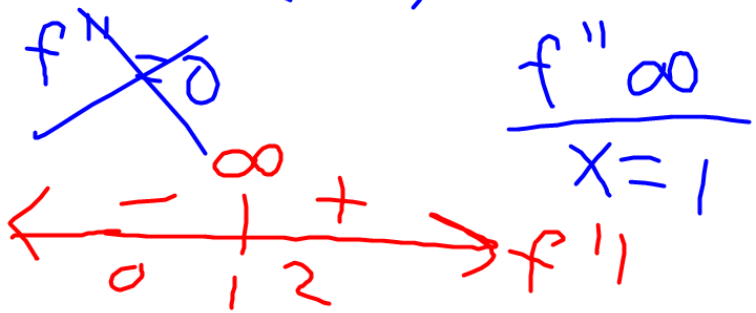
$f' = 0$: $-1 \neq 0$

$f' \infty$: $x = 1$

Dec $(-\infty, 1) \cup (1, \infty)$

$f'' \neq 0$

$f'' \infty$: $x = 1$



CU $(1,\infty)$ CD $(-\infty,1)$

d) No IP

e) HA

$$\lim_{x\to\infty} \frac{x}{x-1} = 1$$

$y = 1$

VA

$x-1=0$

$x=1$

f) Intercepts

y int

let $x=0$

$y = \frac{0}{0-1} = 0$

$(0,0)$

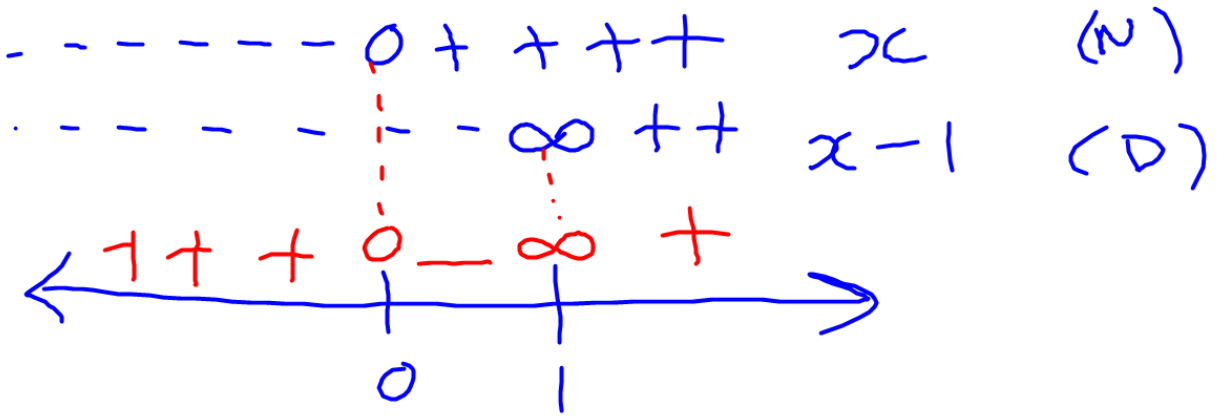
x int

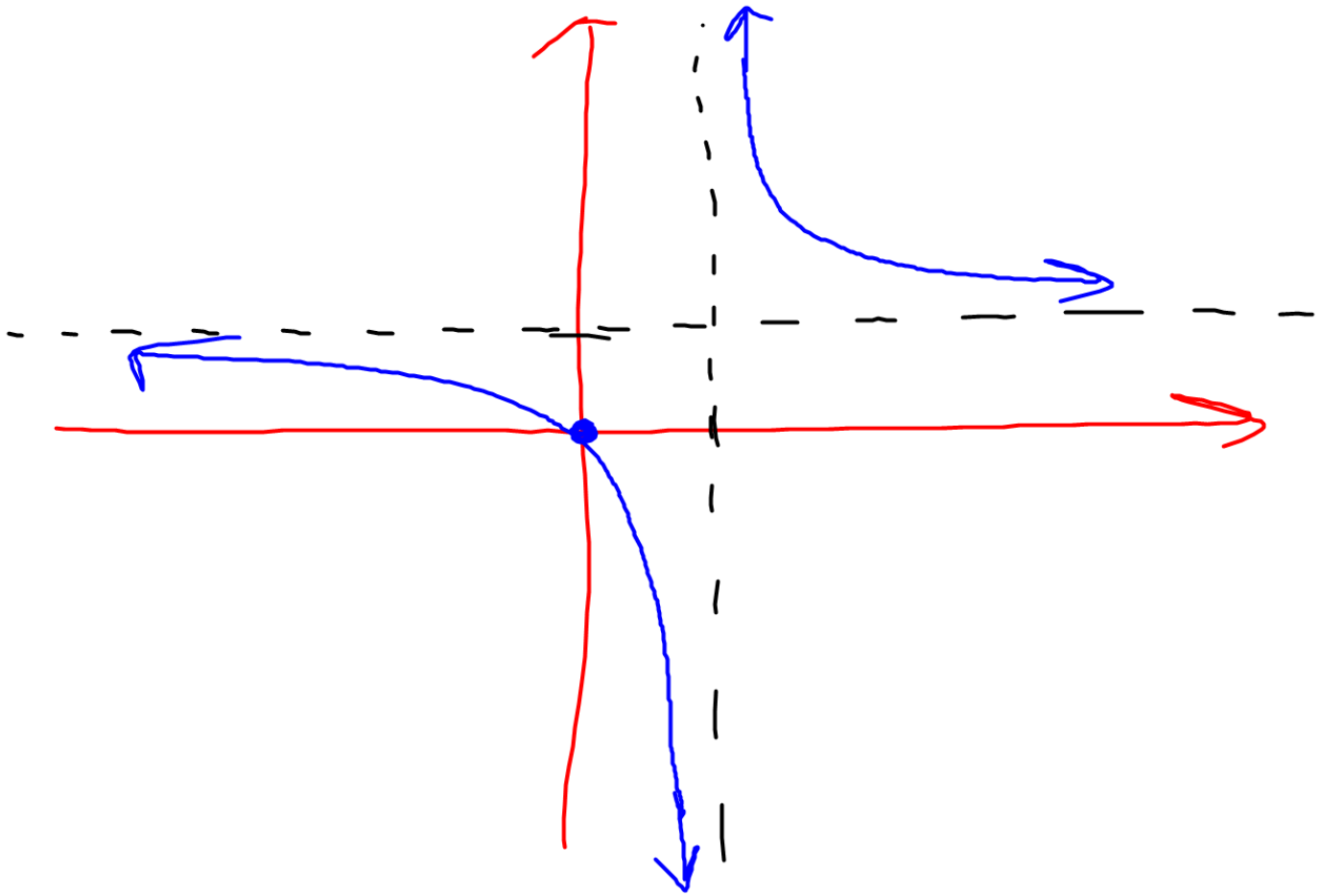
let $y=0$

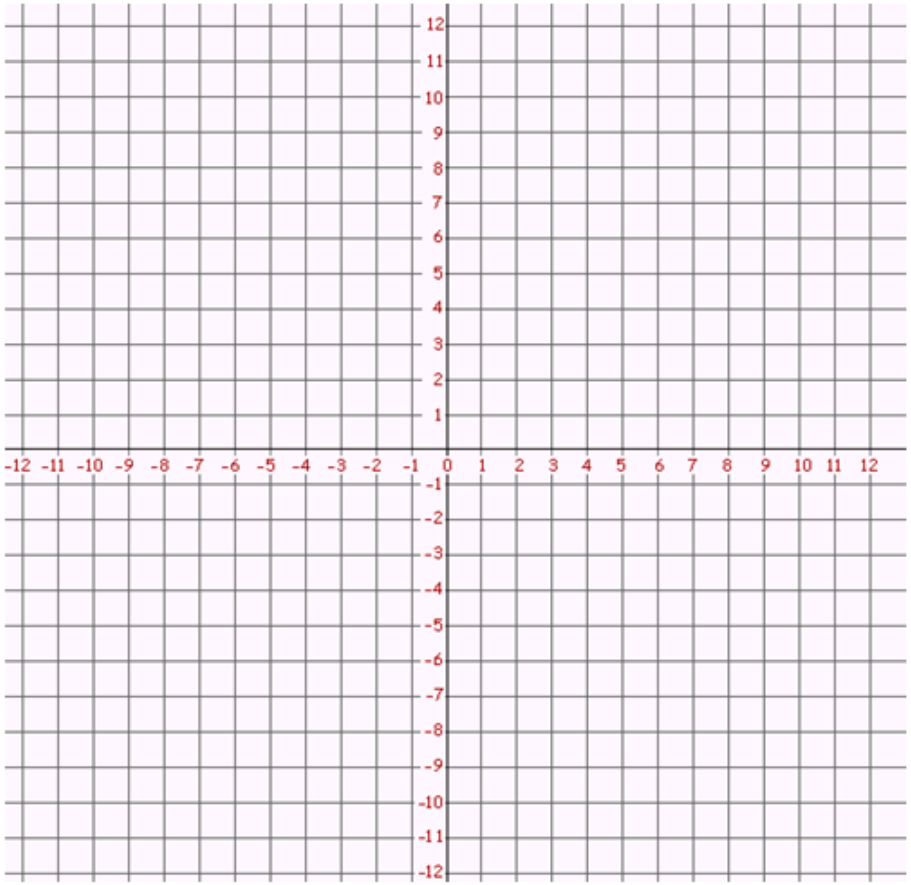
$0 = \frac{x}{x-1}$

$0 = x$ $(0,0)$

Sign Chart



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Example 4 :

For the function $f(x)=\dfrac{1}{x^2-9}$ find :

a) Intervals of increase and decrease

b) Relative max and mins

c) Intervals of concavity

d) Inflection Points

e) Horizontal and Vertical Asymptotes

f) Intercepts

g) Sketch

a) $f' = \frac{-2x}{(x^2-9)^2}$

$f' = 0$

$-2x = 0$

$x = 0$

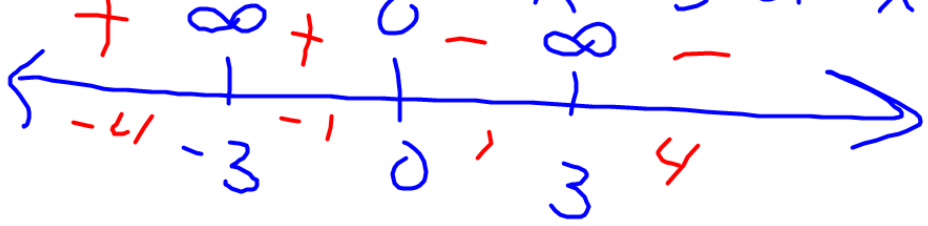
$f' \infty$

$(x^2-9)^2 = 0$

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3$ or $x = -3$



inc

$(-\infty, -3) \cup (-3, 0)$

dec

$(0, 3) \cup (3, \infty)$

b) Max
$f(0) = \frac{1}{-9}$

$(0, -1/9)$

c) $f'' = \frac{6x^2 + 18}{(x^2-9)^3}$

$f'' = 0$

$6x^2 + 18 = 0$

$x^3 = -3$

$f'' \infty$

$x = \pm 3$

CU

$(-\infty, -3) \cup (3, \infty)$

CD

$(-3, 3)$

d) No IPs

e) HA

$$\lim_{x \to \infty} \frac{1}{x^2 - 9} = 0$$

$y = 0$

VA

$x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = 3$ $x = -3$

f) $y = \frac{1}{x^2 - 9} = \frac{1}{(x-3)(x+3)}$

y int

Let $x = 0$

$(0, -\frac{1}{9})$

x int

$0 = \frac{1}{x^2 - 9}$

~~$0 = 1$~~

None

f) $y = \frac{1}{x^2 - 9} = \frac{1}{(x-3)(x+3)}$

y int

Let $x = 0$

$(0, -\frac{1}{9})$

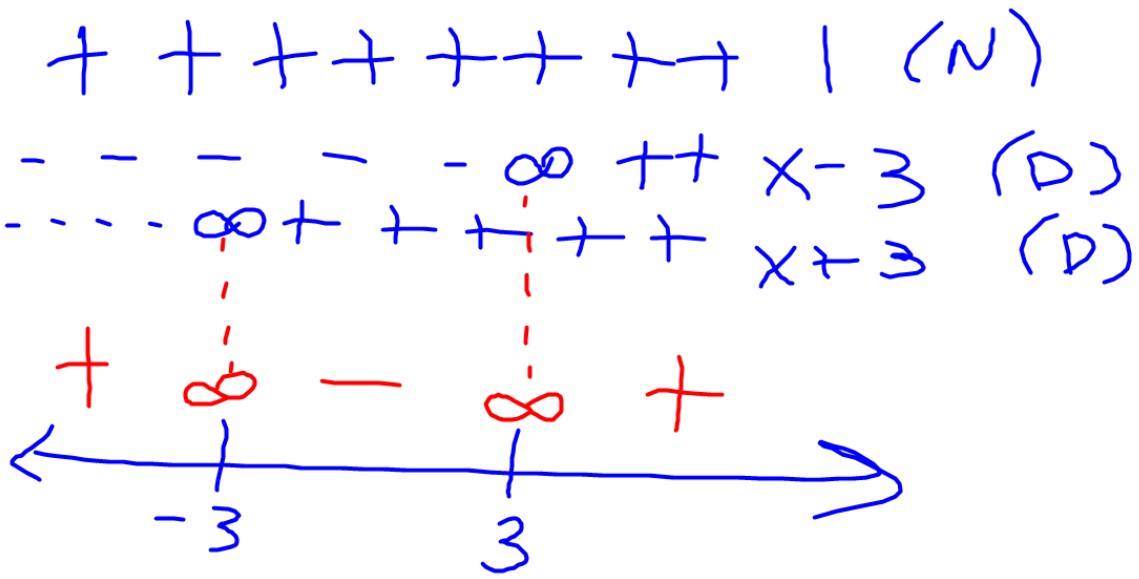
x int

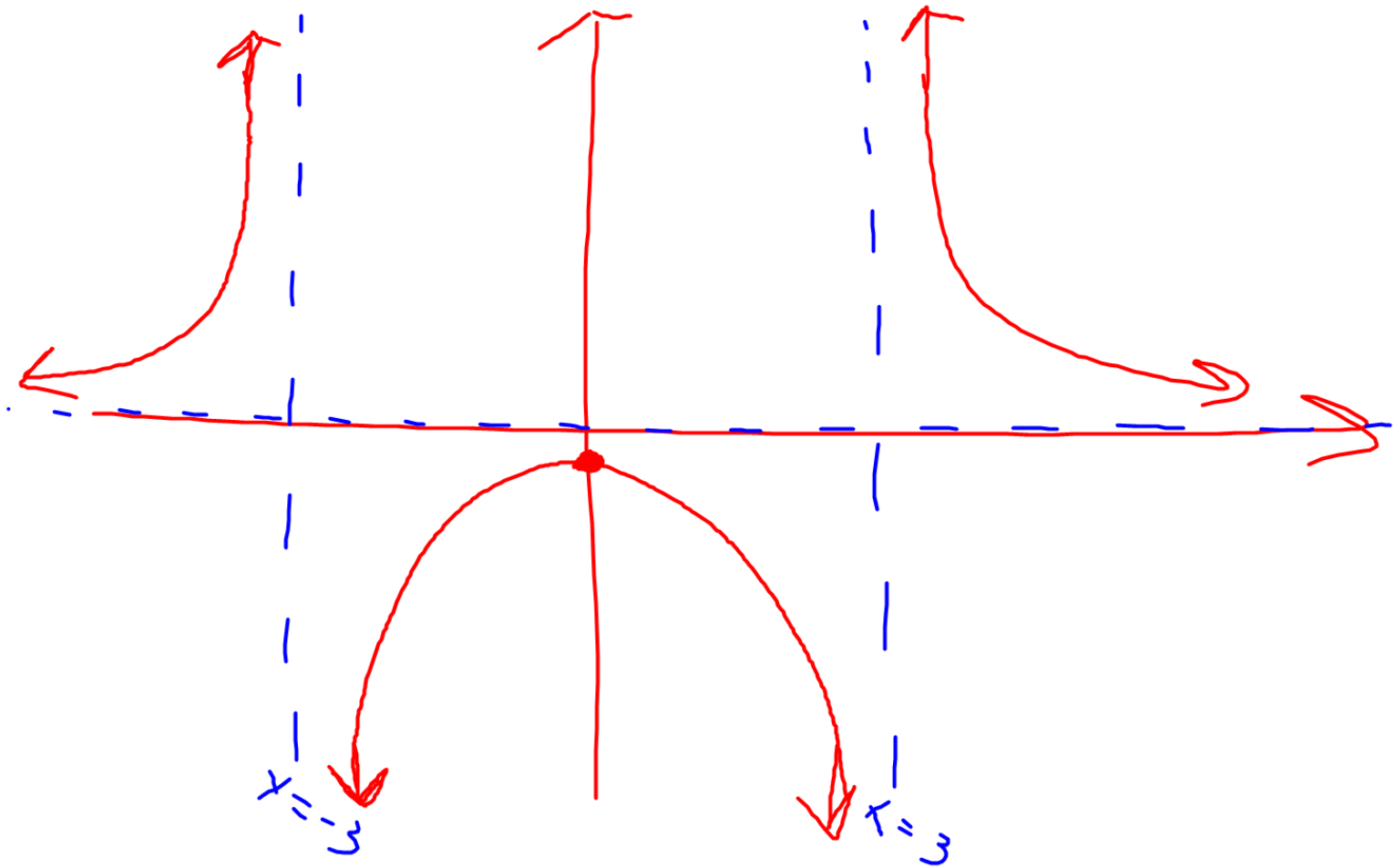
$0 = \frac{1}{x^2 - 9}$

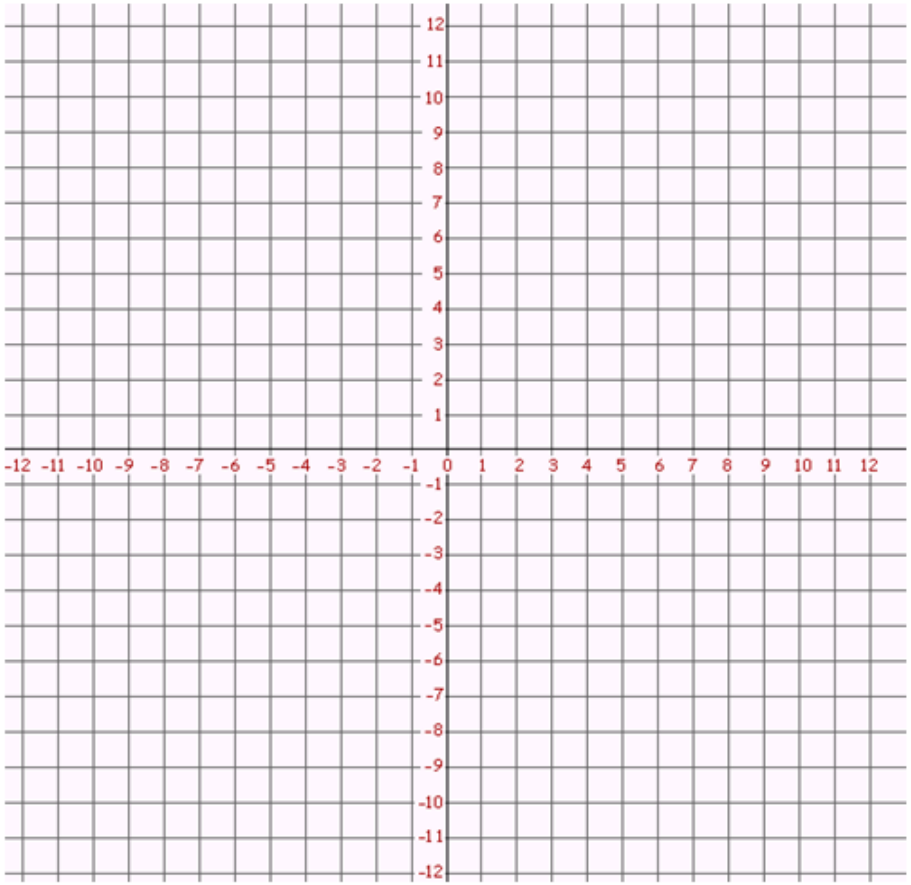
~~$0 = 1$~~

None

Sign Chart




x = -3
x = 3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