5.5 The Trapezoidal Rule

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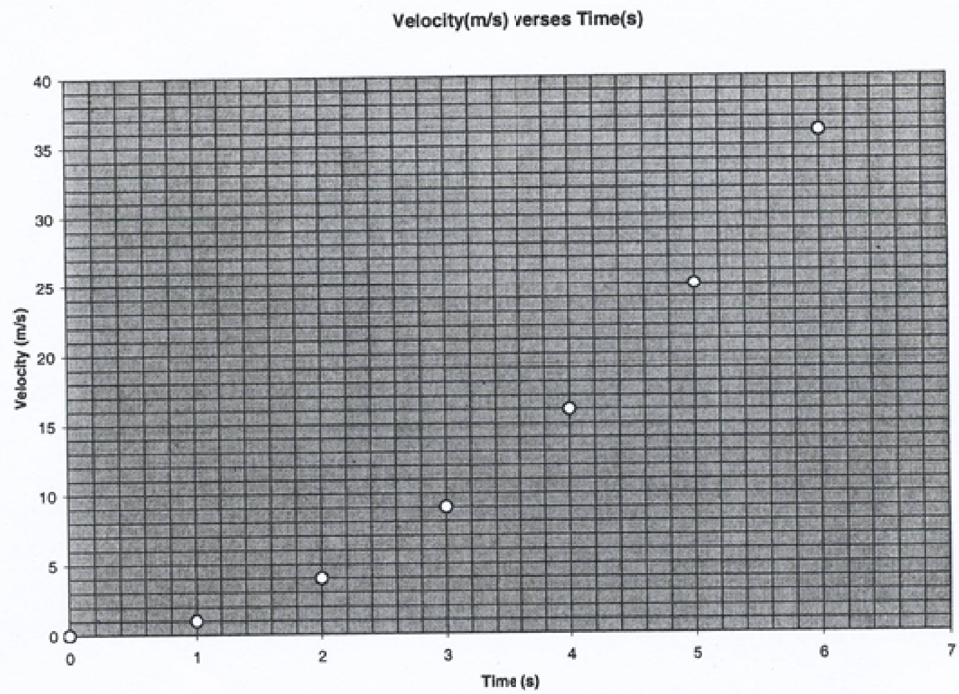
Learning Targets:

1. SWBAT use the Trapezoidal Rule to approximate the value of a definite integral.

2 SWBAT to use the Trapezoidal Rule to answer anumber of different word problems.

3. SWBAT determine when the Trapezoidal Formula can be applied and when it can not.

Assume an object travels with a velocity of $v(t) = t^2$ for the time interval [0,6]. The velocity time graph is shown below.



If we used LRAM to approximate the are it was an underestimate. RRAM resulted in an overestimate. MRAM gave us the best estimate. What if we used trapezoids instead of rectangles?

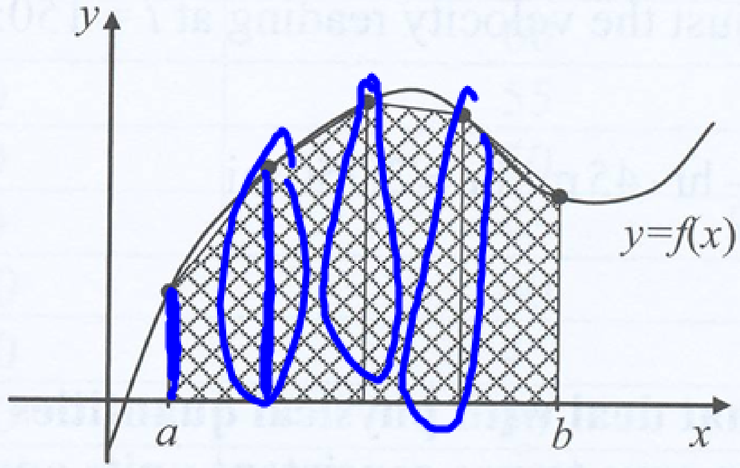
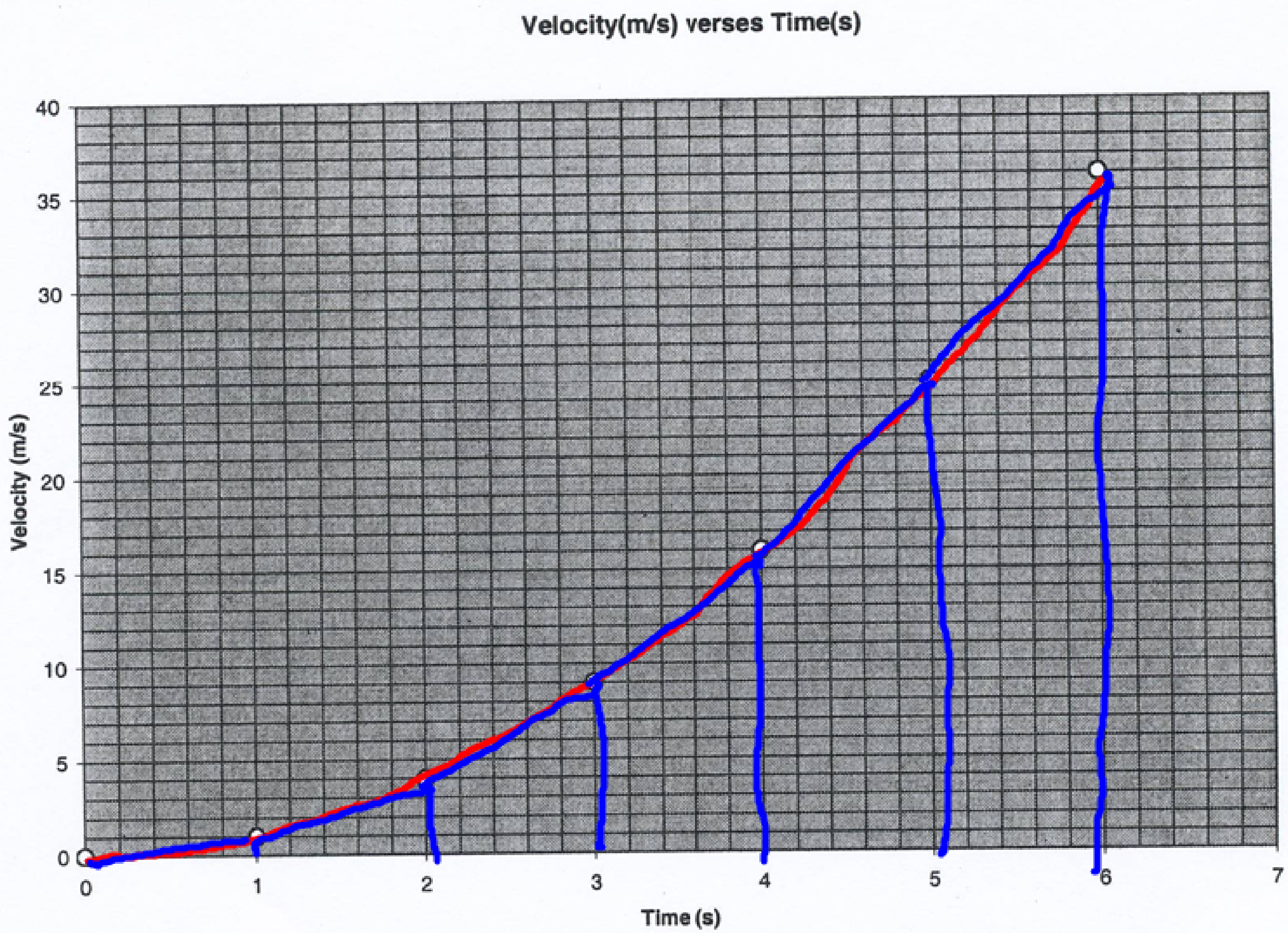


Figure 5-5. Trapezoidal sum approximation of a definite integral

Assume an object travels with a velocity of $v(t) = t^2$ for the time interval [0,6]. The velocity time graph is shown below.



$$\frac{1}{2}b(s+\ell)$$

$$= \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1+4) + \frac{1}{2}(1)(4+9) + \frac{1}{2}(1)(9+16)$$

$$+ \frac{1}{2}(1)(16+25) + \frac{1}{2}(1)(25+36)$$

$$= \frac{1}{2} + \frac{5}{2} + \frac{13}{2} + \frac{25}{2} + \frac{41}{2} + \frac{61}{2} = \frac{146}{2}$$

$$= 73$$

$$\int_0^6 (t^2)\,dt =$$

The Trapezoidal Rule

To approximate $\int_a^b f(x)dx$, use

$$\mathrm{T} = \frac{(b\text{-}a)}{2n}\left(y_o + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right)$$

where a and b are the limits for the integrand and n is the number of intervals.

This formula can only be used if the intervals are of equal width.

Example 1

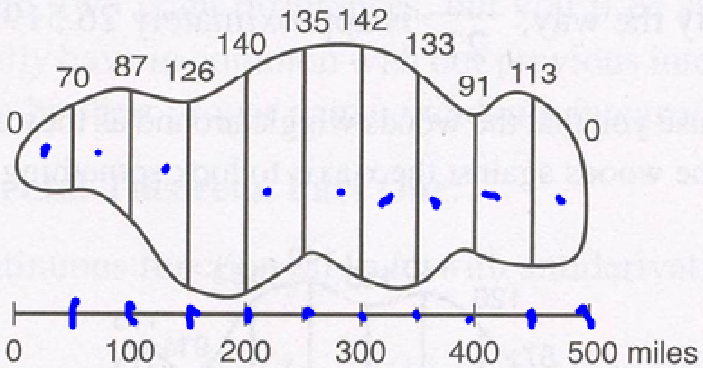
$$T = \frac{b-a}{2n}\left[y_0 + 2y_1 + \cdots\cdots y_n\right]$$

a)

(Based on the Stephen King book *The Girl who Loved Tom Gordon*) Nine-year-old Trisha McFarland is hopelessly lost in the woods, and the efforts of those looking for her have turned up nothing. The graph below shows the woods in which she is lost and measurements of the width of the woods (in miles) at regular intervals.

(a) Use the Trapezoidal Rule to approximate the area of the woods.

$$\frac{500-0}{2(10)}\left[0 + 2(70) + 2(87) + 2(126) + 2(140) + 2(135) + 2(142) + 2(133) + 2(91) + 2(113) + 0\right]$$



(b) Assuming that one person can search 15 mi² in one day, how many people will it take to scour the entire woods in a single day?

$$= 25[0 + 140 + 174 + 252 + 280 + 270 + 284 + 266$$
$$+ 182 + 226 + 0]$$

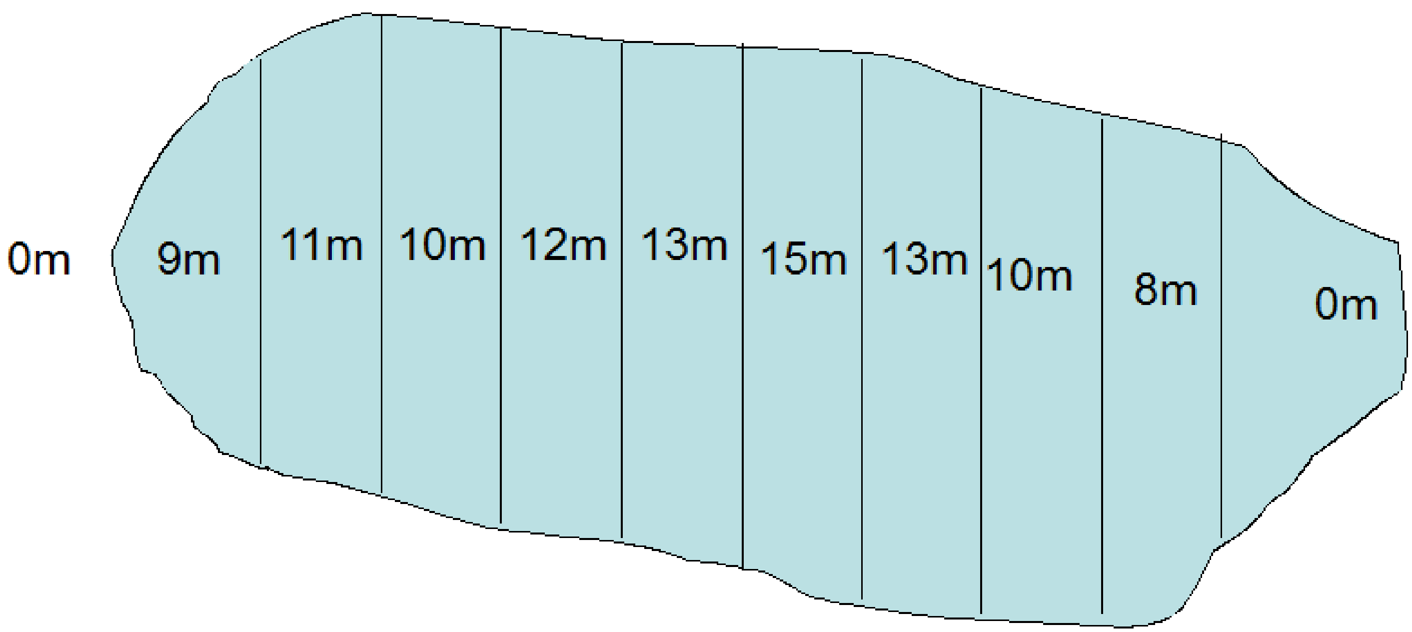
$$= 25[2074]$$

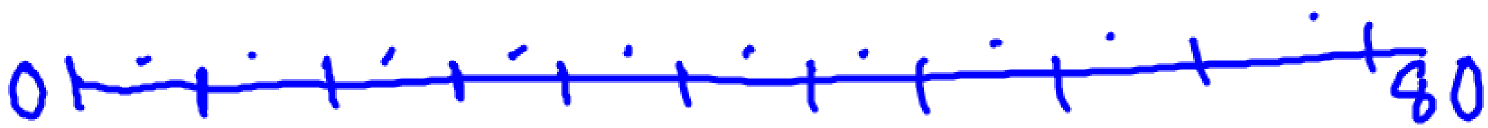
$$= 51\,850 \text{ miles}^2$$

b) $\dfrac{51\,850}{15}$

$$\simeq 3457$$

Example 2: A pond has an average depth of 5m. Measurements of the width of the pond were taken every 8m from left to right. Use the trapezoidal rule to approximate the volume of water in the pond.





$$\text{Volume} = 5\left[\frac{80-0}{2(10)}\left[0+18+22+20+24+26+30+26+20+16+0\right]\right]$$

$$= 5(4)(202)$$

$$= 4040\,m^3$$

Example 3

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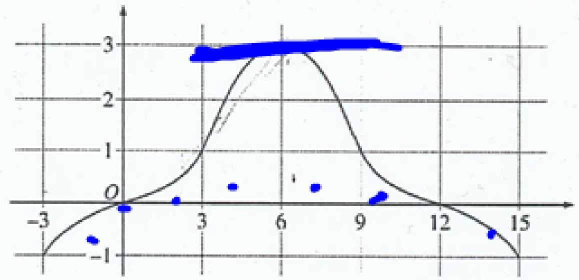
Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let

$g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

g'

$$g(x) = 5 + \int_6^x f(t)\,dt$$

a) $$g(6) = 5 + \int_6^6 f(t)\,dt = 5$$

$g'(6) = f(x) = f(6) = 3$

$g''(6) = f'(6) = 0$

b) $(-3, 0) \cup (12, 15)$ $\quad g' < 0$

c) $(6, 15)$ $\quad g'$ dec

d) $$\int_{-3}^{15} f(t)\,dt = \frac{15-(-3)}{2(6)}\left[-1 + 2(0) + 2(1) + 2(3) + 2(1) + 2(0) + (-1)\right]$$

$$= \frac{3}{2}\left[-\cancel{1} + 2 + 6 + \cancel{2} \cancel{-1}\right]$$

$$= 3/2\,(8) = 12$$

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

2. A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

   (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

   (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

   (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

   (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

a) $E'(6) \simeq \dfrac{E(7)-E(5)}{7-5} = \dfrac{21-13}{2} = \dfrac{2100-1300}{2}$

400 entries/hr

b) $\dfrac{1}{8}\displaystyle\int_0^8 \boxed{E(t)\,dt}$

$= \dfrac{1}{8\,hr}\left[\frac{1}{2}(2)(4) + \frac{1}{2}(3)(4+13) + \frac{1}{2}(2)(13+21) + \frac{1}{2}(1)(21+23)\right]$

entries · hrs

$= \dfrac{1}{8\,\cancel{hr}}\left[4 + \frac{51}{2} + 34 + 22\right] \simeq 10.688$ entries

10.688 is the average # of entries between $t = 0$ hrs to $t = 8$ hrs.

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

   (a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

   (c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

   (d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

   (a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

   (c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

   (d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

$$\int_{20}^{40} v(t)\,dt = \frac{1}{2}(5)(-10-8) + \frac{1}{2}(7)(-8-4) + \frac{1}{2}(8)(-4+7)]$$

$$= -45 - 42 + 12$$

$$= -75 \text{ m.}$$

-75m represents the displacement of the particle from $t = 20$s to $t = 40$s.

Calculus AB **Section I** Part B

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

$$\int_2^8 f(x)\,dx = \frac{1}{2}(3)(40) + \frac{1}{2}(2)(70) + \frac{1}{2}(1)(60)$$
$$= 60 + 70 + 30 = 160$$

Assignment
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#'s 2,7,8,