

5.2 Transformations of Sinusoidal Functions

Vertical
Shift

Recall from Section 5.1

phase

Horizontal
Shift

$$y = A \sin B(x - C) + D$$

A affects amplitude
B affects period

→ could be reflect
in x axis

Today we will examine the
affects of **C** and **D**

Desmos Exploration

$$y = \sin \theta$$

$$y = \sin\left(\theta - \frac{\pi}{2}\right) + 1$$

From our exploring we have found that in the equation
 $y = A \sin B(x - C) + D$:

C is the **phase or horizontal shift**.

D is the **vertical displacement or shift**

Ex. 1 Graph the following:

$$y = 3 \sin 2 \left(x - \frac{\pi}{2} \right) + 1$$

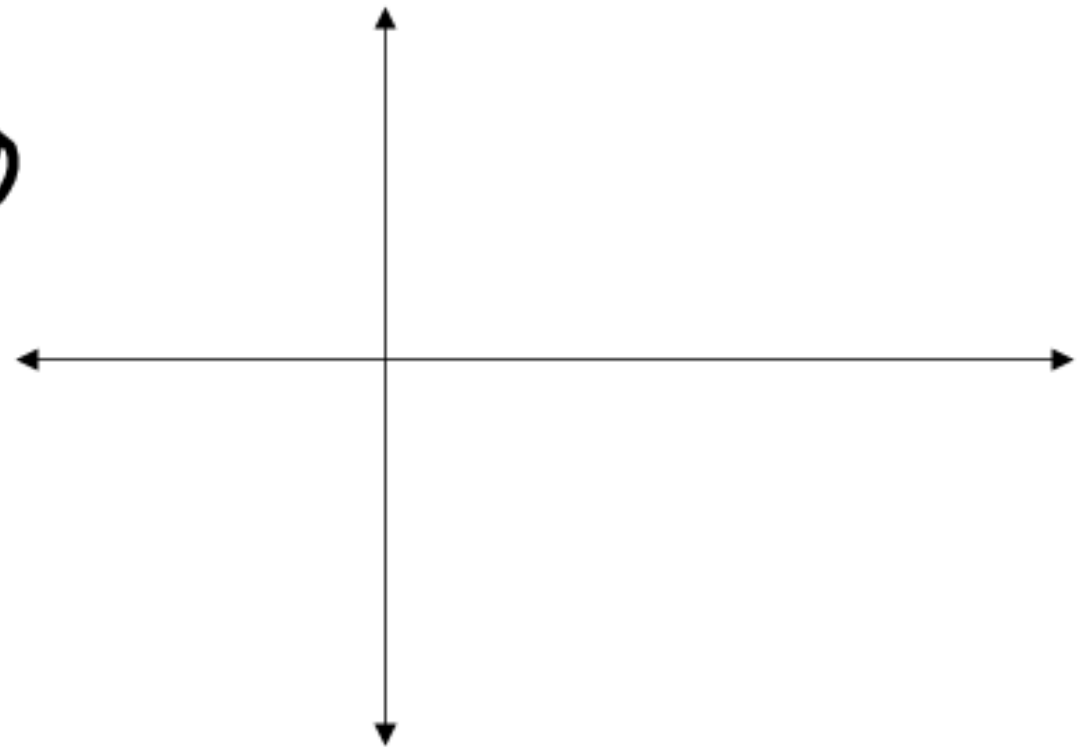
$$\text{amp} = |A|$$
$$= 3$$

②

$$\text{per} = \frac{2\pi}{2} = \pi$$

$$\text{h.s.} = \frac{\pi}{2}$$

$$\text{V.S.} = 1$$



critical #'s

$$\pi \cdot \frac{1}{4} = \left(\frac{\pi}{4}\right)$$

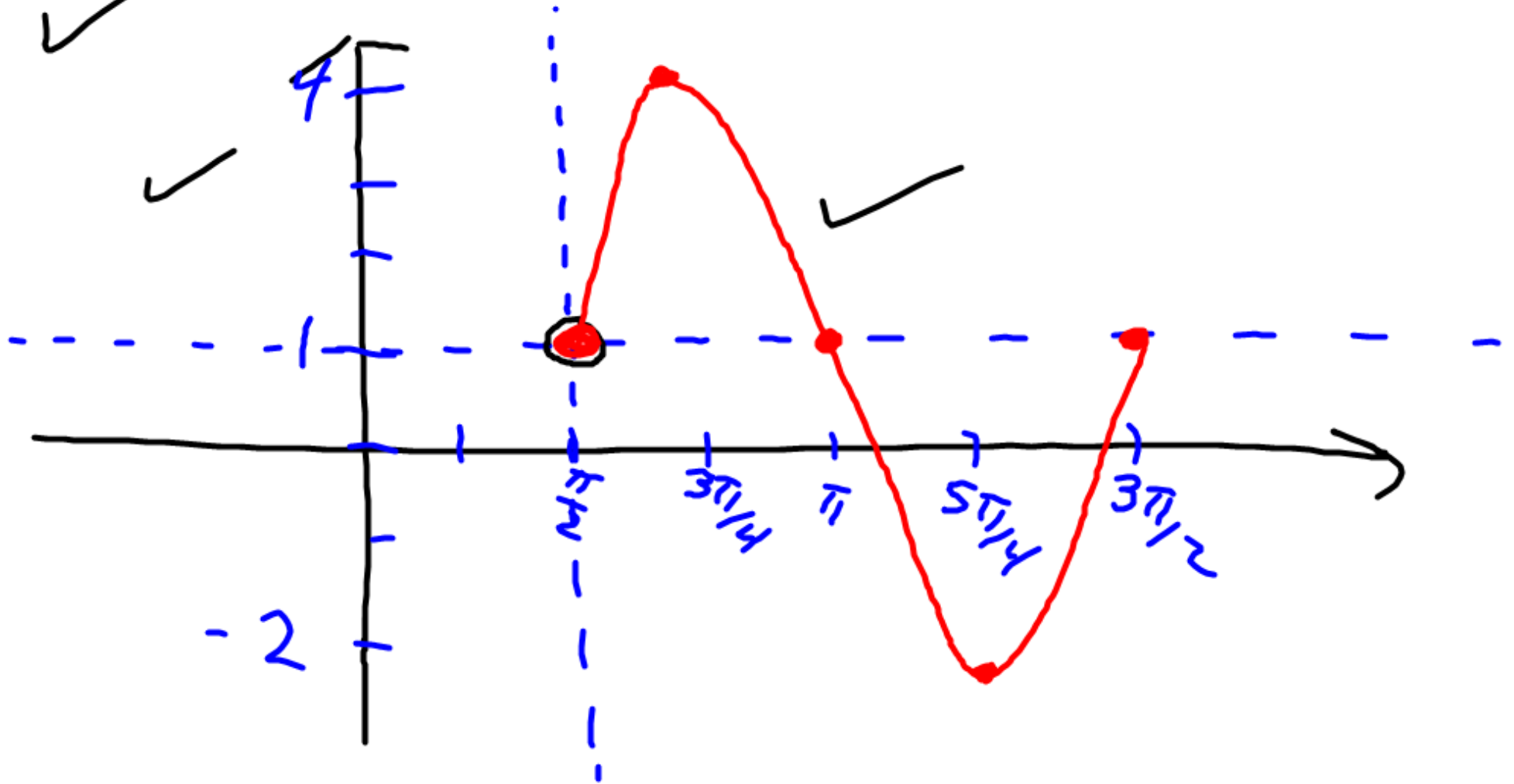
$$\frac{2\pi}{4}$$

$$\frac{3\pi}{4}$$

$$\frac{4\pi}{4}$$

$$\frac{5\pi}{4}$$

$$\frac{6\pi}{4}$$



Ex. 2 Graph the following:

$$y = -2 \cos\left(\frac{1}{2}x + \frac{\pi}{8}\right) - 2$$

$$y = -2 \cos \frac{1}{2} \left(x + \frac{\pi}{4}\right) - 2$$

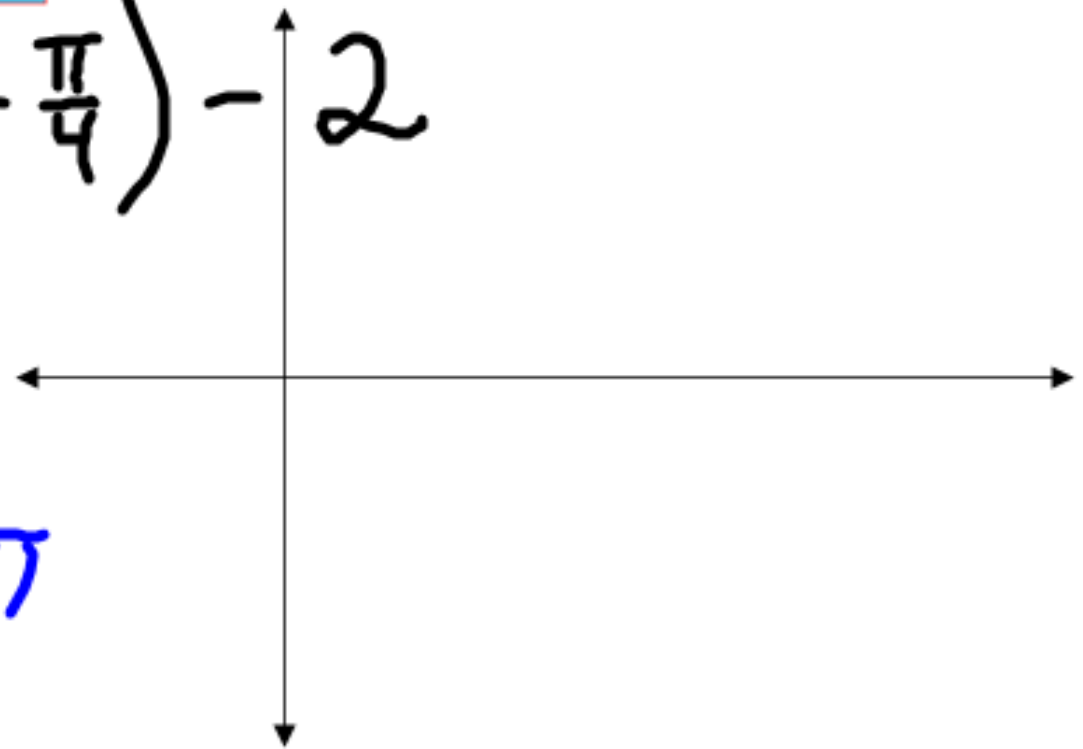
$$\text{amp} = |-2| = 2$$

$$\text{Per} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$$

$$\text{h.s.} = -\pi/4$$

$$\text{v.s.} = -2$$

$$\frac{\pi/8}{2} = \frac{\pi}{4}$$

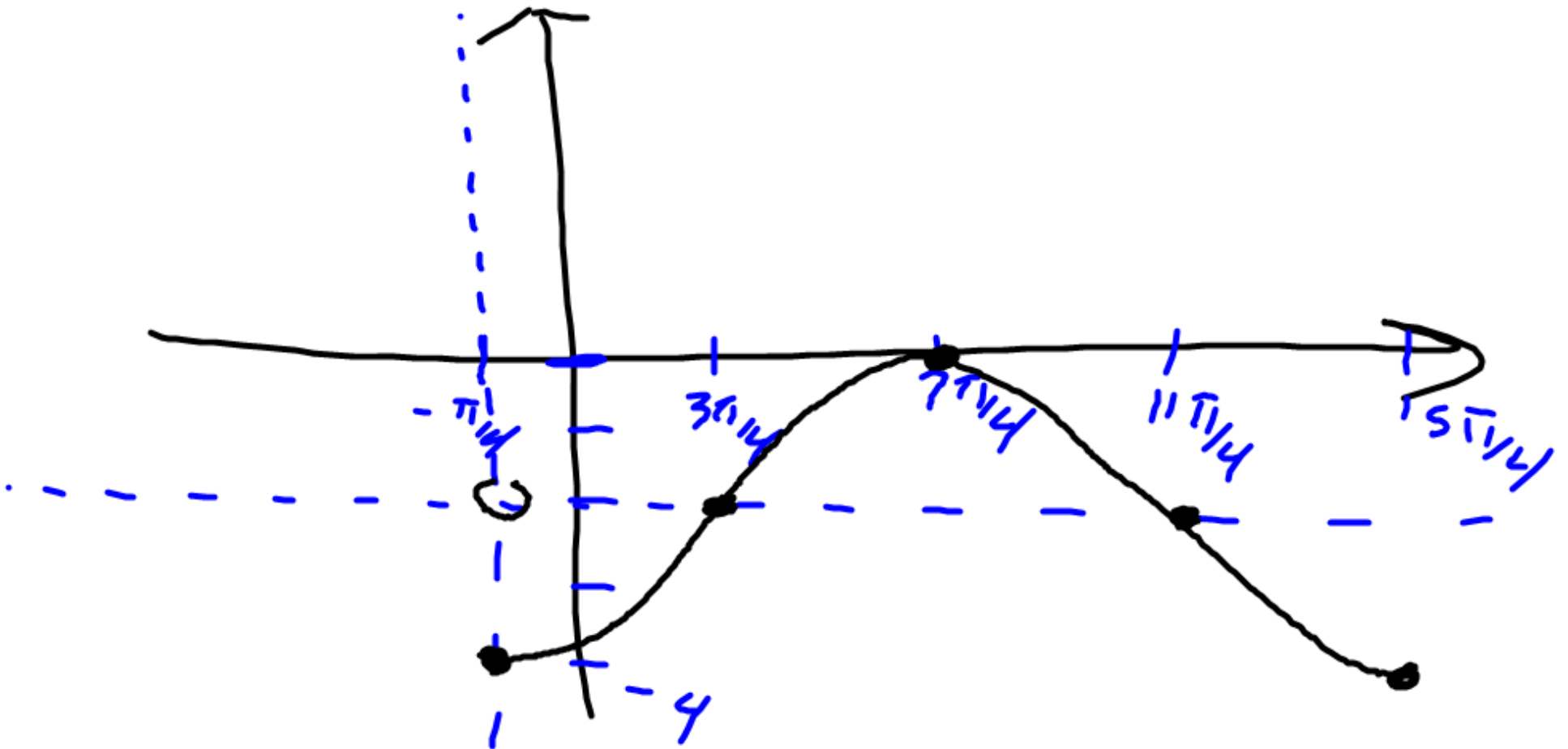


critical #'s

$$4\pi \cdot \frac{1}{4} = \pi$$

$$\frac{4\pi}{4}$$

$-\pi/4$ $3\pi/4$ $7\pi/4$ $11\pi/4$ $15\pi/4$



Ex. 3 Graph the following:

$$y = 1 - 4 \sin\left(2x + \frac{\pi}{3}\right)$$

$$y = -4 \sin 2\left(x + \frac{\pi}{6}\right) + 1$$

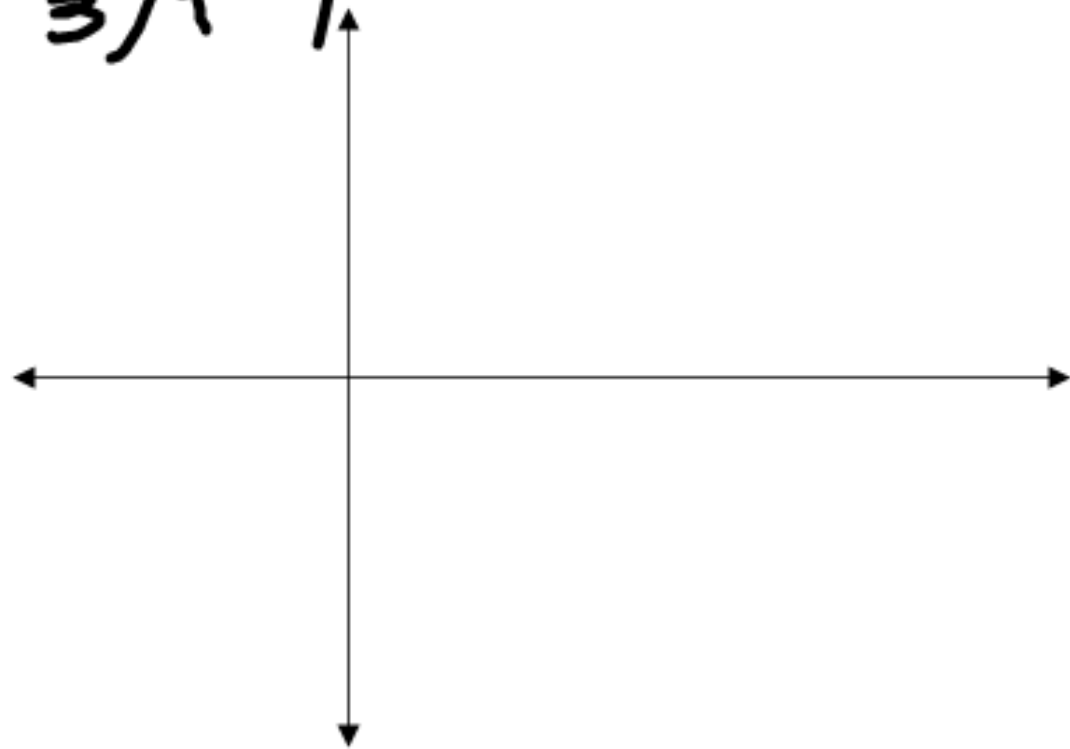
$$y = -4 \sin\left(2x + \frac{\pi}{3}\right) + 1$$

$$\text{amp} = 4$$

$$\text{per} = \frac{2\pi}{2} = \pi$$

$$\text{h.s.} = -\frac{\pi}{6}$$

$$\text{v.s.} = 1$$



critical #'s

$$\pi \cdot \frac{1}{4} = \frac{\pi}{4}$$

$$\frac{-2\pi}{12}$$

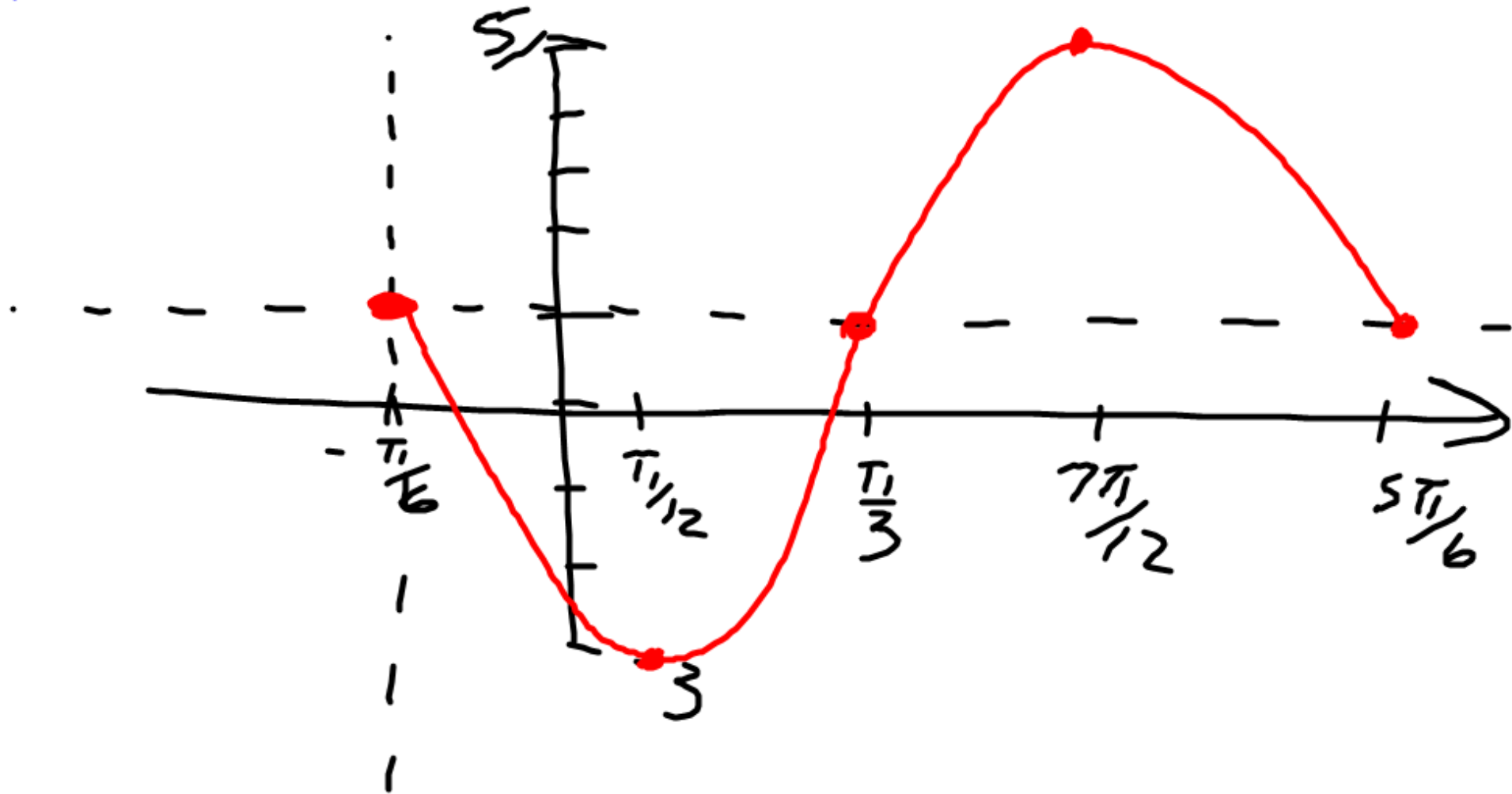
$$\frac{\pi}{12},$$

$$\frac{4\pi}{12}$$

$$\frac{7\pi}{12}$$

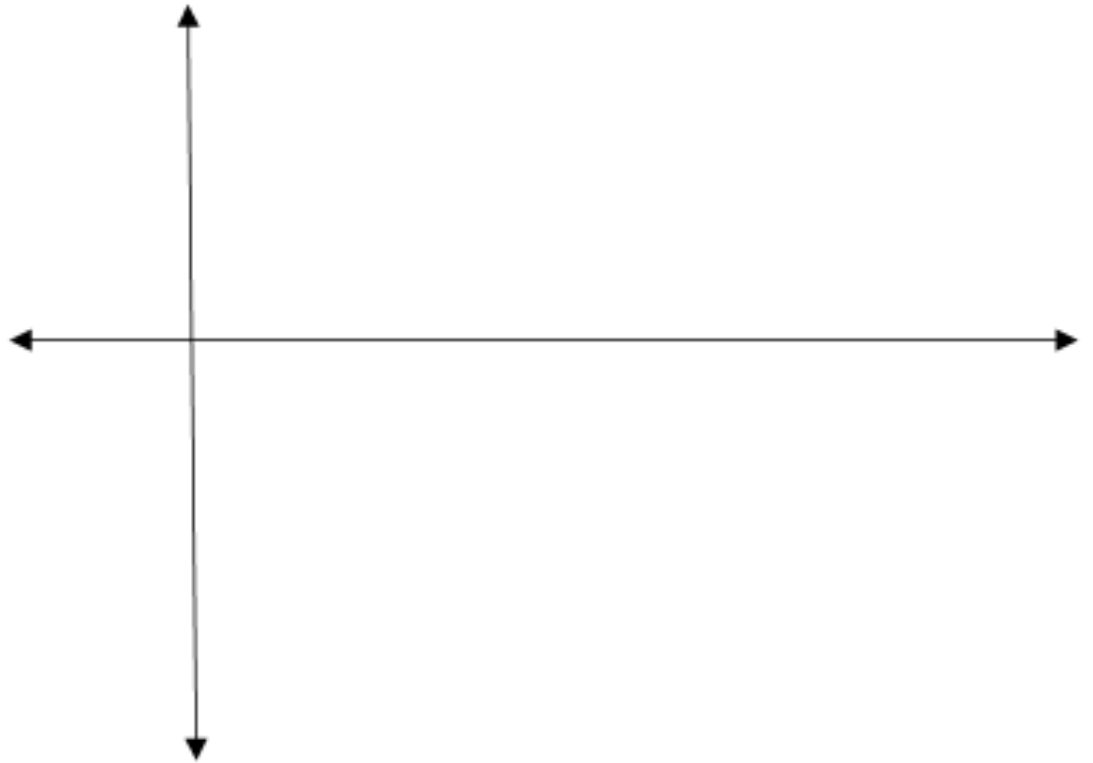
$$\frac{10\pi}{12}$$

$$\frac{3\pi}{12}$$



Your Turn

Sketch the graph of the function $y = 2 \sin \left(\theta - \frac{\pi}{2} \right) + 2$



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1b,c,e, 2b, c,f, 3a 4, 5, 6, 7

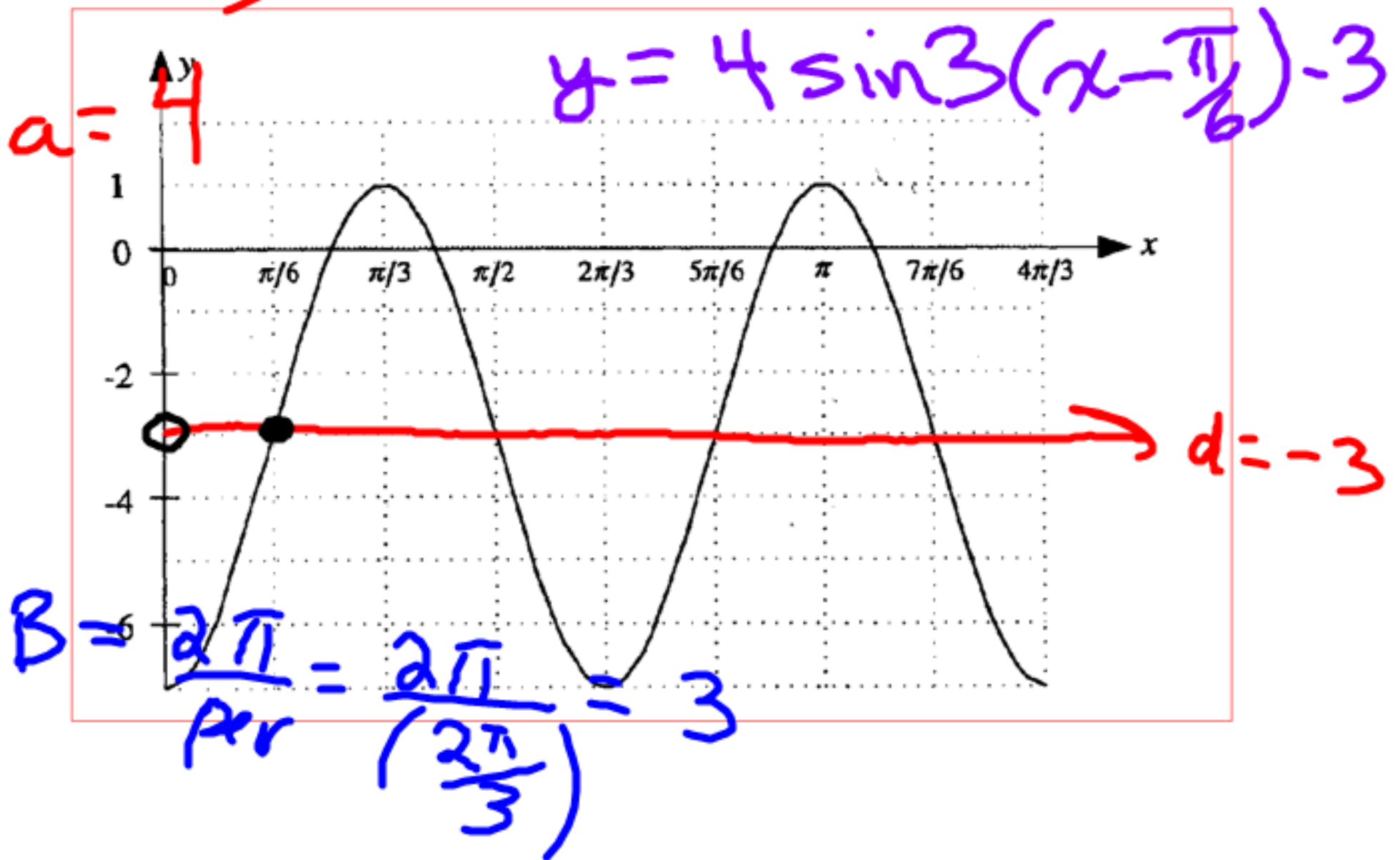
5.2 Transformations of Sinusoidal Functions

Continued.....

In our last lesson we were given the equation of a sinusoidal function and asked to graph it. In today's lesson we will be given the graph of a sinusoidal function and asked to determine the equation that matches the graph.

Ex. 1 Determine the equation of the positive sine graph with a positive phase shift and $a > 0$.

$$y = a \sin b(x - c) + d$$



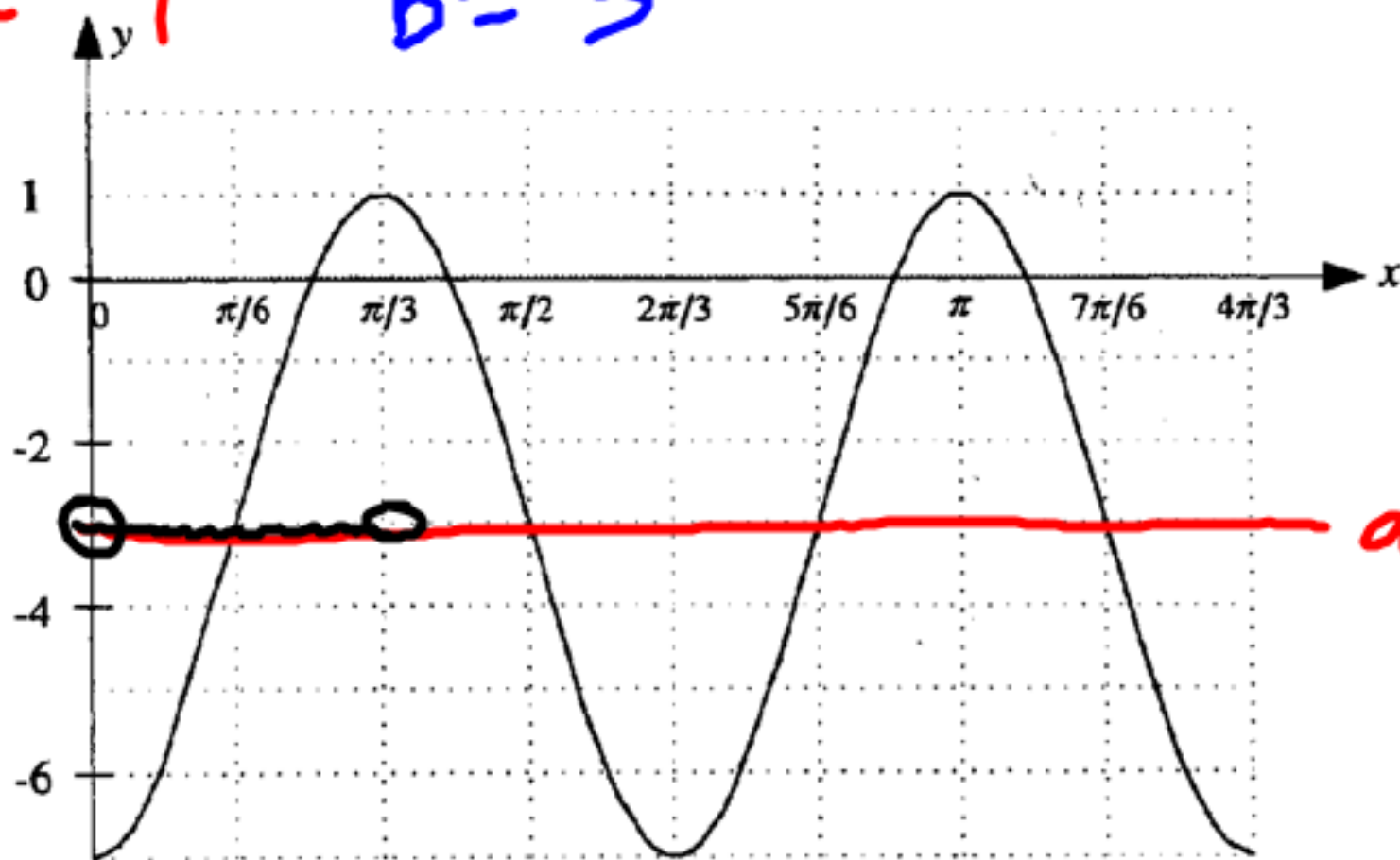
Ex 2 Find the cosine graph of the following assuming $a > 0$ and a positive phase shift.

$$y = a \cos b(x - c) + d$$

$$y = 4 \cos 3\left(x - \frac{\pi}{3}\right) - 3$$

$$a = 4$$

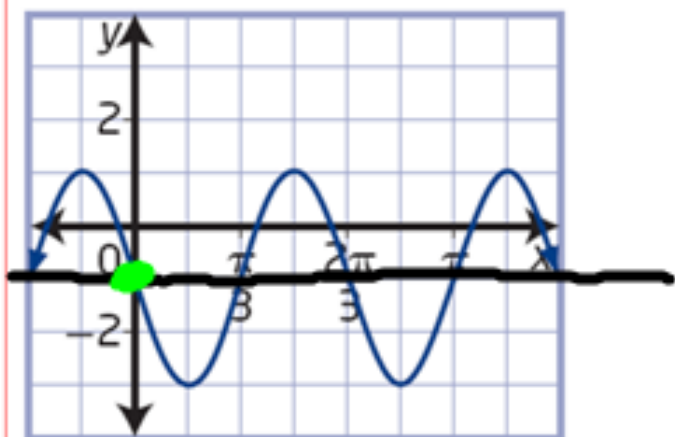
$$b = 3$$



$$d = -3$$

Your Turn

The graph shows the function $y = f(x)$.



positive h-s.

- a) Write the equation of the function in the form $y = a \sin b(x - c) + d$, $a < 0$.
- b) Write the equation of the function in the form $y = a \cos b(x - c) + d$, $a < 0$.

$$y = -2 \sin 3(x) - 1$$

$$y = -2 \cos 3(x - \frac{\pi}{6}) - 1$$

Application of Sinusoidal Functions

$$d) d = 0.6 \cos\left(\frac{2\pi \cdot 7}{13}\right) + 3.7$$

The depth, d , in metres, of the water in the harbour at New Westminster, British Columbia, is approximated by the equation $d(t) = 0.6 \cos \frac{2\pi}{13}t + 3.7$, where t is the time, in hours, after the first high tide.

- a) Graph the function for two cycles starting at $t = 0$.
- b) What is the period of the tide? 13 h
- c) If a boat requires a minimum of 3.5 m of water to launch safely, for how many hours per cycle can the boat safely launch?
- d) What is the depth of the water at 7 h? At what other times is the water level at this depth? Explain your solution.

$$= 3.12 \text{ m}$$

$$\text{amp} = .6$$

$$\text{V.S.} = 3.7$$

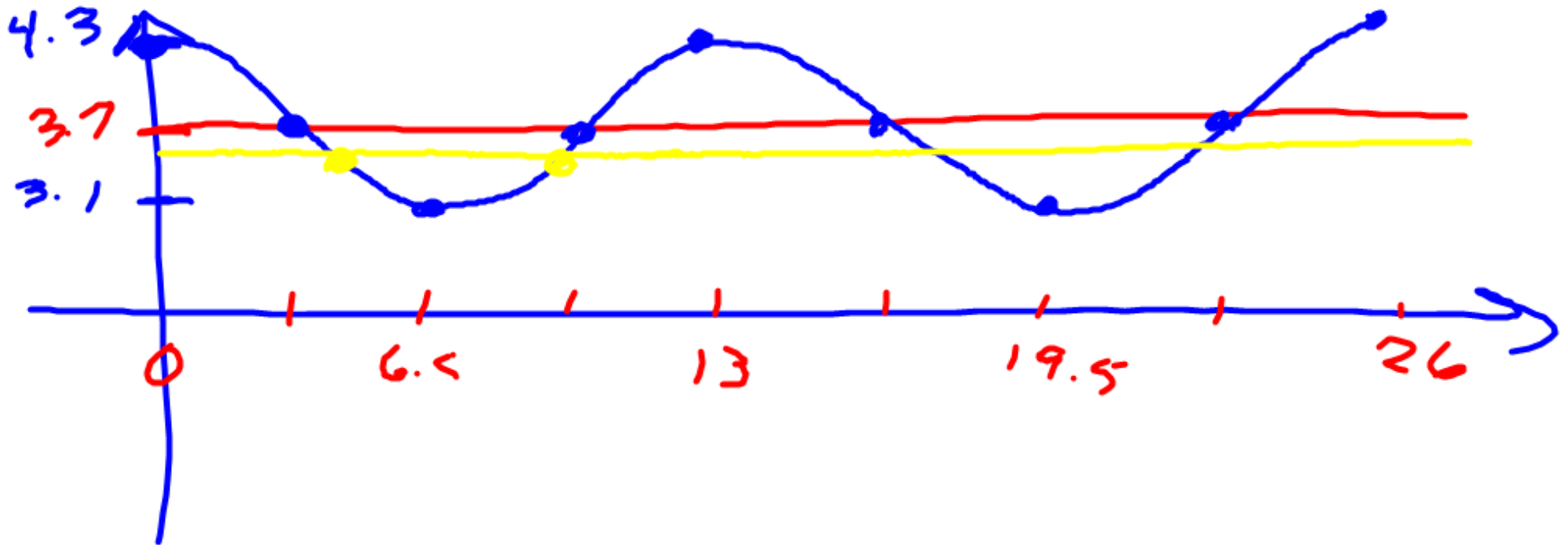
$$\text{per} = \frac{2\pi}{\left(\frac{2\pi}{13}\right)} = 13$$

critical #'s

$$\frac{13}{4} = 3.25$$

0, 3.25, 6.5, 9.75, 13, 16.25, 19.5

22.75, 26

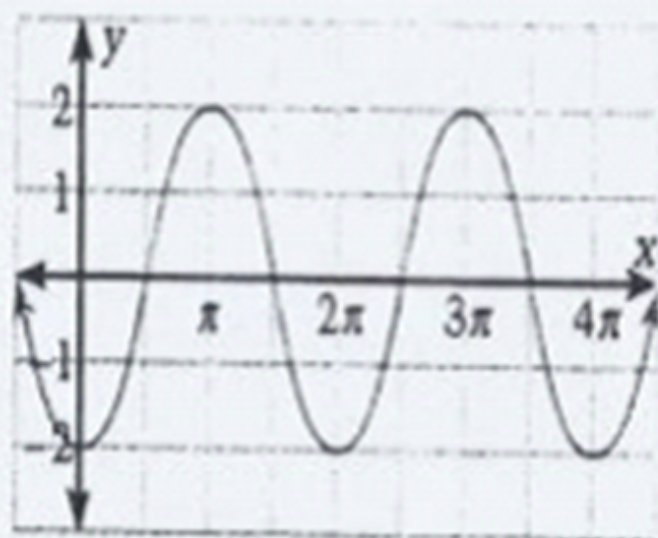


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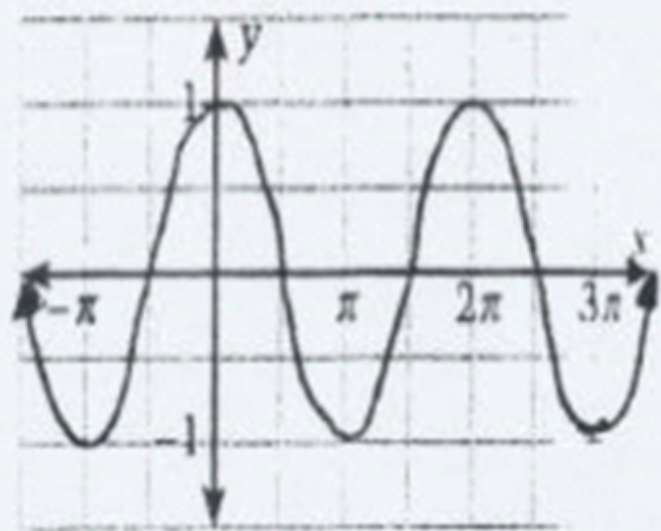
#'s 15, 16, 20

***For #20 need to know 5280 feet in a mile.

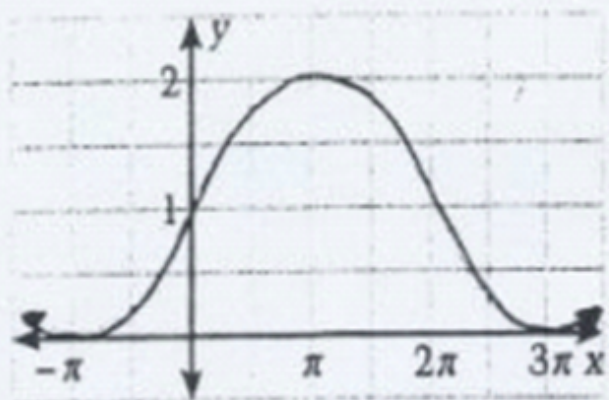
2.



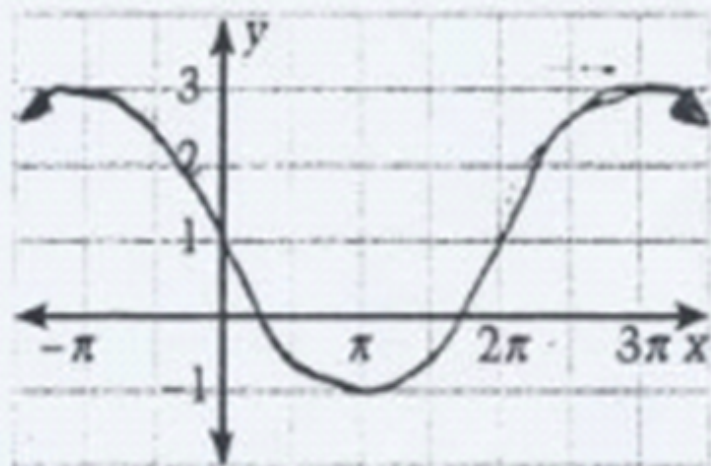
1.



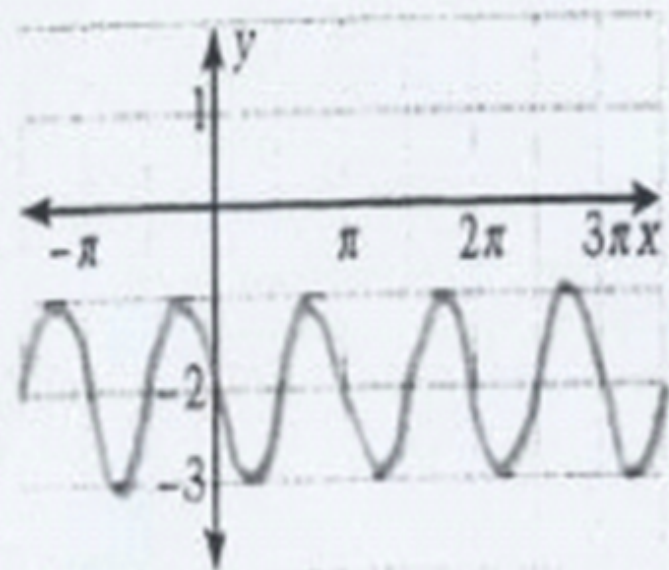
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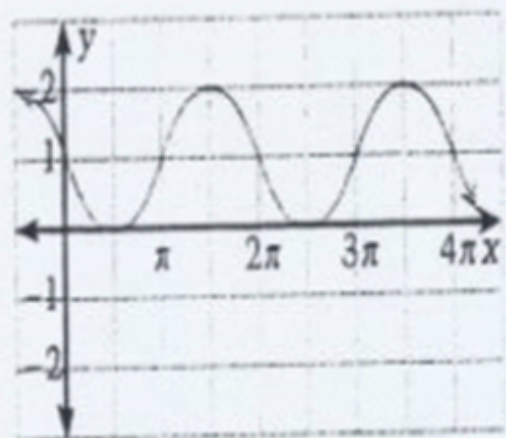
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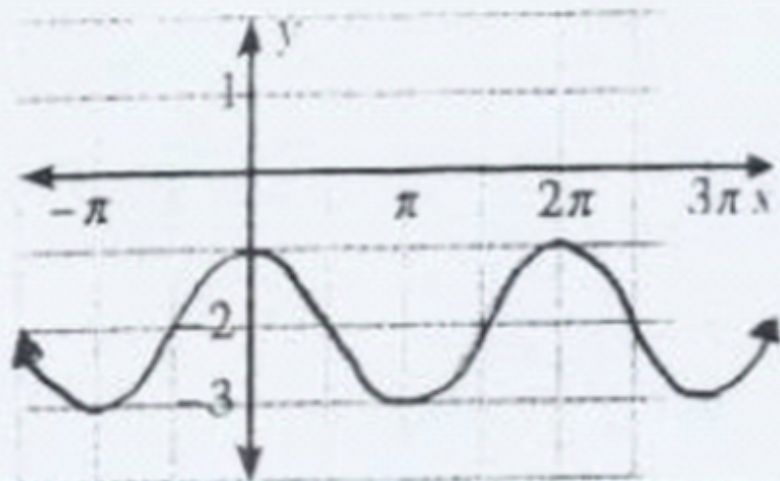
5.



6.



7.



8.

