

# **Unit #5 Graphical Applications Of the Derivative**

## **5.1 Higher Order Derivatives**

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### Learning Targets:

SWBAT take both the first and second derivative of a number of different functions and relations.



In this chapter we will see how **derivatives** will help us in **drawing the graph** of a function.

If  $f(x)$  is a differentiable function, then  $f'(x)$  is known as its first derivative. If  $f'(x)$  is differentiable, then its derivative, the second derivative, is known as  $f''(x)$ .

$$f'(x) = \frac{dy}{dx}$$

$$f''(x) = \frac{d^2 y}{dx^2}$$

Example1: Find the first two derivatives of the following:

$$f(x) = x^6 + 5x^4 - 3x^3 + x$$

$$f'(x) = 6x^5 + 20x^3 - 9x^2 + 1$$

$$f''(x) = 30x^4 + 60x^2 - 18x$$

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Example 2: Find  $\frac{d^2 y}{dx^2}$  for each of the following:

$$\text{a) } y = \frac{2x+1}{x-1}$$

$$y' = \frac{(x-1)(2) - (2x+1)(1)}{(x-1)^2}$$

$$y' = \frac{-3}{(x-1)^2} = -3(x-1)^{-2}$$

$$y'' = 6(x-1)^{-3} = \frac{6}{(x-1)^3}$$

$$\text{b) } y = x^2 \sqrt{x-1}$$

$$y' = \frac{x^2}{1} \left( \frac{1}{2} (x-1)^{-1/2} \right) + (x-1)^{1/2} \cdot 2x$$

$$y' = \frac{x^2}{2(x-1)^{1/2}} + 2x(x-1)^{1/2} \frac{2(x-1)^{1/2}}{2(x-1)^{1/2}}$$

$$y' = \frac{x^2}{2(x-1)^{1/2}} + \frac{4x(x-1)}{2(x-1)^{1/2}}$$

$$y' = \frac{5x^2 - 4x}{2(x-1)^{1/2}}$$

$$y'' = \frac{2(x-1)^{1/2} \underbrace{(10x-4)} - (5x^2-4x)(x-1)^{-1/2}}{4(x-1)}$$

$$y'' = \frac{4(x-1)^{1/2} (5x-2) - \frac{5x^2-4x}{(x-1)^{1/2}}}{4(x-1)}$$

$$y'' = \frac{4(x-1)(5x-2) - (5x^2-4x)}{(x-1)^{3/2}}$$

$$y'' = \frac{20x^2 - 28x + 8 - 5x^2 + 4x}{4(x-1)^{3/2}}$$

$$y'' = \frac{15x^2 - 24x + 8}{4(x-1)^{3/2}}$$



$$\text{c) } x^2 - y^2 = 4$$

$$2x - 2y \frac{dy}{dx} = 0$$

$$\frac{2x}{2y} = \cancel{2y} \frac{dy}{dx}$$

$$\frac{x}{y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2 y}{dx^2} = \frac{y(1) - x \left( \frac{dy}{dx} \right)}{y^2}$$

$$= \frac{y - x \left( \frac{x}{y} \right)}{y^2} = \frac{y - \frac{x^2}{y}}{y^2} = \frac{y^2 - x^2}{y^3}$$

$$\frac{d^2 y}{dx^2} = \frac{y^2 - x^2}{y^3} = -\frac{(x^2 - y^2)}{y^3}$$

$$= \frac{-4}{y^3}$$

Assignment

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