Unit #4(Ch.5) The Definite Integral

5.1 Estimating With Finite Sums
LRAM,MRAM,RRAM

Riemann Sums

5.1 Estimating Finite Sums

Learning Targets:

1. SWBAT find the approximate area under a curve using a Left Hand Riemann Sum.

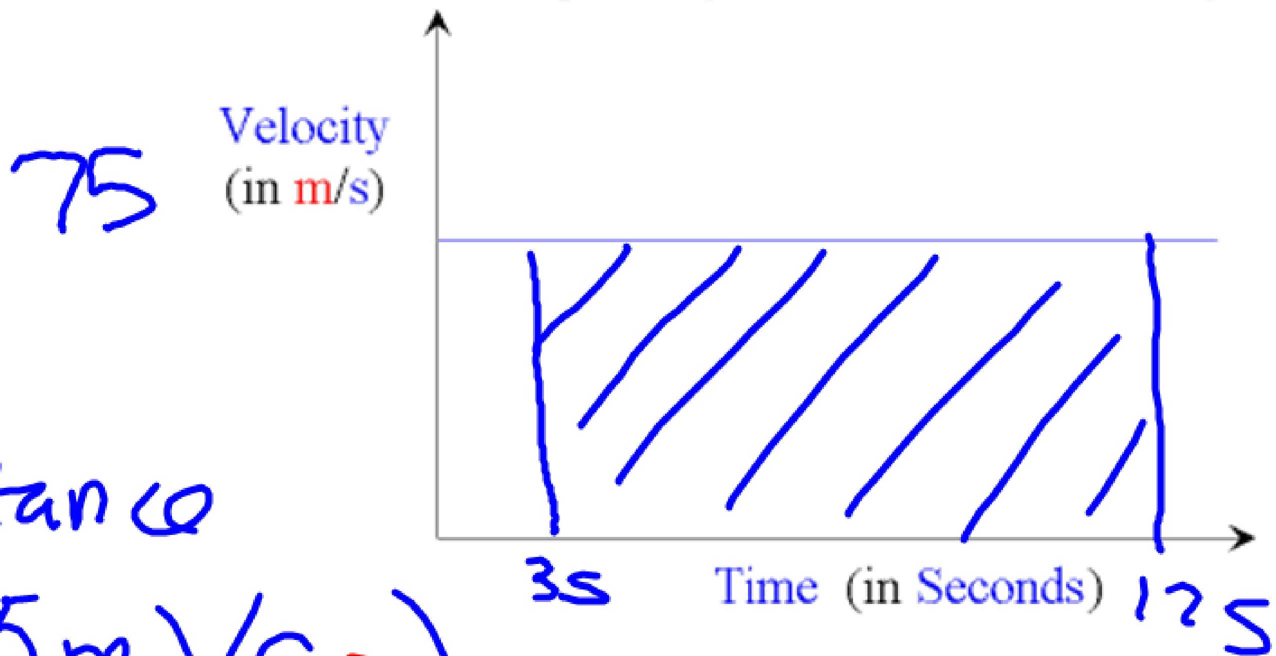
2 SWBAT find the approximate area under a curve using a Right Hand Riemann Sum.

3. SWBAT find the approximate area under a curve using a Mid-Point Riemann Sum.

University of Houston Intro to Area Under Curve

A car travels with a constant velocity of 75 m/s from 3s to 12s. What is the total distance travelled by the car over this time interval?

Velocity - Time graph
showing an object with constant velocity.

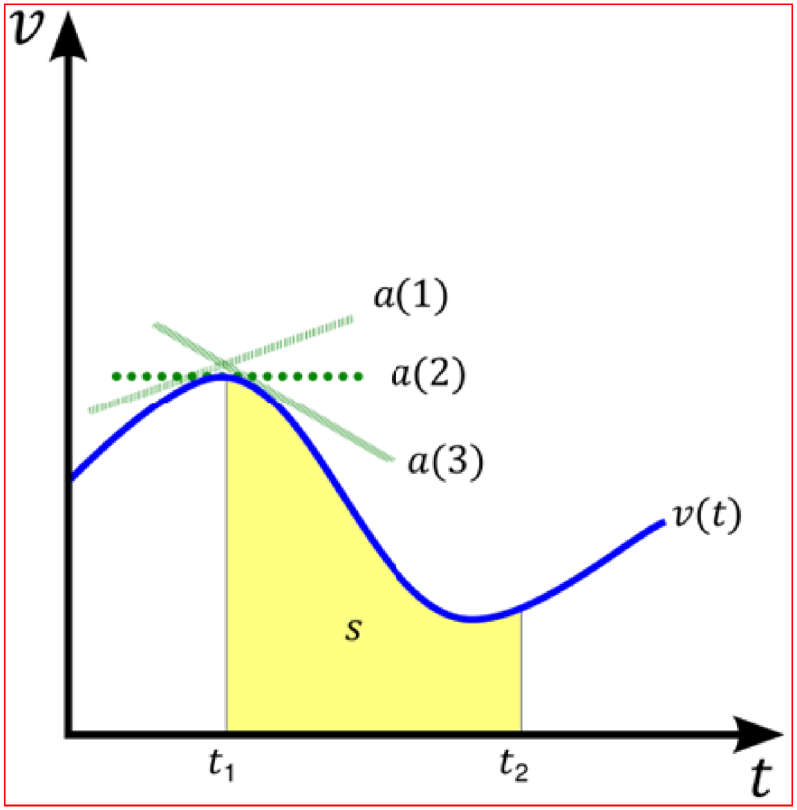


Distance

$$= \left(75 \frac{m}{\cancel{s}}\right)(9\,\cancel{s})$$

$$= 675\,m$$

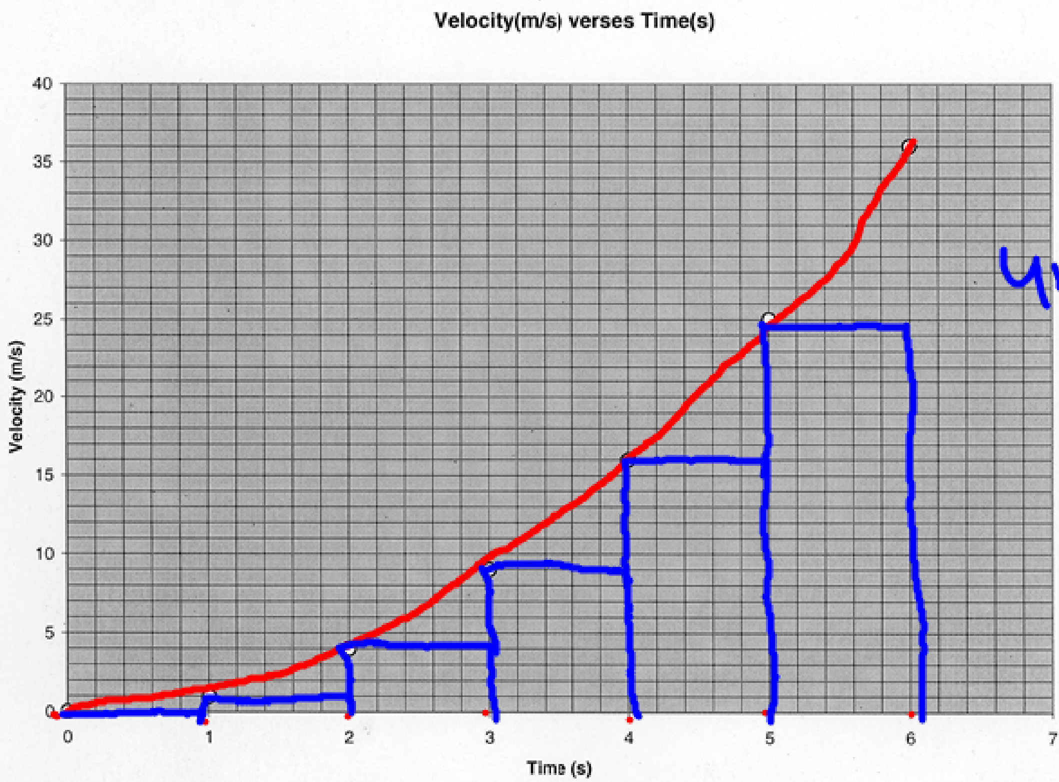
How would we calculate distance if the velocity of the car is not constant?



We will use the process of **Rectangle Approximation Method (RAM)** to estimate the area under the curve.

LRAM

Assume an object travels with a velocity of $v(t) = t^2$ for the time interval [0,6]. The velocity time graph is shown below.



under approximation

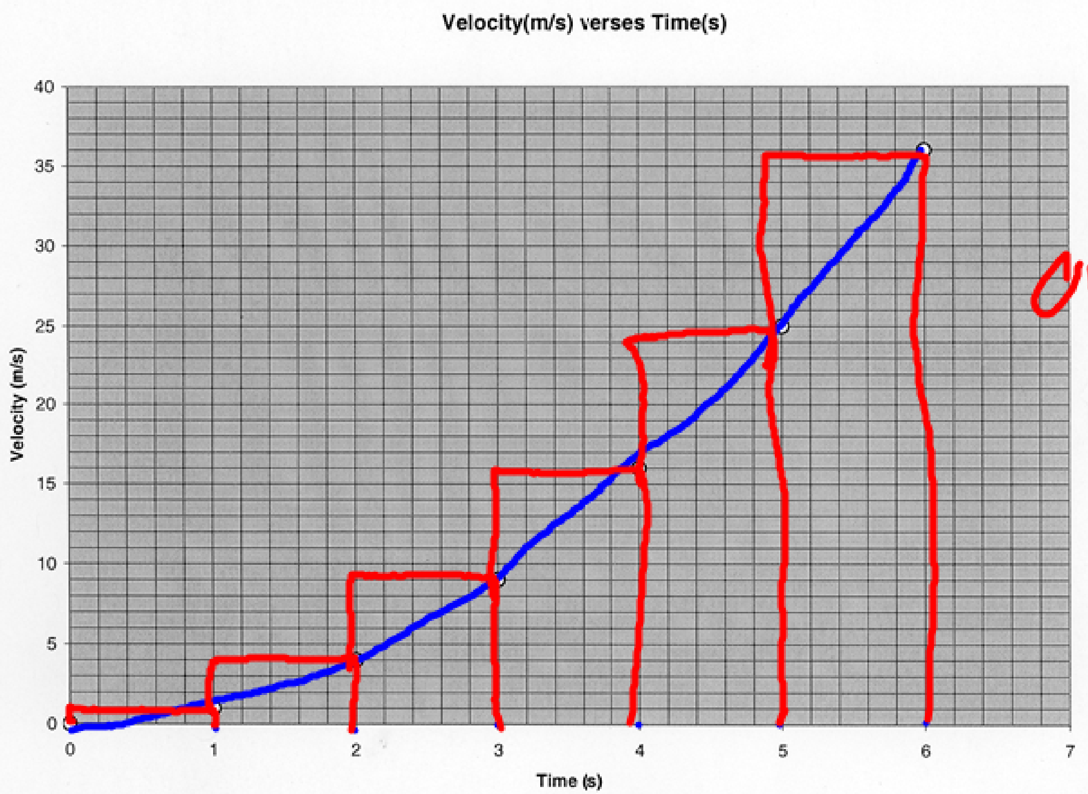
Use LRAM to approximate the area under the curve which will approximate the distance traveled in the first 6 seconds.

$$\int_0^6 v(t)\,dt = 1\text{s}\left(0\tfrac{m}{s} + 1\tfrac{m}{s} + 4\tfrac{m}{s} + 9\tfrac{m}{s} + 16\tfrac{m}{s} + 25\tfrac{m}{s}\right)$$

$$= 55\,\text{m}$$

RRAM

Assume an object travels with a velocity of $v(t) = t^2$ for the time interval [0,6]. The velocity time graph is shown below.



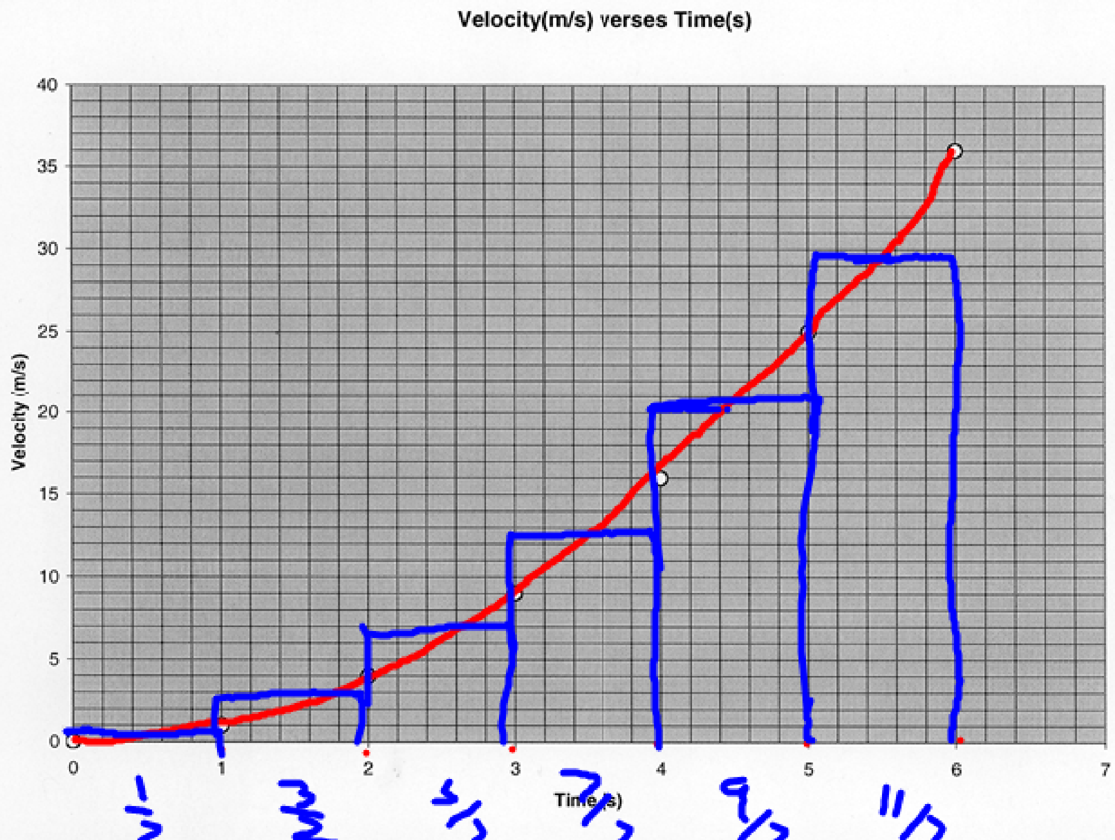
overapproximation

Use RRAM to approximate the area under the curve which will approximate the distance traveled in the first 6 seconds.

$$\int_0^6 v(t)\,dt = 1s\left(1\tfrac{m}{s} + 4m/s + 9m/s + 16m/s + 25m/s + 36m/s\right)$$
$$= 91m.$$

MRAM

Assume an object travels with a velocity of $v(t) = t^2$ for the time interval [0,6]. The velocity time graph is shown below.



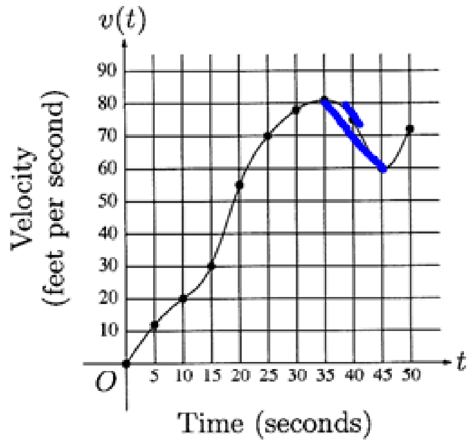
Use MRAM to approximate the area under the curve which will approximate the distance traveled in the first 6 seconds.

$$\int_0^6 v(t)\,dt = 1s\left(\frac{1}{4}m/s + \frac{9}{4}m/s + \frac{25}{4}m/s + \frac{49}{4}m/s + \frac{81}{4}m/s + \frac{121}{4}m/s\right)$$

$$= 71.5m$$

AP Exam Free Response Question

1998 Calculus AB Scoring Guidelines



(10s)

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

a) $(0, 35) \cup (45, 50)$ that is where $V(t)$ increasing.

b)

$$\text{ave accel} = \frac{V(50) - V(0)}{50 - 0}$$

$$= \frac{72\,ft/s - 0\,ft/s}{50s}$$

$$= \frac{36}{25}\,ft/s^2$$

c) $$\simeq a(40) = \frac{V(45) - V(35)}{45 - 35} = \frac{60\,ft/s - 81\,ft/s}{10s}$$

$$= \frac{-21}{10}\,ft/s^2$$

a) $\int_0^{50} v(t)\,dt =$

$= (10\,s)(12\,ft/s + 30\,ft/s + 70\,ft/s + 81\,ft/s + 60\,ft/s)$

$= 2530\,ft$

AP Exam Free Response Question part (a)

AB–3 / BC–3 **1999**

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24–hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24–hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

a) $\int_0^{24} R(t)\,dt = (6h)(10.4\,g/h + 11.2\,g/h + 11.3\,g/h + 10.2\,g/h)$

$= 258.6$ gallons

Amount of H_2O that flowed through the pipe from $t = 0h$ to $t = 24h$.

b)

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

a) $\simeq C'(3.5) = \dfrac{C(4) - C(3)}{4-3}$

$$= \frac{(12.8 - 11.2)\,\text{oz}}{1\ \text{min}}$$

$$= 1.6\ \text{oz/min}$$

b) $\dfrac{C(4)-C(2)}{4-2}$

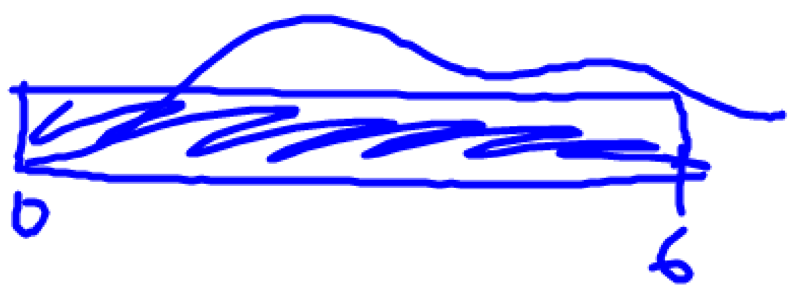
$= \dfrac{12.8-8.8}{2} = \dfrac{4}{2} = 2$

since $C(t)$ is diff. and closed by the MVT there must be some time t $2 < t < 4$ such that $c'(t) = 2$.

c) $\frac{1}{6}\int_0^6 C(t)\,dt$

$$= \frac{1}{6\text{min}}\left[(2\text{min})(5.3 + 11.2 + 13.8 \text{ oz})\right]$$

$$= 10.1 \text{ oz}$$



t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

1. Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$ hours, as shown in the table above. For $0 < t \le 2.4$, $v(t) > 0$.

   (a) Use the data in the table to approximate Ruth's acceleration at time $t = 1.4$ hours. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate $\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

$(L(40))^2 = 225$

$L(40) = 15$

AP® CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 5

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

(a) Use the data in the table to approximate Ben's acceleration at time $t = 5$ seconds. Indicate units of measure.

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

(c) For $40 \le t \le 60$, must there be a time t when Ben's velocity is 2 meters per second? Justify your answer.

(d) A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

$(L(40))^2 = 144 + (B(40))^2$
$= 144 + 81$

a) $\simeq a(5) = \dfrac{V(10)-V(0)}{10-0}$

$= \dfrac{2.3-2.0}{10} \dfrac{m/s}{s}$

$= 0.03\, m/s^2$

b) $\int_0^{60} |V(t)|\,dt$ Distance travelled in the time interval 0 s to 60 s.

LRAM

$$\int_0^{60} |v(t)|\, dt$$

$$= (10\,s)(2.0\,m/s) + (30\,s)(2.3\,m/s)$$

$$+ (20\,s)(2.5\,m/s)$$

$$= 20\,m + 69\,m + 50\,m = 139\,m$$

c) ave vel = $\dfrac{S(60) - S(40)}{60 - 40}$

$$= \frac{49 - 9}{20} = 2\,m/s$$

Since function is diff, $\therefore$ cont. and it is closed MVT applies $\therefore$ there is time between $40 < t < 60$ such that velocity $= 2\,m/s$.

d) $(L(t))^2 = (12)^2 + (B(t))^2$

$$2L(t)\frac{dL}{dt} = 2B(t)\frac{dB}{dt}$$

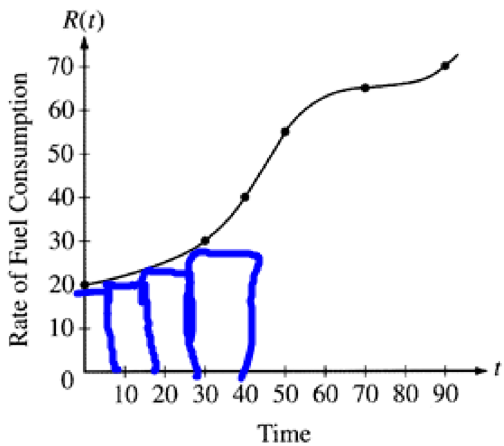
$$2(L(40))\frac{dL}{dt} = 2B(40)\left.\frac{dB}{dt}\right|_{40}$$

$$2(5)\frac{dL}{dt} = 2(9)(2.5)$$

$$\frac{dL}{dt} = \frac{45}{30} \text{ m/min}$$

AP Exam Free Response Question part (c)

2003 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

   (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   (c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   (d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

c) $\displaystyle\int_0^{90} R(t)\,dt$

$= (30\text{ min})(20\text{ g/min}) + (10\text{ min})(30\text{ g/min})$
$+ (10\text{ m})(40\text{ g/min}) + (20\text{ min})(55\text{ g/min})$
$+ (20\text{ min})(65\text{ g/min})$

$= 1540$ gallons

a) $R'(45) = \dfrac{R(50) - R(40)}{50 - 40}$

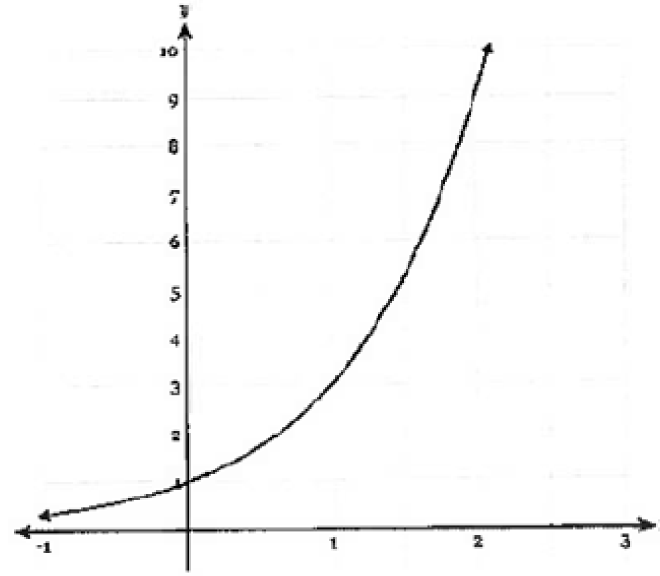
$= \dfrac{55 - 40}{10} = \dfrac{3}{2}$ gallon/min^2

b)

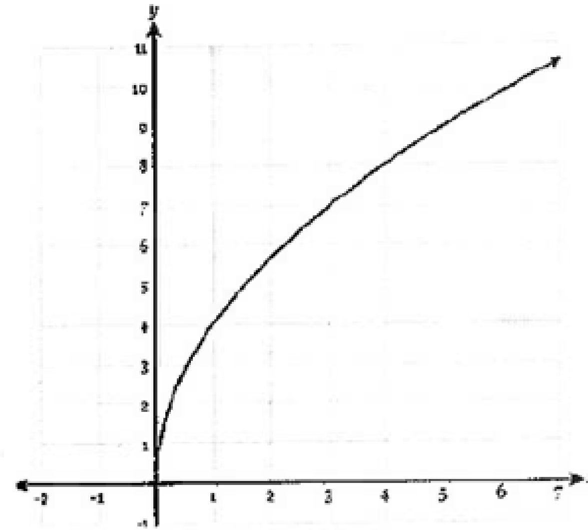
Assignment Handout

1. Let f be the function graphed below. If four subintervals of equal length are used, draw rectangles whose area represents a midpoint Riemann sum approximation of $\int_0^2 f(x)\,dx$.

Approximate the value of the integral using the midpoint Riemann sum.

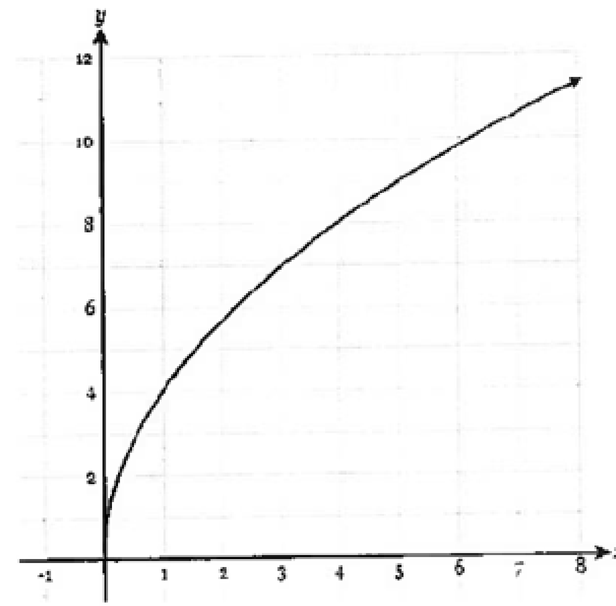


2. Let f be the function graphed below. If four subintervals of equal length are used, draw rectangles whose area represents a right Riemann sum approximation of $\int_2^6 f(x)\,dx$.



Approximate the value of the integral using the right Riemann sum.

3. Let f be the function graphed below. If four subintervals of equal length are used, draw rectangles whose area represents a left Riemann sum approximation of $\int_3^7 f(x)\,dx$.

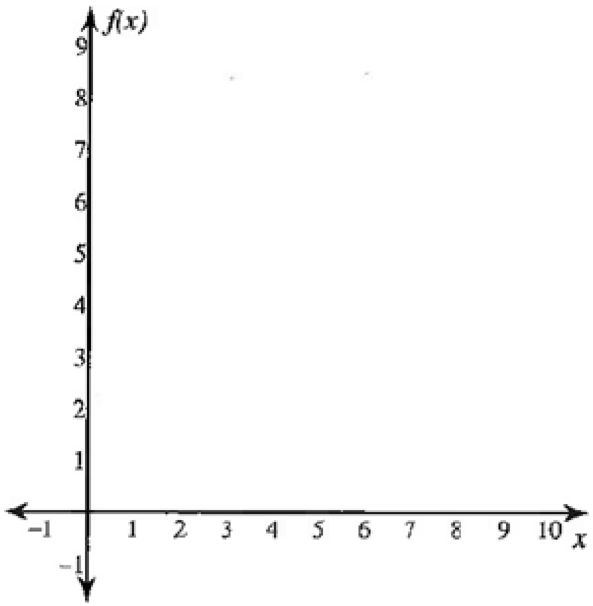


Approximate the value of the integral using the left Riemann sum.

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table. You may use the provided graph to sketch the function data and Riemann sums.

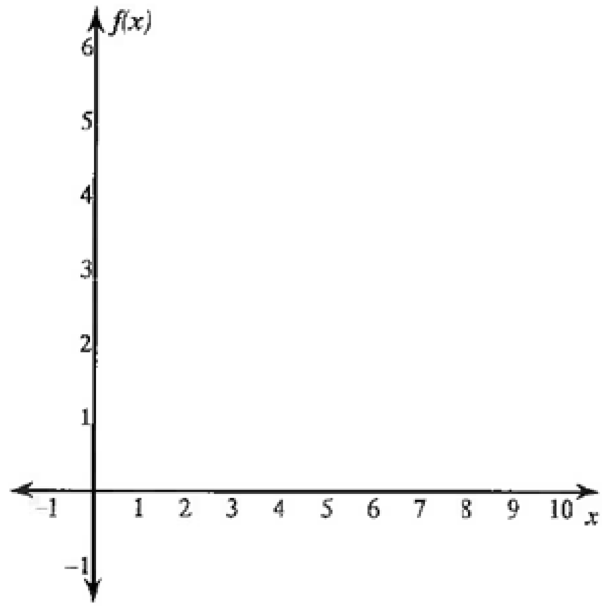
1) $\int_0^{10} f(x)\,dx$

x	0	2	5	7	10
$f(x)$	2	3	5	7	8



2) $\int_0^{10} f(x)\,dx$

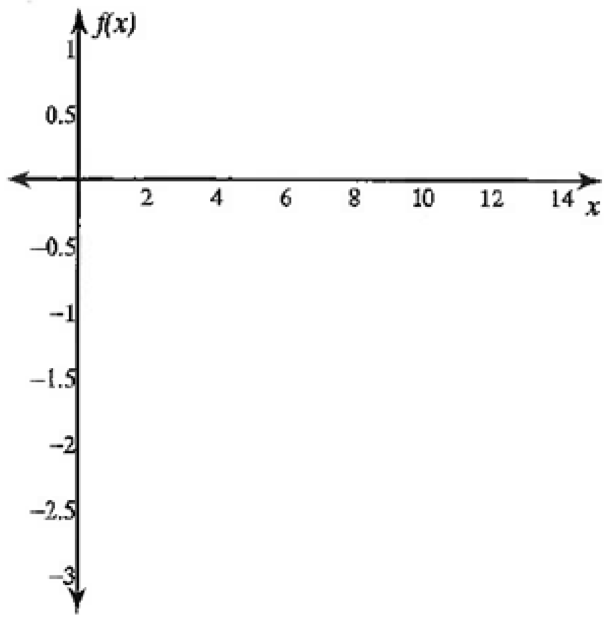
x	0	4	6	7	10
$f(x)$	5	3	2	3	5



For each problem, use a right-hand Riemann sum to approximate the integral based off of the values in the table. You may use the provided graph to sketch the function data and Riemann sums.

3) $\int_0^{14} f(x)\,dx$

x	0	3	5	9	13	14
$f(x)$	−1	−2	−1	0	−1	0



4) $\int_0^{19} f(x)\,dx$

x	0	4	9	10	12	19
$f(x)$	−3	−5	−4	−2	−1	1