$y' = f'g + fg'$ $\quad y' = \frac{f'g - fg'}{g^2}$

4.9 Combining Differentation Rules P 205 1–19 (19/20 1st step and y')

1. $f(x) = 2x^3 + 15x^2 - 36x - 12$

$f'(x) = 6x^2 + 30x - 36$

$f'(x) = 6(x^2 + 5x - 6)$

$F'(x) = 6(x-1)(x+6)$

2. $f(x) = -2x^{-3} - \frac{1}{2}x^{-2} + x^{-1} + 11$

$F'(x) = 6x^{-4} + x^{-3} - 1x^{-2}$

$F'(x) = x^{-4}(6 + x - x^2)$

$f'(x) = x^{-4}(3-x)(2+x)$

$f'(x) = -x^{-4}(x-3)(x+2)$

3. $y = \frac{1}{x} + 4x = x^{-1} + 4x$

$y' = -1x^{-2} + 4$

$y' = x^{-2}(-1 + 4x^2)$

$y' = x^{-2}(4x^2 - 1)$

$y' = x^{-2}(2x+1)(2x-1)$

4. $y = \sqrt{\frac{x}{5}} + \frac{5}{\sqrt{x}} - \frac{x}{\sqrt{5}}$

$y = \frac{1}{\sqrt{5}}x^{1/2} + 5x^{-1/2} - \frac{1}{\sqrt{5}}x$

$y' = \frac{1}{2\sqrt{5}}x^{-1/2} - \frac{5}{2}x^{-3/2} - \frac{1}{\sqrt{5}}$

5. $f(x) = (2x-3)^3(x+1)^2$

$f'(x) = 3(2x-3)^2(2)(x+1)^2 + (2x-3)^3 2(x+1)$

$f'(x) = (2x-3)^2(x+1)[6(x+1) + 2(2x-3)]$

$f'(x) = (2x-3)^2(x+1)[6x + 6 + 4x - 6]$

$F'(x) = 10x(x+1)(2x-3)^2$

6. $F(x) = x^2\sqrt{1-x^2}$

$F'(x) = 2x(1-x^2)^{1/2} + x^2\left(\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\right)$

$F'(x) = x(1-x^2)^{-1/2}[2(1-x^2)^1 - x^2]$

$f'(x) = x(1-x^2)^{-1/2}[2 - 2x^2 - x^2]$

$F'(x) = x(1-x^2)^{-1/2}(2-3x^2)$

$F'(x) = -x(3x^2-2)(1-x^2)^{-1/2}$

7. $y = (x-2)\sqrt{x^2-3x-1}$

$y' = (1)(x^2-3x-1)^{1/2} + (x-2)\left(\frac{1}{2}(x^2-3x-1)^{-1/2}(2x-3)\right)$

$y' = (x^2-3x-1)^{-1/2}\left[(x^2-3x-1) + \frac{1}{2}(2x-3)(x-2)\right]$

$y' = \frac{1}{2}(x^2-3x-1)^{-1/2}[2(x^2-3x-1) + (2x^2-7x+6)]$

$y' = \frac{1}{2}(x^2-3x-1)^{-1/2}[2x^2-6x-2+2x^2-7x+6]$

$y' = \frac{1}{2}(x^2-3x-1)^{-1/2}[4x^2-13x+4]$

4.9 - Continued

8. $y = 4\sqrt{x-1} - 6\sqrt{x+1}$

$y = 4(x-1)^{1/2} - 6(x+1)^{1/2}$

$y' = 2(x-1)^{-1/2} - 3(x+1)^{-1/2}$

9. $F(x) = \dfrac{x^2-3x}{x^2+3}$

$F'(x) = \dfrac{(2x-3)(x^2+3) - (x^2-3x)(2x)}{(x^2+3)^2}$

$F'(x) = \dfrac{2x^3+6x-3x^2-9-2x^3+6x^2}{(x^2+3)^2}$

$F'(x) = \dfrac{3x^2+6x-9}{(x^2+3)^2}$

$F'(x) = \dfrac{3(x^2+2x-3)}{(x^2+3)^2}$

$F'(x) = \dfrac{3(x+3)(x-1)}{(x^2+3)^2}$

10. $f(x) = \dfrac{6}{\sqrt[3]{x^3-2}}$

$f'(x) = \dfrac{(0)(x^3-2)^{1/3} - (6)\left(\frac{1}{3}\right)(x^3-2)^{-2/3}(3x^2)}{(x^3-2)^{2/3}}$

$f'(x) = \dfrac{-6x^2(x^3-2)^{-2/3}}{(x^3-2)^{2/3}}$

$f'(x) = -6x^2(x^3-2)^{-4/3}$

11. $y = x^3(2x-1)(3x+2)$

$y = (2x^4-x^3)(3x+2)$

$y' = (8x^3-3x^2)(3x+2) + (2x^4-x^3)(3)$

$y' = 24x^4+16x^3-9x^3-6x^2+6x^4-3x^3$

$y' = 30x^4+4x^3-6x^2$

$y' = 2x^2(15x^2+2x-3)$

12. $y = \dfrac{x(2x-3)}{x^2+2}$

$y = \dfrac{2x^2-3x}{x^2+2}$

$y' = \dfrac{(4x-3)(x^2+2) - (2x^2-3x)(2x)}{(x^2+2)^2}$

$y' = \dfrac{4x^3+8x-3x^2-6-4x^3+6x^2}{(x^2+2)^2}$

$y' = \dfrac{3x^2+8x-6}{(x^2+2)^2}$

4.9 Continued

13 $f(x) = \left(\frac{2x}{x+2}\right)^{-2} = \frac{(2x)^{-2}}{(x+2)^{-2}}$

$F(x) = \frac{(x+2)^2}{4x^2}$

$F'(x) = \frac{2(x+2)^1(4x^2) - (x+2)^2(8x)}{(4x^2)^2}$

$= \frac{8x(x+2)[x - (x+2)]}{16x^4}$

$= \frac{8x(x+2)[-2]}{16x^4}$

$= \frac{-16x(x+2)}{16x^4}$

$= \frac{-(x+2)}{x^3}$

14 $f(x) = \frac{(x+1)^2}{x^2-2}$

$F'(x) = \frac{2(x+1)^1(x^2-2) - (x+1)^2(2x)}{(x^2-2)^2}$

$f'(x) = \frac{2(x+1)[(x^2-2) - (x+1)(x)]}{(x^2-2)^2}$

$f'(x) = \frac{2(x+1)(x^2-2 - x^2 - x)}{(x^2-2)^2}$

$f'(x) = \frac{2(x+1)(-x-2)}{(x^2-2)^2}$

$f'(x) = \frac{-2(x+1)(x+2)}{(x^2-2)^2}$

15 $y = \frac{\sqrt{x}}{x^2+1}$

$y' = \frac{\frac{1}{2}x^{-1/2}(x^2+1) - x^{\frac{1}{2}}(2x)}{(x^2+1)^2}$

$y' = \frac{\frac{1}{2}x^{-1/2}[x^2+1 - x^1(4x)]}{(x^2+1)^2}$

$y' = \frac{\frac{1}{2}x^{-1/2}[-3x^2+1]}{(x^2+1)^2} = \frac{(1-3x^2)}{2x^{1/2}(x^2+1)^2}$

4.9 Continued

16. $f(x) = \frac{\sqrt{3-x}}{x^4} = \frac{(3-x)^{1/2}}{x^4}$

$f'(x) = \frac{\frac{1}{2}(3-x)^{-1/2}(-1)(x^4) - (3-x)^{1/2}4x^3}{x^8}$

$f'(x) = \frac{\frac{1}{2}x^3(3-x)^{-1/2}\left[-x^1 - (3-x)^1(8)\right]}{x^8}$

$f'(x) = \frac{\frac{1}{2}x^3(3-x)^{-1/2}\left[-x - 24 + 8x\right]}{x^8}$

$f'(x) = \frac{1}{2}x^{-5}(3-x)^{-1/2}\left[7x - 24\right]$

17. $F(x) = \frac{1}{(x^2-2)\sqrt{2x+3}}$

$F(x) = (x^2-2)^{-1}(2x+3)^{-1/2}$

$f'(x) = -1(x^2-2)^{-2}(2x)(2x+3)^{-1/2} + (x^2-2)^{-1}(-\frac{1}{2})(2x+3)^{-3/2}(2)$

$f'(x) = -2x(x^2-2)^{-2}(2x+3)^{-1/2} - 1(x^2-2)^{-1}(2x+3)^{-3/2}$

$f'(x) = (x^2-2)^{-2}(2x+3)^{-3/2}\left[-2x(2x+3)^1 - (x^2-2)^1\right]$

$f'(x) = (x^2-2)^{-2}(2x+3)^{-3/2}\left[-4x^2 - 6x - x^2 + 2\right]$

$f'(x) = (x^2-2)^{-2}(2x+3)^{-3/2}\left[-5x^2 - 6x + 2\right]$

$f'(x) = -\left[5x^2 + 6x - 2\right)(x^2-2)^{-2}(2x+3)^{-3/2}$

19. $f(x) = \sqrt{\frac{x+4}{x-4}}$

$f(x) = \frac{(x+4)^{1/2}}{(x-4)^{1/2}}$

$f'(x) = \frac{\frac{1}{2}(x+4)^{-1/2}(x-4)^{1/2} - (x+4)^{1/2}(\frac{1}{2})(x-4)^{-1/2}}{(x-4)^{1/2(2)}}$

$f'(x) = \frac{\frac{1}{2}(x+4)^{-1/2}(x-4)^{-1/2}\left[(x-4) - (x+4)\right]}{(x-4)}$

$f'(x) = \frac{\frac{1}{2}(x+4)^{-1/2}(x-4)^{-1/2}(-8)}{(x-4)}$

$f'(x) = -4(x+4)^{-1/2}(x-4)^{-3/2}$

4.9 Continued

19. $y = \dfrac{x^4(2x-1)^3}{(x+2)^2}$ (f', g)

$$y' = \frac{\left[4x^3(2x-1)^3 + x^4(3)(2x-1)^2(2)\right](x+2)^2 - \left[x^4(2x-1)^3(2)(x+2)\right]}{(x+2)^4}$$

FIRST STEP ONLY

20. $y = \dfrac{x\sqrt{x+3}}{x-2}$ $\qquad y = \dfrac{x(x+3)^{1/2}}{x-2}$ $\qquad f = x(x+3)^{1/2}$ $\qquad g = x-2$ (f', g, f, g')

$$y' = \frac{\left[1(x+3)^{1/2} + x\left(\frac{1}{2}\right)(x+3)^{-1/2}\right](x-2) - \left(x\sqrt{x+3}\right)(1)}{(x-2)^2}$$

FIRST STEP ONLY

OR

19 $y = x^4(2x-1)^3(x+2)^{-2}$ (f, g, h)

$$y' = \underset{f'\,g\,h}{4x^3(2x-1)^3(x+2)^{-2}} + \underset{F\,g'\,h}{x^4(3)(2x-1)^2(2)(x+2)^{-2}} + \underset{F\,g\,h'}{x^4(2x-1)^3(-2)(x+2)^{-3}}$$

FIRST STEP

20. $y = x\sqrt{x+3}\,(x-2)^{-1}$ (F, g, h)

$$y' = 1(x+3)^{1/2}(x-2)^{-1} + x\left(\tfrac{1}{2}\right)(x+3)^{-1/2}(x-2)^{-1} + x(x+3)^{1/2}(-1)(x-2)^{-2}$$

FIRST STEP