4.8 The Chain Rule P. 200 1-28

1. $y = (3x-4)^6$
$y' = 6(3x-4)^5(3)$
$y' = 18(3x-4)^5$

2. $y = (5-2x)^{11}$
$y' = 11(5-2x)^{10}(-2)$
$y' = -22(5-2x)^{10}$

3. $y = (x+7)^6$
$y' = 6(x+7)^5(1)$
$y' = 6(x+7)^5$

4. $y = (2-x)^5$
$y' = 5(2-x)^4(-1)$
$y' = -5(2-x)^4$

5. $f(x) = (x^2+3)^3$
$f'(x) = 3(x^2+3)^2 \cdot (2x)$
$f'(x) = 6x(x^2+3)^2$

6. $F(x) = (3x^2-5)^4$
$f'(x) = 4(3x^2-5)^3(6x)$
$f'(x) = 24x(3x^2-5)^3$

7. $y = (12-2x^3)^5$
$y' = 5(12-2x^3)^4(-6x^2)$
$y' = -30x^2(12-2x^3)^4$

8. $F(x) = (x^2-2x-3)^4$
$f'(x) = 4(x^2-2x-3)^3(2x-2)$
$F'(x) = 4(x^2-2x-3)^3(2)(x-1)$
$F'(x) = 8(x-1)(x^2-2x-3)^3$

9. $f(x) = (x^4-8x)^7$
$f'(x) = 7(x^4-8x)^6 \cdot (4x^3-8)$
$f'(x) = 7x^6(x^3-8)^6 4(x^3-2)$
$F'(x) = 28x^6(x^3-2)(x^3-8)^6$

10. $y = (3x-x^2)^3$
$y' = 3(3x-x^2)^2(3-2x)$
$y' = 3x^2(3-x)^2(3-2x)$
$y' = 3x^2(3-2x)(3-x)^2$

11. $y = (x^2+4)^{-3}$
$y' = -3(x^2+4)^{-4}(2x)$
$y' = -6x(x^2+4)^{-4}$
he has x^3 (wrong)

12. $y = (x^2+6x-3)^{-2}$
$y' = -2(x^2+6x-3)^{-3}(2x+6)$
$y' = -2(x^2+6x-3)^{-3}2(x+3)$
$y' = -4(x+3)(x^2+6x-3)^{-3}$

13. $f(x) = \dfrac{3}{x^2+9}$
$f'(x) = 3(x^2+9)^{-1}$
$f'(x) = -3(x^2+9)^{-2}(2x)$
$f'(x) = \dfrac{-6x}{(x^2+9)^2}$

14. $F(x) = \dfrac{1}{5x^2+10x+4}$
$f(x) = (5x^2+10x+4)^{-1}$
$f'(x) = -1(5x^2+10x+4)^{-2}(10x+10)$
$f'(x) = \dfrac{-10(x+1)}{(5x^2+10x+4)^2}$

4.8 Continued

15. $f(x) = \sqrt{x^2+6x}$

$f'(x) = \frac{1}{2}(x^2+6x)^{-1/2} \cdot (2x+6)$

$f'(x) = (x+3)(x^2+6x)^{-1/2}$

16. $f(x) = -3\sqrt{2x^2-4x-1}$

$f'(x) = -3(\frac{1}{2})(2x^2-4x-1)^{-1/2}(4x-4)$

$f'(x) = -3(2x^2-4x-1)^{-1/2}(x-1)$

$f'(x) = -6(x-1)(2x^2-4x-1)^{-1/2}$

17. $y = \frac{6}{5}\sqrt[3]{x-2}$

$y = \frac{6}{5}(x-2)^{1/3}$

$y' = \frac{2}{5}(x-2)^{-2/3}$

18. $y = 9\sqrt[3]{(x^2-4)^2}$

$y = 9(x^2-4)^{2/3}$

$y' = 6(x^2-4)^{-1/3} \cdot (2x)$

$y' = 12x(x^2-4)^{-1/3}$

19. $y = \frac{10}{\sqrt{2x-1}}$

$y = 10(2x-1)^{-1/2}$

$y' = -5(2x-1)^{-3/2} \cdot 2$

$y' = -10(2x-1)^{-3/2}$

20. $y = \frac{-8}{\sqrt[4]{2x^2-8x+1}}$

$y = -8(2x^2-8x+1)^{-1/4}$

$y' = 2(2x^2-8x+1)^{-5/4} \cdot (4x-8)$

$y' = 8(x-2)(2x^2-8x+1)^{-5/4}$

21. $f(x) = \left(x + \frac{1}{x}\right)^{1/2}$

$f'(x) = \frac{1}{2}(x+\frac{1}{x})^{-1/2} \cdot (1-x^{-2})$

$f'(x) = \frac{1}{2}(1-x^{-2})(x+\frac{1}{x})^{-1/2}$

22. $f(x) = 12(x^3-6x)^6$

$f'(x) = 72(x^3-6x)^5 \cdot (3x^2-6)$

$f'(x) = 216(x^2-2)(x^3-6x)^5$

$f'(x) = 216x^5(x^2-2)(x^2-6)^5$

23. $f(x) = 6\sqrt{\frac{1}{2}x-4}$

$f(x) = 6(\frac{1}{2}x-4)^{1/2}$

$f'(x) = 3(\frac{1}{2}x-4)^{-1/2} \cdot \frac{1}{2}$

$f'(x) = \frac{3}{2}(\frac{1}{2}x-4)^{-1/2}$

24. $f(x) = \frac{5}{3}\sqrt[5]{(5x^2+10x-1)^3}$

$f(x) = \frac{5}{3}(5x^2+10x-1)^{3/5}$

$f'(x) = (5x^2+10x-1)^{-2/5} \cdot (10x+10)$

$f'(x) = 10(x+1)(5x^2+10x-1)^{-2/5}$

25. $f(x) = 2\sqrt{x^2+13}$ $\quad (6, f(6))$

$f(x) = 2(x^2+13)^{1/2}$ $\quad f(6) = 2\sqrt{6^2+13} = 14$

$f'(x) = (x^2+13)^{-1/2} \cdot 2x$ $\quad (6, 14)$

$f'(x) = 2x(x^2+13)^{-1/2}$

$f'(6) = 2(6)(6^2+13)^{-1/2}$

$f'(6) = \frac{12}{\sqrt{49}} = \left(\frac{12}{7}\right)$ ← slope

4.8 Continued

26. $f(x) = \sqrt[3]{x^2-4x+3}$

$f(x) = (x^2-4x+3)^{1/3}$

$f'(x) = \frac{1}{3}(x^2-4x+3)^{-2/3} \cdot (2x-4)$

$f'(x) = \frac{2}{3}(x-2)(x^2-4x+3)^{-2/3}$

$f'(x) = \frac{2(x-2)}{3(x^2-4x+3)^{2/3}}$

a) horizontal $m=0$

$0 = \frac{2(x-2)}{3(x^2-4x+3)^{2/3}}$ $\quad 0 = 2x-4$ $\quad x=2$ $\quad (2,-1)$

b) vertical $y=0$ (set denom = 0)

$0 = 3(x^2-4x+3)^{2/3}$

$0 = x^2-4x+3$

$0 = (x-1)(x-3)$

$x=3$ $\quad x=1$ $\quad (1,0)$ $(3,0)$

27. $f(x) = \frac{5}{3x^2+6x}$

a) Quotient Rule

$f'(x) = \frac{(0)(3x^2+6x) - (5)(6x+6)}{(3x^2+6x)^2}$

$= \frac{-30x-30}{(3x^2+6x)^2}$

$= \frac{-30(x+1)}{9x^2(x+2)^2}$

$= \frac{-10(x+1)}{3x^2(x+2)^2}$

b) Chain Rule

$f(x) = 5(3x^2+6x)^{-1}$

$f'(x) = -5(3x^2+6x)^{-2} \cdot 6x+6$

$f'(x) = \frac{-30x-30}{(3x^2+6x)^2}$

$f'(x) = \frac{-30(x+1)}{9x^2(x+2)^2}$

$f'(x) = \frac{-10(x+1)}{3x^2(x+2)^2}$

28. a) $(f \circ g)'(2) = F'(2)$

$F'(x) = f'(g(x))(g'(x))$

$F'(2) = f'(g(2))(g'(2))$

$F'(2) = f'(1)(-3)$

$f'(2) = (6)(-3)$

$f'(2) = -18$

b) $(g \circ f)'(3) = G'(3)$

$G'(3) = g'(f(3))(f'(3))$

$G'(3) = (g'(1))(5)$

$G'(3) = (-4)(5)$

$G'(3) = -20$