

$$y' = \frac{f'g - fg'}{g^2} \quad -1-$$

4.7 The Quotient Rule

P. 1951-17

1. $y = \frac{2x}{x+1}$

$$y' = \frac{2(x+1) - 2x(1)}{(x+1)^2}$$

$$y' = \frac{2x+2-2x}{(x+1)^2}$$

$$y' = \frac{2}{(x+1)^2}$$

2. $y = \frac{x^2}{2x-3}$

$$y' = \frac{2x(2x-3) - x^2(2)}{(2x-3)^2}$$

$$y' = \frac{4x^2 - 6x - 2x^2}{(2x-3)^2}$$

$$y' = \frac{2x^2 - 6x}{(2x-3)^2}$$

3. $f(x) = \frac{x+4}{x-4}$

$$f'(x) = \frac{(1)(x-4) - (x+4)(1)}{(x-4)^2}$$

$$f'(x) = \frac{x-4-x-4}{(x-4)^2}$$

$$f'(x) = \frac{-8}{(x-4)^2}$$

4. $f(x) = \frac{2x-3}{2x+3}$

$$f'(x) = \frac{2(2x+3) - (2x-3)(2)}{(2x+3)^2}$$

$$f'(x) = \frac{4x+6-4x+6}{(2x+3)^2}$$

$$f'(x) = \frac{12}{(2x+3)^2}$$

5. $y = \frac{x-4}{x^2}$

$$y' = \frac{(1)(x^2) - (x-4)(2x)}{x^4}$$

$$y' = \frac{x^2 - 2x^2 + 8x}{x^4}$$

$$y' = \frac{-x^2 + 8x}{x^4}$$

$$y' = \frac{-x+8}{x^3}$$

6. $y = \frac{1}{x^2+2x+3}$

$$y' = \frac{(0)(x^2+2x+3) - 1(2x+2)}{(x^2+2x+3)^2}$$

$$y' = \frac{-2x-2}{(x^2+2x+3)^2}$$

7. $f(x) = \frac{4-2x}{1-x}$

$$f'(x) = \frac{(-2)(1-x) - (4-2x)(-1)}{(1-x)^2}$$

$$f'(x) = \frac{-2+2x+4-2x}{(1-x)^2}$$

$$f'(x) = \frac{2}{(1-x)^2} \quad \ll (x-1)^2?$$

$$y' = \frac{f'g - fg'}{g^2} \quad -2-$$

4.7 - Continued

8 $f(x) = \frac{x^2-4}{x^2+1}$

$$f'(x) = \frac{(2x)(x^2+1) - (x^2-4)(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2x^3+2x-2x^3+8x}{(x^2+1)^2}$$

$$f'(x) = \frac{10x}{(x^2+1)^2}$$

9. $f(x) = \frac{2}{\sqrt{x+1}}$

$$f'(x) = \frac{(0)(\sqrt{x+1}) - 2(\frac{1}{2}x^{-1/2})}{(\sqrt{x+1})^2}$$

$$f'(x) = \frac{-x^{-1/2}}{(\sqrt{x+1})^2}$$

$$f'(x) = \frac{-1}{\sqrt{x}(\sqrt{x+1})^2}$$

10. $y = \frac{\sqrt{x}}{x+2}$

$$y' = \frac{(\frac{1}{2}x^{-1/2})(x+2) - (\sqrt{x})(1)}{(x+2)^2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2} + x^{-1/2} - x^{1/2}}{(x+2)^2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(x+2-2x)}{(x+2)^2}$$

$$y' = \frac{2-x}{2\sqrt{x}(x+2)^2}$$

11. $y = \frac{x}{\sqrt{x+1}}$

$$y' = \frac{(1)(\sqrt{x+1}) - x(\frac{1}{2}x^{-1/2})}{(\sqrt{x+1})^2}$$

$$y' = \frac{x^{1/2} + 1 - \frac{1}{2}x^{1/2}}{(\sqrt{x+1})^2}$$

$$y' = \frac{\frac{1}{2}x^{1/2} + 1}{(\sqrt{x+1})^2}$$

$$y' = \frac{\frac{1}{2}(x^{1/2}+2)}{(\sqrt{x+1})^2}$$

$$y' = \frac{(\sqrt{x}+2)}{2(\sqrt{x+1})^2}$$

12. $f(x) = \frac{\sqrt{x}+2}{\sqrt{x}-2}$

$$f'(x) = \frac{(\frac{1}{2}x^{-1/2})(\sqrt{x}-2) - (\sqrt{x}+2)(\frac{1}{2}x^{-1/2})}{(\sqrt{x}-2)^2}$$

$$f'(x) = \frac{\frac{1}{2} - x^{-1/2} - \frac{1}{2} - x^{-1/2}}{(\sqrt{x}-2)^2}$$

$$f'(x) = \frac{-2x^{-1/2}}{(\sqrt{x}-2)^2}$$

$$f'(x) = \frac{-2}{\sqrt{x}(\sqrt{x}-2)^2}$$

$$y' = \frac{F'g - Fg'}{g^2}$$

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4.7 Continued

$$13 \textcircled{1} y = \frac{x^2}{x+2} \quad (2,1)$$

$$m = \frac{3}{4} \quad (2,1)$$

$$y' = \frac{2x(x+2) - x^2(1)}{(x+2)^2}$$

$$\textcircled{3} \begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{3}{4}(x - 2) \end{aligned}$$

$$y' = \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$4(y-1) = 3(x-2)$$

$$4y - 4 = 3x - 6$$

$$y' = \frac{x^2 + 4x}{(x+2)^2}$$

$$\textcircled{0 = 3x - 4y - 2}$$

$$\textcircled{2} y' = \frac{(2)^2 + 4(2)}{(2+2)^2}$$

$$y' = \frac{4+8}{16} = \frac{12}{16} = \frac{3}{4} \leftarrow \text{slope}$$

$$14. f(x) = \frac{\sqrt{x}}{2-x}, \quad (4,-1)$$

$$f'(x) = \frac{(\frac{1}{2}x^{-1/2})(2-x) - (\sqrt{x})(-1)}{(2-x)^2}$$

$$\textcircled{3} \begin{aligned} y - y_1 &= m(x - x_1) \\ y + 1 &= \frac{3}{8}(x - 4) \end{aligned}$$

$$f'(x) = \frac{x^{-1/2} - \frac{1}{2}x^{1/2} + x^{1/2}}{(2-x)^2}$$

$$8(y+1) = 3(x-4)$$

$$f'(x) = \frac{x^{-1/2} + \frac{1}{2}x^{1/2}}{(2-x)^2}$$

$$8y + 8 = 3x - 12$$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(2+x)}{(2-x)^2}$$

$$\textcircled{0 = 3x - 8y - 20}$$

$$f'(x) = \frac{(2+x)}{2\sqrt{x}(2-x)^2}$$

$$\textcircled{2} f'(4) = \frac{(2+4)}{2\sqrt{4}(2-4)^2}$$

$$= \frac{6}{4(-2)^2}$$

$$= \frac{6}{16} = \frac{3}{8} \leftarrow \text{slope}$$

$$y' = \frac{f'g - fg'}{g^2}$$

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4.7 - Continued

15. ① $y = \frac{12}{x^2+2}, (-1, 4)$

$$y' = \frac{(0)(x^2+2) - (12)(2x)}{(x^2+2)^2}$$

$$y' = \frac{-24x}{(x^2+2)^2}$$

② $y' = \frac{-24(-1)}{((-1)^2+2)^2}$

$$y' = \frac{24}{9} = \frac{8}{3}$$

③ $y - y_1 = m(x - x_1)$

$$y - 4 = \frac{8}{3}(x + 1)$$

$$3(y - 4) = 8(x + 1)$$

$$3y - 12 = 8x + 8$$

$$0 = 8x - 3y + 20$$

16. $y = \frac{15x^6}{3x^2}$

$$y = 5x^4$$

$$y' = 20x^3$$

$$y' = \frac{90x^5(3x^2) - (15x^6)(6x)}{(3x^2)^2}$$

$$y' = \frac{270x^7 - 90x^7}{9x^4}$$

$$y' = \frac{180x^7}{9x^4}$$

$$y' = 20x^3$$

17. $f(x) = \frac{x^3 - 9}{x - 2}$

$$f(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)}$$

$$f(x) = x^2 + 2x + 4$$

$$f'(x) = 2x + 2$$

$$f'(x) = \frac{(3x^2)(x-2) - (x^3-9)(1)}{(x-2)^2}$$

$$= \frac{3x^3 - 6x^2 - x^3 + 9}{(x-2)^2}$$

$$= \frac{2x^3 - 6x^2 + 9}{(x-2)^2}$$

$$= \frac{2(x^3 - 3x^2 + 4)}{(x-2)^2}$$

$$= \frac{2(x+1)(x-2)^2}{(x-2)^2}$$

$$= 2(x+1)$$

$$= 2x + 2$$

$$-1 \begin{vmatrix} 1 & -3 & 0 & 4 \\ 2 & -1 & 4 & -9 \\ 1 & -4 & 4 & 10 \end{vmatrix}$$

$$(x+1)(x^2-4x+4) = (x+1)(x-2)(x-2)$$