$y' = f'g + fg'$

4.6 The Product Rule

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1. $y = (9x+11)(3x-4)$
$y' = 9(3x-4) + (9x+11)(3)$
$y' = 27x - 36 + 27x + 33$
$y' = \boxed{54x - 3}$ ✓

2. $F(x) = (x^2-4x)(2x+7)$
$F'(x) = (2x-4)(2x+7) + (x^2-4x)(2)$
$f'(x) = 4x^2 + 6x - 28 + 2x^2 - 8x$
$f'(x) = \boxed{6x^2 - 2x - 28}$

3. $F(x) = (x^2-6x+5)(x^2-3x-2)$
$F'(x) = (2x-6)(x^2-3x-2) + (x^2-6x+5)(2x-3)$
$F'(x) = 2x^3 - 6x^2 - 4x - 6x^2 + 18x + 12 + 2x^3 - 3x^2 - 12x^2 + 18x + 10x - 15$
$F'(x) = \boxed{4x^3 - 27x^2 + 42x - 3}$

4. $y = (6x^4)(5x^2+8x-4)$
$y' = (24x^3)(5x^2+8x-4) + (6x^4)(10x+8)$
$y' = 120x^5 + 192x^4 - 96x^3 + 60x^5 + 48x^4$
$y' = \boxed{180x^5 + 240x^4 - 96x^3}$

5. $y = (-3x^{-2})(2x^5 + 4x^3)$
$y' = (6x^{-3})(2x^5 + 4x^3) + (-3x^{-2})(10x^4 + 12x^2)$
$y' = 12x^2 + 24 - 30x^2 - 36$
$y' = \boxed{-18x^2 - 12}$

6. $F(x) = 6x$ or $6 \cdot x$
$F'(x) = (0)(x) + (6)(1)$
$f'(x) = \boxed{6}$

7. $F(x) = (x - x^{-1})(x + x^{-1})$
$F'(x) = (1 + x^{-2})(x + x^{-1}) + (x - x^{-1})(1 - x^{-2})$
$f'(x) = x + x^{-1} + x^{-1} + x^{-3} + x - x^{-1} - x^{-1} + x^{-3}$
$F'(x) = \boxed{2x + 2x^{-3}}$

8. $y = (2x)(3x-4)(5x+2)$
$y = (6x^2 - 8x)(5x+2)$
$y' = (12x-8)(5x+2) + (6x^2-8x)(5)$
$y' = 60x^2 + 24x - 40x - 16 + 30x^2 - 40x$
$y' = \boxed{90x^2 - 56x - 16}$

$y' = f'g + fg'$

4.6 - Continued

9. $y = x^2(2x+3)(x^2-x)$
$y = (2x^3+3x^2)(x^2-x)$
$y' = (6x^2+6x)(x^2-x) + (2x^3+3x^2)(2x-1)$
$y' = 6x^4 - 6x^3 + 6x^3 - 6x^2 + 4x^4 - 2x^3 + 6x^3 - 3x^2$
$y' = \boxed{10x^4 + 4x^3 - 9x^2}$

10. $F(x) = (x^2-1)^2$
$f(x) = (x^2-1)(x^2-1)$
$f'(x) = (2x)(x^2-1) + (x^2-1)(2x)$
$f'(x) = 2x^3 - 2x + 2x^3 - 2x$
$F'(x) = \boxed{4x^3 - 4x}$

$(fgh)' = f'gh + fg'h + fgh'$

11. $F(x) = (2x-1)(x^2+3)(2x^4)$
$F'(x) = 2(x^2+3)(2x^4) + (2x-1)(2x)(2x^4) + (2x-1)(x^2+3)(8x^3)$

12. $f(x) = (2x^2-6x-1)(x^2+4x+8)$
$f'(x) = (4x-6)(x^2+4x+8) + (2x^2-6x-1)(2x+4)$

13. $y = (4x^{1/2})(2x^3 - x^{-3})(5x^{-1/3} + 2x^2)$
$y' = (2x^{-1/2})(2x^3 - x^{-3})(5x^{-1/3} + 2x^2) + (4x^{1/2})(6x^2 + 3x^{-4})(5x^{-1/3} + 2x^2) +$
$(4x^{1/2})(2x^3 - x^{-3})\left(-\frac{5}{3}x^{-4/3} + 4x\right)$

14. $y = (6 - \sqrt{x})(2x + 9\sqrt[3]{x})(-x^2 + \sqrt[4]{x^3})$
$y = (6 - x^{1/2})(2x + 9x^{1/3})(-x^2 + x^{3/4})$
$y' = \left(-\frac{1}{2}x^{-1/2}\right)(2x + 9x^{1/3})(-x^2 + x^{3/4}) + (6 - x^{1/2})(2 + 3x^{-2/3})(-x^2 + x^{3/4}) +$
$(6 - x^{1/2})(2x + 9x^{1/3})\left(-2x + \frac{3}{4}x^{-1/4}\right)$

15. $y = \left(\frac{x}{4} - \frac{4}{x}\right)\left(\frac{x^2}{6} + \frac{6}{\sqrt[3]{x}}\right)$
$y = \left(\frac{1}{4}x - 4x^{-1}\right)\left(\frac{1}{6}x^2 + 6x^{-1/3}\right)$
$y' = \left(\frac{1}{4} + 4x^{-2}\right)\left(\frac{1}{6}x^2 + 6x^{-1/3}\right) + \left(\frac{1}{4}x - 4x^{-1}\right)\left(\frac{1}{3}x - 2x^{-4/3}\right)$

$y' = f'g + fg'$

16. $y = (6x^2 - 5x - 4)(x^3 + 4x^2)$ at $x = 1$

$y' = (12x - 5)(x^3 + 4x^2) + (6x^2 - 5x - 4)(3x^2 + 8x)$

$y' = (12(1) - 5)(1^3 + 4(1)^2) + (6(1)^2 - 5(1) - 4)(3(1)^2 + 8(1))$

$y' = (7)(5) + (-3)(11)$

$y' = 35 - 33 = \boxed{2}$ ← slope at $x = 1$

17. $y = (x - 3)(2x^2 - x - 1)(x^3 + 3x^2 + 2)$

$y' = (1)(2x^2 - x - 1)(x^3 + 3x^2 + 2) + (x - 3)(4x - 1)(x^3 + 3x^2 + 2) + (x - 3)(2x^2 - x - 1)(3x^2 + 6$

$y' = (1)(2(0)^2 - (0) - 1)(0^3 + 3(0)^2 + 2) + (0 - 3)(4(0) - 1)(0^3 + 3(0)^2 + 2) + (0 - 3)(2(0)^2 - 0 - 1)(3(0)^2 + 6$

$y' = \quad -2 \quad + \quad 6 \quad + 0$

$y' = \boxed{4}$ ← slope at $x = 0$

18. $f(x) = (x - 2)(x - 1)(x)(x + 3)$ at $x = -1$

$f'(x) = (1)(x - 1)(x)(x + 3) + (x - 2)(1)(x)(x + 3) + (x - 2)(x - 1)(1)(x + 3) + (x - 2)(x - 1)(x)(1)$

$f'(-1) = (1)(-1 - 1)(-1)(-1 + 3) + (-1 - 2)(1)(-1)(-1 + 3) + (-1 - 2)(-1 - 1)(1)(-1 + 3) + (-1 - 2)(-1 - 1)(-1)(1)$

$= \quad 4 \quad + \quad 6 \quad + \quad 12 \quad + 6$

$= \boxed{16}$ ← slope at $x = -1$

19. $f(x) = -x^2(x^2 + 4x)(3x - 2)$ $\quad x = -1$

$f'(x) = (-2x)(x^2 + 4x)(3x - 2) + (-x^2)(2x + 4)(3x - 2) + (-x^2)(x^2 + 4x)(3$

$f'(-1) = (-2(-1))((-1)^2 + 4(-1))(3(-1) - 2) + (-(-1)^2)(2(-1) + 4)(3(-1) - 2) + (-(-1)^2)((-1)^2 + 4(-1))(3$

$= (+2)(-3)(-5) \quad + \quad (-1)(2)(-5) \quad + \quad (-1)(-3)(-3)$

$= \quad 30 \quad + \quad 10 \quad + 9$

$= \quad 49$ ← slope at $x = -1$

$f(-1) = -(-1)^2((-1)^2 + 4(-1))(3(-1) - 2)$

$= (-1)(-3)(-5) = -15 \quad (-1, -15)$ ← point

$y - y_1 = m(x - x_1)$

$y + 15 = 49(x + 1)$

$y + 15 = 49x + 49$

$y = 49x + 34$

$\boxed{y = 49x + 34}$