4.5 Elementary Differentiation Rules P.186 1-5

1. a) $f(x) = 3x^2 - 7x + 11$
$f'(x) = 6x - 7$

b) $y = -4x^3 + 6x^2 - 3x + 2$
$y' = -12x^2 + 12x - 3$

c) $y = \frac{3}{4}x^4 - \frac{5}{3}x^3 + \frac{7}{2}x^2 - x - 6$
$y' = 3x^3 - 5x^2 + 7x - 1$

d) $f(x) = \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + \frac{2}{3}x + \frac{3}{4}$
$f'(x) = x^4 - x^3 + x^2 - x + \frac{2}{3}$

e) $y = 6x^{2/3} - 4x^{1/2} + 3\pi$
$y' = 4x^{-1/3} - 2x^{-1/2}$

f) $f(x) = 4 - \frac{2}{x} + \frac{5}{x^2} - \frac{7}{x3}$
$f'(x) = 2x^{-2} - 10x^{-3} + 21x^{-4}$

g) $f(x) = (x-8)^2$
$f(x) = x^2 - 16x + 64$
$F'(x) = 2x - 16$

h) $y = (2x+1)^3$ $(2x+1)(4x^2+4x+1)$ $8x^3+8x^2+2x+4x^2+4x+1$
$y = 8x^3 + 12x^2 + 6x + 1$
$y' = 24x^2 + 24x + 6$

i) $y = (3x-4)(2x+5)$
$y = 6x^2 + 7x - 20$
$y' = 12x + 7$

j) $f(x) = x^3(2x-1)(2x+1)$
$F(x) = x^3(4x^2-1)$
$F(x) = 4x^5 - x^3$
$F'(x) = 20x^4 - 3x^2$

k) $f(x) = \frac{2x^4 - 3x^3}{x}$
$f(x) = 2x^3 - 3x^2$
$f'(x) = 6x^2 - 6x$

l) $F(x) = \frac{x^2 + 4x - 8}{2x^4}$
$f(x) = \frac{1}{2}x^{-2} + 2x^{-3} - 4x^{-4}$
$F'(x) = -1x^{-3} - 6x^{-4} + 16x^{-5}$

m) $y = 8\sqrt{x} - 6\sqrt[3]{x} - 4\sqrt[4]{x} + 2\sqrt{2}$
$y = 8x^{1/2} - 6x^{1/3} - 4x^{1/4} + 2\sqrt{2}$
$y' = 4x^{-1/2} - 2x^{-2/3} - 1x^{-3/4}$

n) $y = \sqrt{2x} + \sqrt[3]{3x} + \sqrt[4]{5x} + \sqrt{7}$
$y = \sqrt{2}x^{1/2} + \sqrt[3]{3}x^{1/3} + \sqrt[4]{5}x^{1/4} + \sqrt{7}$
$y' = \frac{1}{2}\sqrt{2}x^{-1/2} + \frac{1}{3}\sqrt[3]{3}x^{-2/3} + \frac{1}{4}\sqrt[4]{5}x^{-3/4}$

o) $y = \sqrt{\frac{x}{2}} - \sqrt[3]{\frac{x}{3}}$
$y = \frac{1}{\sqrt{2}}x^{1/2} - \frac{1}{\sqrt[3]{3}}x^{1/3}$
$y' = \frac{1}{2\sqrt{2}}x^{-1/2} - \frac{1}{3\sqrt[3]{3}}x^{-2/3}$
↑ he has a "2"

p) $f(x) = \frac{3}{x^3} + \frac{8}{\sqrt{x}} - \sqrt{3x}$
$f(x) = 3x^{-3} + 8x^{-1/2} - \sqrt{3}x^{1/2}$
$F'(x) = -9x^{-4} - 4x^{-3/2} + \frac{\sqrt{3}}{2}x^{-1/2}$

4.5 - Continued

2 a) $y = m^4 - 6m^2 - 8$

$\frac{dy}{dm} = 4m^3 - 12m$

b) $V = \frac{4}{3}\pi r^3$

$\frac{dv}{dr} = 4\pi r^2$

c) $f(x) = \frac{1}{x^4} - \frac{2}{x^3} + \frac{3}{x^2}$

$f(x) = x^{-4} - 2x^{-3} + 3x^{-2}$

$f'(x) = -4x^{-5} + 6x^{-4} - 6x^{-3}$

d) $y = 2a^{-2} + 3a - 4$

$y' = -4a^{-3} + 3$

e) $y = 2c^{10} - 5c^2$

$y' = 20c^9 - 10c$

f) $x = 4y^2 - 6y + 11$

$\frac{dx}{dy} = 8y - 6$

3 a) $f(x) = 2x^2 - 5x + 3$ at $x = -2$

$f'(x) = 4x - 5$

$f'(-2) = 4(-2) - 5$

$= \boxed{-13}$

b) $y = \frac{1}{6}x^3 - \frac{3}{4}x^2 + 2x - 7$ at x

$y' = \frac{1}{2}x^2 - \frac{3}{2}x + 2$

$y' = \frac{1}{2}(4)^2 - \frac{3}{2}(4) + 2$

$= \boxed{4}$

c) $y = 3\sqrt{x} - 2\sqrt[3]{x} + 6$ at $x = 64$

$y = 3x^{1/2} - 2x^{1/3} + 6$

$y' = \frac{3}{2}x^{-1/2} - \frac{2}{3}x^{-2/3}$

$= \frac{3}{2}(64)^{-1/2} - \frac{2}{3}(64)^{-2/3}$

$= \frac{3}{2}\left(\frac{1}{8}\right) - \frac{2}{3}\left(\frac{1}{16}\right)$

$= \frac{3}{16} - \frac{2}{48} = \boxed{\frac{7}{48}}$

d) $f(x) = \frac{x-3}{x}$ at $x = -3$

$f'(x) = 1 - 3x^{-1}$

$f'(x) = 3x^{-2}$

$f'(-3) = 3(-3)^{-2}$

$= 3\left(\frac{1}{9}\right)$

$= \boxed{\frac{1}{3}}$

4 a) $f(x) = -x^3 + 8x^2$ at $(5, f(5))$

① $f'(x) = -3x^2 + 16x$

$f'(5) = -3(5)^2 + 16(5)$

$= \boxed{5}$ ← slope

② $(5, 75)$

$-(5)^3 + 8(-5)^2 = -125 + 200$

$= 75$

③ $y - y_1 = m(x - x_1)$

$y - 75 = 5(x - 5)$

$y - 75 = 5x - 25$

$\boxed{y = 5x + 50}$

4.5 Continued

4 b) $y = \frac{3}{x} - \frac{2}{x^2} + 7$ at $(-2, f(-2))$ ② $(-2, 5)$

$\frac{3}{-2} - \frac{2}{(-2)^2} + 7 = 5$

$-\frac{6}{4} - \frac{2}{4} + \frac{28}{4}$

$y = 3x^{-1} - 2x^{-2} + 7$

① $y' = -3x^{-2} + 4x^{-3}$

$= -3(-2)^{-2} + 4(-2)^{-3}$

$= \frac{-3}{4} + \frac{4}{-8} = \boxed{\frac{-5}{4}}$ ← slope

③ $y - y_1 = m(x - x_1)$

$y - 5 = \frac{-5}{4}(x + 2)$ $\quad \boxed{y = \frac{-5}{4}x + \frac{5}{2}}$

$4(y - 5) = -5(x + 2)$

$4y - 20 = -5x - 10$

$4y = -5x + 10$

c) $y = (x^2 - 2x + 3)^2$

$(x^2 - 2x + 3)(x^2 - 2x + 3)$

$y = x^4 - 4x^3 + 10x^2 - 12x + 9$

$x^4 - 2x^3 + 3x^2 - 2x^3 + 4x^2 - 6x + 3x^2 - 6x + 9$

$x^4 - 4x^3 + 10x^2 - 12x + 9$

① $y' = 4x^3 - 12x^2 + 20x - 12$

$y' = 4(-1)^3 - 12(-1)^2 + 20(-1) - 12$

$y' = -4 - 12 - 20 - 12 = \boxed{-48}$

② $P + (-1, f(-1))$ $\quad (-1, 36)$

$((-1)^2 - 2(-1) + 3)^2$

$(1 + 2 + 3)^2 = 36$

③ $y - y_1 = m(x - x_1)$

$y - 36 = -48(x + 1)$ $\quad \boxed{y = -48x - 12}$

$y - 36 = -48x - 48$

5 a) $f(x) = 3x^2 - 12x + 5$

① $f'(x) = 6x - 12$

$0 = 6x - 12$

$12 = 6x$

$2 = x$

② $f(2) = 3(2)^2 - 12(2) + 5$

$= 12 - 24 + 5 = -7$

$\boxed{(2, -7)}$

b) $y = x^3 - 6x^2 + 9x - 1$

① $y' = 3x^2 - 12x + 9$

$0 = 3x^2 - 12x + 9$

$0 = x^2 - 4x + 3$

$0 = (x - 1)(x - 3)$

$x = 1 \quad x = 3$

② $y = (1)^3 - 6(1)^2 + 9(1) - 1$

$y = 1 - 6 + 9 - 1 = 3$

$\boxed{(1, 3)}$

③ $y = (3)^3 - 6(3)^2 + 9(3) - 1$

$= 27 - 54 + 27 - 1 = -1$

$\boxed{(3, -1)}$

c) $f(x) = 2x^3 + 3x^2 + 30x - 40$

$f'(x) = 6x^2 + 6x + 30$

$0 = 6x^2 + 6x + 30$

$0 = x^2 + x + 5$

↑ no real roots $\quad D = b^2 - 4ac$

$= (1)^2 - 4(1)(5)$

$= -19$

$\boxed{\infty \text{ no answers}}$