4.4 Determining Slopes of Tangent Lines at the General Point

p. 176 1-3

1. a) $f(x) = 2x^2 - 3x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 - 3(x+h)] - [2(x)^2 - 3x]}{h}$$

$$= \lim_{h \to 0} \frac{[2(x^2 + 2xh + h^2) - 3x - 3h] - [2x^2 - 3x]}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$$

$$= \lim_{h \to 0} 4x + 2h - 3$$

$$= 4x + 2(0) - 3$$

$$f'(x) = 4x - 3$$

$f'(x) = 4x - 3$
$f'(-3) = -15$
$f'(-1) = -7$
$f'(0) = -3$
$f'(1) = 1$
$f'(3) = 9$

$(x+h)(x^2 + 2xh + h^2)$
$= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$
$= x^3 + 3x^2h + 3xh^2 + h^3$

b) $f(x) = x^3 + 5x^2 - 7x - 2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^3 + 5(x+h)^2 - 7(x+h) - 2] - [x^3 + 5x^2 - 7x - 2]}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 5x^2 + 10xh + 5h^2 - 7x - 7h - 2 - x^3 - 5x^2 + 7x + 2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 10xh + 5h^2 - 7h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 10x + 5h - 7$$

$$= 3x^2 + 3x(0) + (0)^2 + 10x + 5(0) - 7$$

$$f'(x) = 3x^2 + 10x - 7$$

$f'(x) = 3x^2 + 10x - 7$
$f'(-3) = -10$
$f'(-1) = -14$
$f'(0) = -7$
$f'(1) = 6$
$f'(3) = 50$

4.4 - Continued

1 c) $f(x) = -3x - 8$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[-3(x+h) - 8] - [-3x - 8]}{h}$$

$$= \lim_{h\to 0} \frac{-3x - 3h - 8 + 3x + 8}{h}$$

$$= \lim_{h\to 0} \frac{-3h}{h}$$

$$= \lim_{h\to 0} -3$$

$$f'(x) = -3$$

$f'(x) = -3$
$f'(-3) = -3$
$f'(-1) = -3$
$f'(0) = -3$
$f'(1) = -3$
$f'(3) = -3$

d) $f(x) = 10$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = 0$$

$f'(x) = 0$ $f'(0) = 0$
$f'(-3) = 0$ $f'(1) = 0$
$f'(-1) = 0$ $f'(3) = 0$

e) $y = -3x^4 + x$

$(x^2 + 2xh + h^2)(x^2 + 2xh + h^2)$
$x^4 + 2x^3h + x^2h^2 + 2x^3h + 4x^2h^2 + 2xh^3$
$+ x^2h^2 + 2xh^3 + h^4$
$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[-3(x+h)^4 + (x+h)] - [-3x^4 + x]}{h}$$

$$= \lim_{h\to 0} \frac{-3x^4 - 12x^3h - 18x^2h^2 - 12xh^3 - 3h^4 + x + h + 3x^4 - x}{h}$$

$$= \lim_{h\to 0} \frac{-12x^3h - 18x^2h^2 - 12xh^3 - 3h^4 + h}{h}$$

$$= \lim_{h\to 0} -12x^3 - 18x^2h - 12xh^2 - 3h^3 + 1$$

$$= -12x^3 - 18x^2(0) - 12x(0)^2 - 3(0) + 1$$

$$f'(x) = -12x^3 + 1$$

$f'(x) = -12x^3 + 1$
$f'(-3) = 325$
$f'(-1) = 13$
$f'(0) = 1$
$f'(1) = -11$
$f'(3) = -323$

4.4 Contined

1\. f) $y = 4\sqrt{x}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} \cdot \frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{16(x+h) - 16(x)}{h(4\sqrt{x+h} + 4\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{16x + 16h - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{16}{4\sqrt{x+h} + 4\sqrt{x}}$$

$$= \frac{16}{4\sqrt{x+0} + 4\sqrt{x}}$$

$$= \frac{16}{8\sqrt{x}}$$

$$f'(x) = \frac{2}{\sqrt{x}} = \frac{2\sqrt{x}}{x}$$

$f'(x) = \frac{2\sqrt{x}}{x}$

$f'(-3)$ = not in domain

$f'(-1)$ = not in domain

$f'(0)$ = does not exist because tangent line vertic

$f'(1) = 2$

$f'(3) = \frac{2\sqrt{3}}{3}$

g) $y = \frac{2}{x^2}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x^2}{x^2}\frac{2}{(x+h)^2} - \frac{2}{x^2}\frac{(x+h)^2}{(x+h)^2}}{h}$$

$$= \lim_{h\to 0} \left(\frac{2x^2 - 2(x+h)^2}{x^2(x+h)^2}\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{2x^2 - 2x^2 - 4xh + 2h^2}{x^2(x+h)^2}\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{-4x + 2h}{x^2(x+h)^2}$$

$$= \frac{-4x + 2(0)}{x^2(x+0)^2} = \frac{-4x}{x^4}$$

$$= \frac{-4}{x^3}$$

$F'(x) = \frac{-4}{x^3}$

$F'(-3) = \frac{-4}{-27} = \frac{4}{27}$

$F'(-1) = \frac{-4}{-1} = 4$

$F'(0) = \frac{-4}{0}$ = und.

$f'(1) = \frac{-4}{1} = -4$

$f'(3) = \frac{-4}{27}$

4.4 Continued

1 h) $y = |x|$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{|x+h| - |x|}{h}$$

① $= \lim_{h \to 0} \frac{(x+h) - (x)}{h}$

$= \lim_{h \to 0} \frac{h}{h}$

$= \lim_{h \to 0} 1$

$= 1$

② $\lim_{h \to 0} \frac{-(x+h) - (x)}{h}$

$= \lim_{h \to 0} \frac{-x - h + x}{h}$

$= \lim_{h \to 0} -1$

$= -1$

$f'(-3) = -1$
$f'(-1) = -1$
$f'(0) =$ does not exist
$f'(1) = 1$
$f'(3) = 1$

2. a) $f(x) = x^2 + 2x - 3$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 + 2(x+h) - 3] - [x^2 + 2x - 3]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} 2x + h + 2$$

$$= 2x + 0 + 2$$

$$f'(x) = 2x + 2$$

b) ① $(-2, f(-2))$
$(-2, -3)$

$f(-2) = (-2)^2 + 2(-2) - 3$
$= 4 - 4 - 3$
$= -3$

slope → plug in $x = -2$
$2(-2) + 2$
$= -4 + 2 = -2$

$y - y_1 = m(x - x_1)$ if $m = -2$, $(-2, -3)$
$y + 3 = -2(x + 2)$
$y + 3 = -2x - 4$
$y = -2x - 7$

4.4 continued

2 b) ② $(1, f(1))$ $\quad f(1) = (1)^2 + 2(1) - 3$

$(1, 0)$ $\quad = 1 + 2 - 3$

slope → plug in $x = 1$ $\quad = 0$

$2(1) + 2$ $\quad y - y_1 = m(x - x_1)$ if $m = 4$ $(1, 0)$

$= 4$ $\quad y - 0 = 4(x - 1)$

$y = 4x - 4$

c) ① $y = -2x - 7$

② $y = 4x - 4$

$-2x - 7 = 4x - 4$

$-3 = 6x$

$-\frac{1}{2} = x$

$y = -2(-\frac{1}{2}) - 7$

$= 1 - 7 = -6$

$\left(-\frac{1}{2}, -6\right)$

3. $f(x) = -2x^2 + 8x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[-2(x+h)^2 + 8(x+h)] - [-2x^2 + 8x]}{h}$$

$$= \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 8x + 8h + 2x^2 - 8x}{h}$$

$$= \lim_{h \to 0} \frac{-4xh - 2h^2 + 8h}{h}$$

$$= \lim_{h \to 0} -4x - 2h + 8$$

$$= -4x - 2(0) + 8$$

$$= -4x + 8$$

Slope $\quad -8 = -4x + 8$ $\quad$ pt: $f(4) = -2(4)^2 + 8(4)$

$-16 = -4x$ $\quad = -32 + 32$

$4 = x$ $\quad = 0$

$(4, 0)$