4.3 Determining Slope of Tangent Lines at Specific Points

Pg 167 1-6

1. $f(x) = x^2$ $P(2, f(2))$
$Q(2+h, f(2+h))$

① $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(2+h) - f(2)}{2+h-2}$

$= \dfrac{[(2+h)^2] - [(2)^2]}{h}$

$= \dfrac{4 + 4h + h^2 - 4}{h}$

$= 4 + h$

② $\lim\limits_{h \to 0} 4 + h$

$= 4 + 0 = 4$

③ $y - y_1 = m(x - x_1)$ Pt $(2, f(2))$

$y - 4 = 4(x - 2)$ Pt $(2, 4)$

$y - 4 = 4x - 8$ $m = 4$

$y = 4x - 4$

$(-2+h)(4 - 4h + h^2)$

$= -8 + 8h - 2h^2 + 4h - 4h^2 + h^3$

$= h^3 - 6h^2 + 12h - 8$

2. $f(x) = x^3$, $P(-2, f(-2))$
$Q(-2+h, f(-2+h))$

① $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(-2+h) - f(-2)}{-2+h-(-2)}$

$= \dfrac{[(-2+h)^3] - [(-2)^3]}{h}$

$= \dfrac{h^3 - 6h^2 + 12h - 8 + 8}{h}$

$= h^2 + 6h + 12$

② $\lim\limits_{h \to 0} h^2 - 6h + 12$

$= (0)^2 - 6(0) + 12$

$= 12$

③ $y - y_1 = m(x - x_1)$ Pt $(-2, f(-2))$

$y + 8 = 12(x + 2)$ Pt $(-2, -8)$

$y + 8 = 12x + 24$ $m = 12$

$y = 12x + 16$

4.3 - Continued

3. $f(x) = x^2 - 4x$ $\quad P(1, f(1))$

$Q(1+h, f(1+h))$

① $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(1+h) - f(1)}{1+h-1}$

$= \dfrac{(1+h)^2 - 4(1+h) - (1^2 - 4(1))}{h}$

$= \dfrac{1 + 2h + h^2 - 4h - 4 + 3}{h}$

$= \dfrac{h^2 - 2h}{h}$

$= h - 2$

② $\lim\limits_{h \to 0} h - 2$

$= 0 - 2 = -2$

③ $y - y_1 = m(x - x_1)$ $\quad Pt(1, f(1))$

$y + 3 = -2(x - 1)$ $\quad Pt(1, -3)$

$y + 3 = -2x + 2$ $\quad m = -2$

$y = -2x - 1$

4. $f(x) = 6x - 2x^3$ $\quad P(-1, f(-1))$

$Q(-1+h, f(-1+h))$

$(-1+h)(1 - 2h + h^2)$

$-1 + 2h - h^2 + h - 2h^2 + h^3$

$= h^3 - 3h^2 + 3h - 1$

① $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(-1+h) - f(-1)}{-1+h-(-1)}$

$= \dfrac{6(-1+h) - 2(-1+h)^3 - (6(-1) - 2(-1)^3)}{h}$

$= \dfrac{-6 + 6h - 2(h^3 - 3h^2 + 3h - 1) - [-6 + 2]}{h}$

$= \dfrac{-6 + 6h - 2h^3 + 6h^2 - 6h + 2 + 4}{h}$

$= \dfrac{-2h^3 + 6h^2}{h}$

$= -2h^2 + 6h$

② $\lim\limits_{h \to 0} -2h^2 + 6h = 0$

③ $(-1, -4)$ $\quad m = 0$

$6(-1) - 2(-1)^3$

$-6 + 2 = -4$

$Y = -4$

4.3 - Continued

5. $f(x) = \frac{4}{x}$, $P(-2, f(-2))$
$P((-2+h), f(-2+h))$

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{f(-2+h) - f(-2)}{-2+h-(-2)}$

$= \frac{\left(\frac{4}{-2+h}\right) - \left(\frac{4}{-2}\right)}{h}$

$= \left(\frac{4}{-2+h} + 2\left(\frac{-2+h}{-2+h}\right)\right)\left(\frac{1}{h}\right)$

$= \left(\frac{4-4+2h}{-2+h}\right)\left(\frac{1}{h}\right)$

$= \left(\frac{+2h}{-2+h}\right)\left(\frac{1}{h}\right)$

$= \frac{+2}{-2+h}$

② $\lim_{h \to 0} \frac{+2}{-2+h} = \frac{+2}{-2+0} = -1$

③ $y - y_1 = m(x - x_1)$ Pt $(-2, f(-2))$
$y + 2 = -1(x+2)$ Pt $(-2, -2)$
$y + 2 = -1x - 2$ $m = -1$
$y = -1x - 4$

6. $f(x) = \sqrt{x+3}$, $P(1, f(1))$
$Q((1+h), f(1+h))$

① $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{f(1+h) - f(1)}{1+h-1}$

$= \frac{\sqrt{(1+h)+3} - \sqrt{1+3}}{h}$

$= \frac{\sqrt{h+4} - 2}{h} \circ \frac{\sqrt{h+4} + 2}{\sqrt{h+4} + 2}$

$= \frac{h+4-4}{h(\sqrt{h+4}+2)}$

$= \frac{1}{\sqrt{h+4}+2}$

② $\lim_{h \to 0} \frac{1}{\sqrt{h+4}+2}$

$= \frac{1}{2+2}$

$= \frac{1}{4}$

③ $y - y_1 = m(x - x_1)$ Pt $(1, f(1))$
$y - 2 = \frac{1}{4}(x - 1)$ Pt $(1, 2)$
$4y - 8 = x - 1$ $m = \frac{1}{4}$
$4y = x + 7$
$y = \frac{1}{4}x + \frac{7}{4}$