4.11 - CHAPTER REVIEW

1. $f(x) = -3x + 7$ a) x increases by 11, y decreases by 33 $\left(\frac{\Delta y}{11} = -3\right)$
b) y increases by 21, y decreases by 7 $\left(\frac{21}{\Delta x} = -3\right)$

2. $f(x) = \frac{1}{4}x + 6$ → rate of change is $\frac{1}{4}$

3. a) $y = 2x^3$ $(1,2)$ $(1.001, 2.006)$
$y = 2(1.001)^3$ $m = \frac{2.006 - 2}{1.001 - 1} = \frac{0.006}{0.001} = \frac{6}{1}$ b) 6
$y = 2.006$

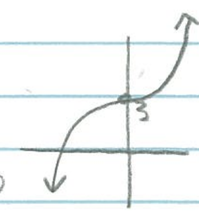
4. If the x-co-ordinate of Q is 4, then Q is one unit to the right of P, so h=1 (change). Secant slope → $6 + 2(1) - (1)^2 = 7$

5. $f(x) = 2x^3 - 7x$

6. $\frac{f(x+h) - f(x)}{h}$ ← slope of secant the pts P & Q not slope of tangent line

7. $f(2) = 1$ means if $x = 2$, $y = 1$
$f'(1) = 2$ means at $x = 1$ the slope of the tangent line is 2

8. $f(0) = 3$ - passes through $(0,3)$
$f'(0) = 0$ ← horizontal slope at $(0,3)$
$f'(x) > 0$ if $x \neq 0$ ← always increasing except at 0



9. The slope of the tangent line at P is larger than the slope of secant line at $\overline{PQ}$

10. See explanation on P. 413

11. a) $f(x) = 6x - x^2$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(6(x+h) - (x+h)^2) - (6x - x^2)}{h}$

$= \lim_{h\to 0} \frac{6x + 6h - x^2 - 2xh - h^2 - 6x + x^2}{h}$

$\lim_{h\to 0} \frac{6h - 2xh - h^2}{h}$

$\lim_{h\to 0} 6 - 2x - h^2 = 6 - 2x - 0^2$

$= \boxed{6 - 2x}$

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11 b) $f(x) = \sqrt{x-2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h} \cdot \frac{\sqrt{x+h-2} + \sqrt{x-2}}{\sqrt{x+h-2} + \sqrt{x-2}}$$

$$= \lim_{h \to 0} \frac{x+h-2-x+2}{h(\sqrt{x+h-2} + \sqrt{x-2})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h-2} + \sqrt{x-2})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-2} + \sqrt{x-2}}$$

$$= \frac{1}{\sqrt{x+0-2} + \sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$$

c) $f(x) = \frac{2}{x^2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{(x+h)^2} - \frac{2}{(x)^2}}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 - 2x^2 - 4xh - 2h^2}{x^2(x+h)^2} \cdot \left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \frac{-4xh - 2h^2}{x^2(x+h)^2} \cdot \left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \frac{-4xh - 2h}{x^2(x+h)^2}$$

$$= \frac{-4x - 0}{x^2(x+0)^2}$$

$$= \frac{-4x}{x^4}$$

$$= \frac{-4}{x^3}$$

4.11- Chapter Review

12 a) $f(x) = x^3 - 12x$ $\quad P(3, f(3)) = (3, -9)$ $\quad (3.001, f(3.001))$

$(3.001, -8.98499)$

$$m = \frac{-9 + 8.98499}{3 - 3.001}$$

$$m = \frac{-0.015009}{-0.001}$$

$m = 15.009001$ ← reasonable estimate of a slope of 15.

b) $$\frac{f(3+h) - f(3)}{3+h-3}$$

$$= \frac{\left((3+h)^3 - 12(3+h)\right) - \left[3^3 - 12(3)\right]}{h}$$

$$= \frac{27 + 27h + 9h^2 + h^3 - 36 - 12h - (27 - 36)}{h}$$

$$= \frac{h^3 + 9h^2 + 15h}{h}$$

$$= h^2 + 9h + 15$$

$$\lim_{h \to 0} h^2 + 9h + 15$$

$$= (0)^2 + 9(0) + 15$$

$= 15$ ← tangent slope.

c) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left((x+h)^3 - 12(x+h)\right) - (x^3 - 12x)}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 12x - 12h - x^3 + 12x}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 12h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 - 12$$

$$= 3x^2 + 3x(0) + (0)^2 - 12$$

$= 3x^2 - 12$ AT $x = 3 \rightarrow 3(3)^2 - 12 = 15$

4.11 - Chapter Review

12 d) $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f'(3) = 3(3)^2 - 12$

$= 15$

13. $f(x) = \tan x$ $\quad (x, f(x))$

$$f'(x) = \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h}$$

14. derivative does not exist at x_3, x_4, x_6, x_8

15. $f(x) = 2^{\sqrt{x}}$ $\quad P(1, 2)$

use two points P + $Q(1.00001, f(1.00001))$ ← close to P

16. For a slope to be $-\frac{50}{1}$. Population went down 50 each year

17. If $f'(3) = 10$ $\quad \frac{\Delta y}{\Delta x} = 10$ $\quad \Delta y = 10\Delta x$

y is changing 10 times as quickly as x

18. a) $f(x) = 20x^{20}$

$f'(x) = 400x^{19}$

b) $f(x) = -8x^{7/8}$

$f'(x) = -7x^{-1/8}$

c) $f(x) = (1.05)^{13}$

$f'(x) = 0$

d) $f(x) = \frac{x^{-8}}{4}$

$f'(x) = -2x^{-9}$

e) $y = \sqrt[5]{x}$

$y = x^{1/5}$

$y' = \frac{1}{5}x^{-4/5}$

f) $y = \sqrt{6x}$

$y = \sqrt{6}x^{1/2}$

$y' = \frac{1}{2}\sqrt{6}x^{-1/2}$

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g) $f(x) = -\frac{2}{3}x^3 + \frac{5}{2}x^2 - 8x$

$f'(x) = -2x^2 + 5x - 8$

h) $f(x) = \frac{2}{x^2} - \frac{2}{x} + 2 - 2x + 2x^2$

$f'(x) = -4x^{-3} + 2x^{-2} - 2 + 4x$

i) $f(x) = \sqrt{3x} + \sqrt{3}\,x - \sqrt{\frac{3}{x}}$

$f(x) = \sqrt{3}x^{1/2} + \sqrt{3}x - \sqrt{3}x^{-1/2}$

$f'(x) = \frac{1}{2}\sqrt{3}x^{-1/2} + \sqrt{3} + \frac{1}{2}\sqrt{3}x^{-3/2}$

j) $f(x) = \frac{9}{\sqrt[3]{x}} - \frac{8}{\sqrt[4]{x}}$

$f(x) = 9x^{-1/3} - 8x^{-1/4}$

$f'(x) = -3x^{-4/3} + 2x^{-5/4}$

k) $f(x) = (x^2+2x+1)^2$

$f'(x) = 2(x^2+2x+1)^1 \cdot (2x+2)$

$f'(x) = 4(x+1)(x^2+2x+1)$

or $4x^3+12x^2+12x+4$

l) $f(x) = (x^3-7x)(9x^2+3)$

$f(x) = 9x^5 + 3x^3 - 63x^3 - 21x$

$f'(x) = 45x^4 - 180x^2 - 21$

m) $f(x) = x^{\log_2 8} + 2^8$

$f(x) = x^{\log_2 2^3} + 2^8$

$f(x) = x^3 + 2^8$

$f'(x) = 3x^2$

n) $y = \frac{x^3-4x^2}{2x}$

$y = \frac{1}{2}x^2 - 2x$

$y' = x - 2$

o) $y = \frac{x^3+1}{x+1}$

$y = \frac{\cancel{(x+1)}(x^2-1x+1)}{\cancel{(x+1)}}$

$y = x^2 - x + 1$

$y' = 2x - 1$

19. $f(x) = x^3 + \frac{1}{2}x^2 - 2x + 7$

$f'(x) = 3x^2 + x - 2$

$f'(x) = (3x-2)(x+1)$

$3x^2 + x - 2$

$3x^2 + 3x - 2x - 2$

$3x(x+1) - 2(x+1)$

$(3x-2)(x+1)$

4.11 Chapter Review

20 $f(x) = \frac{x^2 - 7x + 10}{\sqrt{x}}$

$3x^2 - 7x - 10$

$3x^2 - 10x + 3x - 10$

$x(3x-10) + 1(3x-10)$

$(x+1)(3x-10)$

$f(x) = x^{3/2} - 7x^{1/2} + 10x^{-1/2}$

$f'(x) = \frac{3}{2}x^{1/2} - \frac{7}{2}x^{-1/2} - 5x^{-3/2}$

$f'(x) = \frac{1}{2}x^{-3/2}(3x^2 - 7x - 10)$

$f'(x) = \frac{1}{2}x^{-3/2}(3x-10)(x+1)$

21. $f(x) = (x+2)(x-2)(2x+7)$

$f(x) = (x^2-4)(2x+7)$

$f(x) = 2x^3 + 7x^2 - 8x - 28$

$f'(x) = 6x^2 + 14x - 8$

$f'(x) = 2(3x^2 + 7x - 4)$

22. $f(x) = (x+2)(x-2)(2x-7)$

$f(x) = (x^2-4)(2x+7)$ $\qquad f'g + fg'$

$f'(x) = (2x)(2x+7) + (x^2-4)(2)$

$f'(x) = 4x^2 + 14x + 2x^2 - 8$

$f'(x) = 6x^2 + 14x - 8$

$f'(x) = 2(3x^2 + 7x - 4)$

23. $f(x) = (\sqrt{4x-1})(\sqrt[3]{3x+1})$

$f(x) = (4x-1)^{1/2}(3x+1)^{1/3}$

$f'(x) = \frac{1}{2}(4x-1)^{-1/2} \cdot 4\,(3x+1)^{1/3} + (4x-1)^{1/2}(\frac{1}{3})(3x+1)^{-2/3} \cdot 3$

$f'(x) = 2(4x-1)^{-1/2}(3x+1)^{1/3} + (4x-1)^{1/2}(3x+1)^{-2/3}$

$f'(x) = (4x-1)^{-1/2}(3x+1)^{-2/3}(2(3x+1) + (4x-1))$

$f'(x) = (4x-1)^{-1/2}(3x+1)^{-2/3}(10x+1)$

24. $f(x) = (x-1)^2(\sqrt{2x+1})$

$f'(x) = 2(x-1)(2x+1)^{1/2} + (x-1)^2\,\frac{1}{2}(2x+1)^{-1/2} \cdot 2$

$f'(x) = (x-1)(2x+1)^{-1/2}[2(2x+1) + (x-1)]$

$f'(x) = (x-1)(2x+1)^{-1/2}(5x+1)$

$$\frac{f'g - fg'}{g^2}$$

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25. $f(x) = \frac{x}{x^2+1}$

$f'(x) = \frac{1(x^2+1) - (x)(2x)}{(x^2+1)^2}$

$f'(x) = \frac{-x^2+1}{(x^2+1)^2}$

$f'(x) = \frac{(1+x)(1-x)}{(x^2+1)^2}$

26. $f(x) = \frac{x}{x^2+1}$

$f(x) = x(x^2+1)^{-1}$

$f'(x) = (x^2+1)^{-1} + x(-1)(x^2+1)^{-2}(2x)$

$f'(x) = (x^2+1)^{-1} - 2x^2(x^2+1)^{-2}$

$f'(x) = (x^2+1)^{-2}(x^2+1-2x^2)$

$f'(x) = (x^2+1)^{-2}(-x^2+1)$

$f'(x) = \frac{(1-x)(1+x)}{(x^2+1)^2}$

27. $f(x) = \sqrt{\frac{2x-1}{2x+1}}$

$f(x) = \frac{(2x-1)^{1/2}}{(2x+1)^{1/2}}$

$f'(x) = \frac{\frac{1}{2}(2x-1)^{-1/2}(2)(2x+1)^{1/2} - (2x-1)^{1/2}(\frac{1}{2})(2x+1)^{-1/2}(2)}{(2x+1)}$

$f'(x) = \frac{(2x-1)^{-1/2}(2x+1)^{-1/2}((2x+1)-(2x-1))}{2x+1}$

$f'(x) = \frac{(2x-1)^{-1/2}(2)}{(2x+1)^{3/2}}$

28. a) $2x^2 - 5x + y^2 = 4$

$4x - 5 + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-4x+5}{2y}$

$(3,-1)$

$\frac{dy}{dx} = \frac{-4(3)+5}{2(-1)} = \frac{-7}{-2} = \frac{7}{2}$

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28. b) $2x^2 - 5x + y^2 = 4$

$$y^2 = -2x^2 + 5x + 4$$
$$y = (-2x^2 + 5x + 4)^{1/2}$$
$$y' = \frac{1}{2}(-2x^2 + 5x + 4)^{-1/2} \cdot (-4x + 5)$$

$(3, -1)$
$$y' = \frac{1}{2}(-2(3)^2 + 5(3) + 4)^{-1/2}(-4(3) + 5)$$
$$y' = \frac{1}{2}(-18 + 15 + 4)^{-1/2} \quad (-7)$$
$$y' = -\frac{7}{2}$$

check

29 $y^2 - 3xy = -2$

$$2y\frac{dy}{dx} - \left(3y + 3x\frac{dy}{dx}\right) = 0$$
$$(2y - 3x)\frac{dy}{dx} = 3y$$
$$\frac{dy}{dx} = \frac{3y}{2y - 3x} \quad (1, 2)$$

$$\frac{dy}{dx} = \frac{3(2)}{2(2) - 3(1)}$$
$$\frac{dy}{dx} = \frac{6}{1}$$

$$y - 2 = \frac{6}{1}(x - 1)$$
$$y - 2 = 6x - 6$$
$$y = 6x - 4$$

30. $x^2 - xy^2 + y^2 = 4 \quad (4, -2)$

$$2x - \left(y^2 + x\,2y\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$
$$2x - y^2 - 2xy\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{y^2 - 2x}{2y - 2xy}$$

$$\frac{dy}{dx} = \frac{(-2)^2 - 2(4)}{2(-2) - 2(-2)(4)}$$
$$= \frac{-4}{12} = \frac{-1}{3}$$

$$y - y_1 = m(x - x_1)$$
$$(y + 2) = \frac{-1}{3}(x - 4)$$
$$y + 2 = \frac{-1}{3}x + \frac{4}{3}$$
$$y = \frac{-1}{3}x - \frac{2}{3}$$

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31. $x^2 + xy + y^2 = 16$

$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x-y}{x+2y}$

32. $y^2 - 8y - x + 19 = 0$

a) $2y\frac{dy}{dx} - 8\frac{dy}{dx} - 1 = 0$

b) slope undefined!

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y-8}$

33. $m(t) = -0.004t^3 + 0.3t^2$

a) $m(20) = -0.004(20)^3 + 0.3(20)^2$

$= -32 + 120$

$= 88$

b) $m'(t) = -0.012t^2 + 0.6t$

c) $m'(10) = -0.012(10)^2 + 0.6(10)$

$= -1.2 + 6$

$= 4.8$ words/min.

34. $f(x) = x^3 - 3x^2 - 6x + 10$

$f'(x) = 3x^2 - 6x - 6$

slope $\rightarrow 3 = 3x^2 - 6x - 6$

$0 = 3x^2 - 6x - 9$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3 \quad x = -1$

when $x = 3$

$(3)^3 - 3(3)^2 - 6(3) + 10$

$= 27 - 27 - 18 + 10$

$= -8$

$(3, -8)$

$x = -1$

$(-1)^3 - 3(-1)^2 - 6(-1) + 10$

$-1 - 3 + 6 + 10$

$= 12$

$(-1, 12)$

35. $R(1,1) \quad S(4,16)$

$m = \frac{16-1}{4-1} = \frac{15}{3} = 5$

$f(x) = x^2$

$f'(x) = 2x$

$5 = 2x$

$\frac{5}{2} = x$

when $x = \frac{5}{2}$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$\left(\frac{5}{2}, \frac{25}{4}\right)$

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36 $y = 4$ is tangent to $f(x) = x^2 + ax + b$ at $(5, 4)$

slope is 0

$f'(x) = 2x + a$

$0 = 2x + a$

$a = -2x$

$a = -2(5)$

$a = -10$

$4 = (5)^2 - 10(5) + b$

$4 = 25 - 50 + b$

$4 = -25 + b$

$29 = b$

37. Pt $(3, 9)$ $f(x) = x^2$

NOT sure

$\Rightarrow$

Answ $\left(\frac{-1}{12}, \frac{1}{144}\right)$

$F'(x) = 2x$

$f'(3) = 2(3)$

$= 6 \leftarrow$ slope

$= -\frac{1}{6} \leftarrow \perp$ slope

? where to go now?

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{1}{6}(x - 3)$

$6(y - 9) = -1(x - 3)$

$6y - 54 = -1x + 3$

$x + 6y - 59 = 0$

38. $f(x) = x^3 + 12$ $g(x) = x^2 + 8x$ $F(2) = 20$ $g(2) = 20$

$f'(x) = 3x^2$ $g'(x) = 2x + 8$

$f'(2) = 3(2)^2$ $g'(2) = 2(2) + 8$

$f'(2) = 12 \xleftrightarrow[\text{slope}]{\text{same}} g'(2) = 12$

functions pass thru same pt.

39. a) $y = x^3$

$y' = 3x^2$

$f(3) = 3(3)^2$

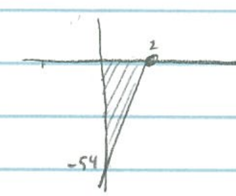
$= 27 \leftarrow$ slope

$y_1 - y_1 = m(x - x_1)$ $(3, 27)$

$y - 27 = 27(x - 3)$

$y - 27 = 27x - 81$

$0 = 27x - y - 54$



x-int $\Rightarrow$ $27x = 54$

$x = 2$

y-int $y = -54$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(2)(54) = 54u^2$

40. $f(x) = \frac{1}{3}x^3 + 4x$

$f'(x) = x^2 + 4$

$2 = x^2 + 4$

$-2 = x^2 \leftarrow$ complex #'s

41. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3a^2 + 2b + c$

$\Rightarrow$ $D - b^2 - 4ac > 0$ to work

why 3?