4.8 The Chain Rule

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Learning Targets:

1. SWBAT find the derivative using the chain rule.
2. SWBAT apply the chaion rule to application problems.

The Chain Rule is used to take the derivative of composite functions

$f(g(x))$

$y = (3x^2 + 4)^3$

Say we want to find $\frac{dy}{dx}$ of the following function:

x variable different

$$y = (2x^2 + 4)^8$$

let $u = 2x^2 + 4$

let $y = u^8$

$\frac{du}{dx} = 4x$

$\frac{dy}{du} = 8u^7$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = 8u^7(4x)$$

$$= 8(2x^2+4)^7(4x)$$

$$= 32x(2x^2+4)^7$$

$$y = (2x^2+4)^8$$

$$y' = 8(2x^2+4)^7 \cdot (4x)$$

$$= 32x(2x^2+4)^7$$

Now to always differentiate a composite function this way would be time consuming. Therefore there must be an easier way!

Ex.2 Find the derivative of the following:

a) $y = \left(x^2 - x + 2\right)^8$

$y' = 8\left(x^2 - x + 2\right)^7 (2x - 1)$

$y' = 8(2x - 1)\left(x^2 - x + 2\right)^7$

The Chain Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $F(x) = f(g(x))$, then $F'(x) = f'(g(x)) \bullet g'(x)$.

b) $y = -2(x^2 + 6x - 4)^5$

$y' = -10(x^2 + 6x - 4)^4 (2x + 6)$

$y' = -20(x + 3)(x^2 + 6x - 4)^4$

c) $y = \left(x^2 - 4\right)^{-3}$

$y' = -3(x^2-4)^{-4}(2x)$

$y' = \dfrac{-6x}{(x^2-4)^4}$

d) $y = \dfrac{2}{5x-9}$

$y = 2(5x-9)^{-1}$

$y' = -2(5x-9)^{-2} \cdot (5)$

$y' = \dfrac{-10}{(5x-9)^2}$

e) $y = \dfrac{8}{\sqrt{6x^2+2}}$

$y = 8(6x^2+2)^{-1/2}$

$y' = -4(6x^2+2)^{-3/2} \cdot 12x$

$= \dfrac{-48x}{(6x^2+2)^{3/2}}$

Example 3 A table of values for f, g, f', and g' is shown.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	5	6	8	4
4	1	3	5	2
5	3	9	7	10

a) If $F(x) = f(g(x))$, find $F'(4)$

$F'(4) = f'(g(4)) \cdot g'(4) = f'(3) \cdot g'(4)$

$= (8)(2)$

$= 16$

b) If $G(x) = g(f(x))$, find $G'(5)$

$$G'(x) = g'(f(x)) \cdot f'(x)$$

$$G'(5) = g'(f(5)) \cdot f'(5)$$

$$= g'(3) \cdot f'(5)$$

$$= (4)(7)$$

$$= 28$$

$$y = \sqrt{x + \sqrt{x}}$$

$$y = \left(x + x^{1/2}\right)^{1/2}$$

$$y' = \frac{1}{2}\left(x + x^{1/2}\right)^{-1/2} \cdot \left(1 + \frac{1}{2}x^{-1/2}\right)$$

The Derivative Song

Assignment
Page 200
#’s 1,2,4,6,8,9,11,13,15,17,18,21