4.7 The Quotient Rule

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Learning Targets:

1. SWBAT find the derivative using the quotient rule.
2. SWBAT apply the quotient rule to application problems.

Quotient Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = \dfrac{f(x)}{g(x)}$, then

$$y' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Low de Hi minus Hi de Low

http://www.youtube.com/watch?v=DdV2UZV7AoA&feature=related

$$\frac{Lo\,DHi - Hi\,DLo}{(Lo)^2}$$

$$= \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

Ex.1 Differentiate:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1}$$

$$F'(x) = \frac{(x^3+1)(2x+2) - (x^2+2x-3)(3x^2)}{(x^3+1)^2}$$

$$= \frac{\cancel{2x^4} + 2x + \cancel{2x^3} + 2 - \cancel{3x^4} - \cancel{6x^3} + 9x^2}{(x^3+1)^2}$$

Ex. 2 Differentiate

$$\frac{2}{\left(\frac{1}{2}\right)}$$

$$y = \frac{\sqrt{x}}{1+2x} = \frac{x^{1/2}}{1+2x}$$

$$\frac{x^{1/2}}{x^{-1/2}}$$

$$y' = \frac{(1+2x)\frac{1}{2}x^{-1/2} - x^{1/2}(2)}{(1+2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2x^{1/2}}{(1+2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2}(1+2x-4x)}{(1+2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2}(1-2x)}{(1+2x)^2}$$

$$= \frac{1-2x}{2x^{1/2}(1+2x)^2}$$

$$y = \frac{x^{2/3}}{x^3+1}$$

$$y' = \frac{(x^3+1)\,\frac{2\,\cancel{3x^{1/3}}}{\cancel{3x^{1/3}}} - x^{2/3}\cdot 3x^2\, 3x^{1/3}}{3x^{1/3}(x^3+1)^2}$$

$$y' = \frac{2(x^3+1) - 9x^3}{3x^{1/3}(x^3+1)^2} = \frac{2x^3+2-9x^3}{3x^{1/3}(x^3+1)^2}$$

$$\frac{-7x^3+2}{3x^{1/3}(x^3+1)^2}$$

Example 3

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}$$

$$f'(2) = \frac{(\cancel{2-1})(2(2)) - ((2)^2-2)(1)}{(2-1)^2}$$

$$= \frac{4-2}{1} = 2$$

Example 4

Find the points on the curve $y = \dfrac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$.

$$4y = -x + 1$$

$$y = -\frac{1}{4}x + \frac{1}{4}$$

$$m = -\frac{1}{4}$$

$$y' = \frac{(x-1)(1) - x(1)}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2}$$

$$-\frac{1}{4} = \frac{-1}{(x-1)^2}$$

$$(x-1)^2 = (4)$$

$$(x-1)^2 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$ OR $x = -1$

$$x = 3$$

$$y = \frac{3}{3-1}$$

$$= \frac{3}{2}$$

$$\left(3, \tfrac{3}{2}\right)$$

$$x = -1$$

$$y = \frac{-1}{-1-1} = \frac{1}{2}$$

$$\left(-1, \tfrac{1}{2}\right)$$

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#'s 1-4, 7-11,13,14,18

$\sqrt[2]{x} = x^{1/2}$

$\sqrt[5]{x^2}$

$x^{2/5}$

Application

21. Some ice cubes were added to a cup of boiling water. The temperature of the water in degrees Celsius, t minutes after the ice cubes were added, can be approximated by the function $T(t) = \dfrac{20t^2 + 100t + 200}{t^2 + t + 2}$. Round your answers to two decimal places where necessary.

(a) Find $T(0)$, $T(1)$, and $T(5)$. Interpret your answers.

(b) Find $T'(t)$.

(c) Find $T'(1)$ and $T'(5)$. Interpret your answers.

(d) Find $\lim\limits_{t \to \infty} \dfrac{20t^2 + 100t + 200}{t^2 + t + 2}$ and interpret your result.