4.5 Negative Exponents and Reciprocals

Lesson Focus

Relation relational exponents and radicals

Review - Power Rules

- $\frac{6^4}{6^2} =$ $4-2$ 6^2 $= 6^2$

- $\frac{4^3}{4^5} =$ 4^{3-5} $= 4^{-2}$

Dividing powers (with same base)

Subtract exponents

Negative Exponents

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Why can't x be 0?

- *Ex:* $2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
- *Ex:* $4^{-2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Fractions with Negative Exponents

- *Ex:* $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{3^2}{2^2} = \frac{9}{4}$

- ***Shortcut – flip the fraction*** *then* ***apply the power to both numerator and denominator***

- *Ex:* $\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^{3} = \frac{5^3}{3^3} = \frac{125}{27}$

Example

Evaluate each power.

a) 3^{-2}

$= \left(\frac{1}{3}\right)^{2}$

$= \frac{1^2}{3^2}$

$= 1/9$

b) $\left(-\frac{3}{4}\right)^{-3}$

$= \left(-\frac{4}{3}\right)^{3}$

$= \frac{(-4)^3}{(3)^3} = \frac{-64}{27}$

c) 0.3^{-4}

$\left(\frac{3}{10}\right)^{-4}$

$\left(\frac{10}{3}\right)^{4}$

$= \frac{10^4}{3^4}$

$= \frac{10\,000}{81}$

Example – Your Turn

Evaluate each power.

a) 7^{-2}

$\left(\frac{1}{7}\right)^{2}$

$= \frac{1}{49}$

b) $\left(\frac{10}{3}\right)^{-3}$

$\left(\frac{3}{10}\right)^{3}$

$= \frac{3^3}{10^3}$

$= \frac{27}{1000}$

c) $(-1.5)^{-3}$

$\left(-\frac{3}{2}\right)^{-3}$

$\left(-\frac{2}{3}\right)^{3}$

$= \frac{(-2)^3}{(3)^3} = -\frac{8}{27}$

Homework

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3, 5, 6, 7, 8

Going Deeper

- *Connect what we did last section (fractional exponents) with this chapter (negative exponents)*
- *Ex:* $4^{-\frac{3}{2}} =$ $\left(\frac{1}{4}\right)^{3/2} = \frac{1^{3/2}}{4^{3/2}} = \frac{1}{(\sqrt{4})^3}$ $= \frac{1}{(2)^3} = \frac{1}{8}$
- *Ex:* $\left(\frac{8}{27}\right)^{-\frac{2}{3}} =$ $\left(\frac{27}{8}\right)^{2/3} = \frac{(27)^{2/3}}{(8)^{2/3}} = \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2} = \frac{9}{4}$

Example

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$

$$\left(\frac{1}{8}\right)^{2/3}$$

$$= \frac{(1)^{2/3}}{(8^{2/3})} = \frac{1}{(\sqrt[3]{8})^2}$$

$$= \frac{1}{4}$$

b) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$$= \left(\frac{16}{9}\right)^{3/2}$$

$$= \frac{16^{3/2}}{9^{3/2}} = \frac{(\sqrt{16})^3}{(\sqrt{9})^3}$$

$$= \frac{(4)^3}{(3)^3}$$

$$= 64/27$$

Example – Your Turn

Evaluate each power without using a calculator.

a) $16^{-\frac{5}{4}}$

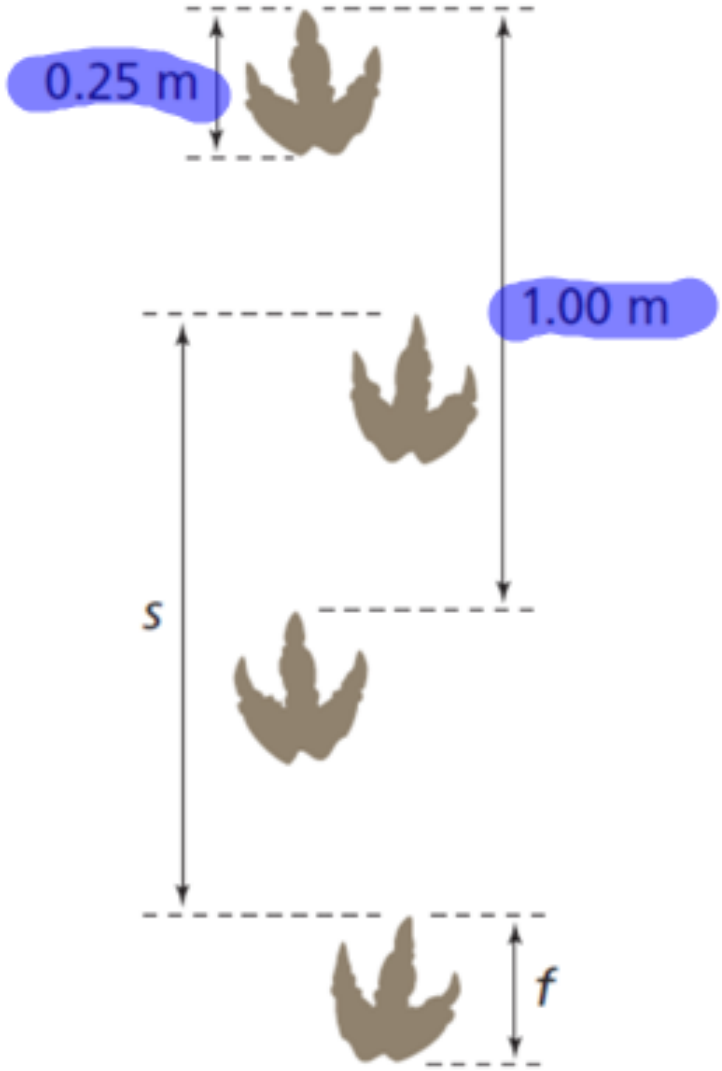
b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$= \left(\frac{36}{25}\right)^{1/2} = \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5}$

Example

Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155 s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, v is the speed in metres per second, s is the distance between successive footprints of the same foot, and f is the foot length in metres.

Use the measurements in the diagram to estimate the speed of the dinosaur.



when $s = 1.5$ and $f = 0.3$.

$$V = 0.155(1.5)^{5/3}(0.3)^{-7/6}$$

$$V = 0.155(1.5)^{5/3}\left(\frac{10}{3}\right)^{7/6}$$

$$= 0.155(1.9656)(4.0740)$$

$$= 1.24 \text{ m/s}$$

Homework

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9, 11, 12, 13, 15, 17

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Handout (Math A30)
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